**[Trang Pham]** - Welcome to the 10-minute information bites of the topics which are considered to be new in the 2023 to 2027 study design. The 10-minute information bites will cover the concepts and the examples of one, vector product; two, planes in the three dimensions, and three, angle between the plane and the line. The vector product, also known as vector cross product arises when we attempt to find a vector, which is perpendicular to two other known vectors. Consider the vectors a and b are given as shown on the slide. Then the vector cross product is given by the formula, again, as shown on the slide. As you can see, this is not an easy formula to remember. There are different methods to derive this formula.

The fact that the cross product is really the determinant of a three by three matrix, so it is worth to use this method rather relying on memorising the formula. The magnitude of vector product is given as the product of the magnitude of vectors a and b and sine theta, where theta is the angle between vectors a and b. Please note that the scalar product, which also known as a dot product and the vector cross product are shown as two different dot points on page 112 in the study design. There are a couple of facts which are worth to note. The first fact is if the cross product of vectors a and b is equals to zero, which is known as a null vector and it is written as zero with a tilde symbol underneath to indicate it is a null vector.

Teachers should be wary that their students don't just write a scalar zero. When vectors A cross B equal null vector implies parallelity is similar to vectors A dot B equal zero implies orthogonality. Note that parallel vectors could be the two vectors either have the same direction or have the exact opposite direction from each other, that is they are not linearly independent or if either one has a zero length. The second fact is if the cross product of vectors a and b does not equal null vector, then the vector found is perpendicular to both vectors a and b, thus normal to the plane containing them. The vectors a and b are linearly independent vectors. A couple of nice properties about the vector product. The first property is known as anti-commutative, which means swapping the position of two arguments of an antisymmetric operation yields a result which is the inverse of the result with unswapped arguments. So the vector product of a and b is equal to negative of the vector product of b and a.

The second property is the magnitude of the vector product of a and b is actually the area of a parallelogram where vectors a and b form two adjacent sides of a parallelogram. We are now looking at planes in three dimensions. So what is the vector equation of a plane? When the normal vector of a plane and a point passing through the plane unknown, then the vector equation of the plane can be found using the formula dot product of vectors r and n equals dot product of vectors a and n, where vector r is the position vector of any point on the plane. Vector a is the position vector of a known point A on the plane and n is a normal vector which is perpendicular to the plane. The formula for the Cartesian equation of a plane is given as n1 times x plus n2 times y plus n3 times z equals to D, where n1, n2 and n3 are the components of the normal vector n and D is a constant. Let's look at example one.

Find the Cartesian equation of the plane, which is perpendicular to the vectors a equal 2i minus 3j plus k and b equal 4i plus 2j take away 3k, which passes through the point three, two and one. First of all, we need to find the normal vector to the plane. The normal vector is equal to the vector product of vectors a and b. There are several methods or formula to find the vector products of two vectors, but I prefer to use the determinant of a three by three matrix method. So let's put the vectors a and b in the form of a matrix with i, j and k in the first row. Those are the unit base vectors. Then the components of vector A in the second row which are two, negative three, and one. and finally, the components of vector b in the last row, which are shown as four, two, and negative three. Now, expanding this determinant along its top row i, j and k by using the following technique.

To obtain the i component, we cover up the i row and the i column which leaves behind a two by two matrix of negative three, one, two, and negative three. Minus the j component, which can be found by cover the j row and the j column, which leaves behind a two by two matrix of two, one, four and negative three, plus the k component which can be found by cover up the k row and the k column, which leaves behind a two by two matrix of two, negative three, four, and two. We are now applying the method of finding the determinant of a two by two matrix and simplifying all that. Hence, the normal vectors is equal to 7i plus 10j plus 16k. From the previous slide, we have found the normal vector, which is the vector product of vectors a and b. We will call this normal vector n, which is equal to 7i plus 10j plus 16k. Using the equation of a plane formula, n1 times x plus n2 times y plus n3 times z equals to D. We have 7x plus 10y plus 16z equal D, where seven, 10 and 16 are the components of the normal vector. To determine the value D, we substitute the point three, two, one into the above Cartesian equation of a plane. So we have seven times three plus 10 times two plus 16 times one, which equals to 57, so D equal 57.

Therefore, the Cartesian equation of the plane, which is perpendicular to the vectors a and b, which passes through the point three, two, and one is 7x plus 10y plus 16z equal 57. We are now looking at the last topic, angle between the plane and the line. So the vector equation of a line is given as r equals to a plus t times d, where r is the position vector at any point on the plane, a is the position vector of a point on the plane, and d is the direction vector of the line. Now, t is called the parameter. Different values of t correspond to different points on the line. To be able to find the angle between the plane and the line, which is marked as 90 degrees minus theta on the diagram where theta is the acute angle between the line and the normal vector to the plane, we need to use the dot product which is r1.r2 equals to the magnitude of r1 times the magnitude of r2 times cos of theta. So looking at the diagram, r1 is actually the normal vector to the plane labelled as n on the diagram and r2 is the direction vector of the line labelled as D on the diagram. Example two. Find the angle between the plane 2x + y + z = 7 and the line given by r equals 11i plus 4j plus 3k plus t times in the brackets i plus 2J minus k where t is an element of real numbers.

Let's have a look at the solution. Recall the Cartesian equation of a plane is n1 times x plus n2 times y plus n3 times z equals D. So the coefficients of the Cartesian equation of the plane are the components of the normal vector to the plan, hence n is equals to 2i plus j plus k and recall the vector equation of a line is r equals to a plus t times d where t is any real number. From the question, we can see the direction vector of the line is d, which is i plus 2j minus k. We let theta be the angle between the normal and the line. Recall that the dot product equals to r1.r2 equals to the magnitude of r1 times the magnitude r2 times cos theta. We now transpose the dot product formula to make cos theta the subject and substitute r1, which is the normal vector n equals to 2i plus j plus k and r2, which is the direction vector of the line.

So d equals to i plus 2j minus k and these vectors were found in the previous slide. So we have cos theta equals to 2i plus j plus k dot with i plus 2j minus k divided by the magnitudes of the vectors n and d, which are the square root of two square plus one squared plus one squared times the square root of one squared plus two squared plus one squared and this simplified to 1/2. Hence the angle between the normal and the line is inverse cos of 1/2, which is equals to 60 degrees. Therefore the angle between the line and the plane is then 90 degrees take away 60 degrees equals to 30 degrees as shown on the diagram. Example three, part A. Find the vector equation of the line through the points A of three, one, negative one and B of five, two, negative six. Let's go through the solutions. The position vector of A relative to O is denoted as OA or just vector a, which is equal to 3i plus j minus k, and the position vector of b relative to O is denoted as OB or just vector B, and it is equals to 5i plus 2j minus 6k. We first need to find the direction vector AB, which is the difference between vectors b and a, so vector AB equals 2i plus j minus 5k.

Now recall the vector equation of a line r equals to a plus t times d where t is an element of real numbers. So we have the equation of the line through point A with direction vector AB is r equals to 3i plus j minus k plus t times in the brackets 2i plus j minus 5k where t is an element of real numbers. An alternative answer when we are using point B is r equals to 5i plus 2j minus 6k plus t times in the brackets 2i plus j minus 5k where t is an element of real numbers. Part b, find the sine of the angle this line makes with the plane given by x plus 2y minus z equals nine. Let's go through the solution. The normal to the plane is denoted as vector n and its components are the coefficients of x, y, and z given in the question. So n equals to i plus 2j minus k. Let theta be the angle between the line with direction vector d and the normal vector to the plane n. By reducing the dot product, we have cos theta equals vector n dot vector d divided by the product of the magnitudes of vectors n and d. Hence, we have cos theta equals to 2i plus j minus 5k dot with i plus 2j minus k divided by the square root of two squared plus one squared plus five squared times the squared root of one squared plus two squared plus one squared. Simplify this and we have three divided by two square root of five.

Note, cos theta is equal to sine pi on two, take away theta and we found cos theta equals to three divided by two square root of five so therefore, the sine of the angle is three divided by two times square root of five. And last but not least, an example where a position vector is given and the question is asking to find when the particle has minimum speed and find that minimum speed. So teachers may have taught similar questions in the previous course already, however, it is worth to note that it might have done as a 2D case whereas the precision vector given in this question includes a k vector component. We won't go through the solution of this example as the solving method is very similar to the 2D case.

And that brings us to the end of the 10-minute information bites on vectors. I hope you have found this presentation useful when planning the 2023 specialist maths course. For further information, please contact Michael MacNeill, VCAA Mathematics Curriculum Manager and his details are as shown on this slide. Thank you so much for your interest and attention.

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