VCE Specialist Mathematics 2023-2027 Units 3&4

Suggested approaches to implementing the 2023-2027 study design

Proof by Contradiction

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VCE Mathematics On-demand Videos

To support the implementation of the 2023-2027 study design for Mathematics, we have developed a series of short on-demand videos outlining approaches that teachers may wish to utilise in the classroom.

The information presented in these on-demand videos has been developed by current VCE teachers, in conjunction with the VCAA, and offer suggestions for ways schools could approach the implementation of the 2023-2027 VCE Mathematics study design.

General Outline

Brief outline of the revised material discussed in this on demand presentation

Proofs

1. Proof by Contradiction

Teaching considerations

Assume that the given statement is false – write a statement which contradicts the statement which is to be proven.

Prove that the written (contradictory) statement is false.

We can then conclude that if the contradictory statement is false, the original statement has been proven to be true.

Teaching example 1

Use proof by contradiction to prove that if n is odd, then n^3+1 is even.

Suggested approach

Assume the opposite: that if *n* is odd, then n^3+1 is odd.

So $n^3+1=2k+1$ where $k \in Z$

and *n* is odd so $n=2p+1$ where $p \in Z$.

Suggested approach continued

R.S. $= 2k + 1$

$$\text{L.S.} = (2p+1)^3 + 1$$
$$= (8p^3 + 12p^2 + 6p + 1) + 1$$
$$= 8p^3 + 12p^2 + 6p + 2$$
$$= 2(4p^3 + 6p^2 + 3p + 1)$$

This is a contradiction as the L.S. is even and the R.S. is odd.

Hence if *n* is odd, then $n^3 + 1$ is even.

Teaching example 2

Use proof by contradiction to prove that $\sqrt{3}+\sqrt{5}>\sqrt{11}$.

Suggested approach

Assume the opposite: $\sqrt{3}+\sqrt{5}\le\sqrt{11}$

As both L.S. and R.S. are both positive, $\text{L.S.}\le\text{R.S.}$ implies that $(\text{L.S.})^2\le(\text{R.S.})^2$.

$$\left(\sqrt{3}+\sqrt{5}\right)^2\le\left(\sqrt{11}\right)^2$$

$$8+2\sqrt{15}\le 11$$

$$2\sqrt{15}\le 3$$

Suggested approach continued

Squaring both sides again gives $60 \le 9$ which is a contradiction.

OR

We use the fact that $\sqrt{15} > 3$ to conclude that $2\sqrt{15} > 3$ which is a contradiction.

Hence, we conclude that $\sqrt{3} + \sqrt{5} > \sqrt{11}$.

Teaching example 3

Use proof by contradiction to prove that $\log_e(5)$ is irrational.

Suggested approach

Assume the opposite: that $\log_e(5)$ is rational.

i.e. $\log_e(5) = \frac{a}{b}$ where $a \in Z, b \in N$ and the fraction is in simplest form.

Suggested approach continued

$$e^{\frac{a}{b}} = 5$$

$$e^{a} = 5^{b}$$

The right side is divisible by 5, the left side is not which is a contradiction.

So $\log_e(5)$ is irrational.

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