**Philip Swedosh -** Welcome to a video On-Demand for Specialist Mathematics. There have been a number of changes to the Specialist Mathematics curriculum. One of the largest changes is the introduction of proofs. My name is Philip Swedosh and in these videos, I will outline the structure required in questions which ask for proof to be performed.

Some of this may feel unfamiliar at first but once the logic and structure are understood most questions are relatively straightforward. In the first video, I spoke about proof by induction questions. In this the second video, we'll meet proof by contradiction. When doing proof by contradiction, the first thing we do is we assume that the given statement is false, so we usually then write a statement which contradicts that statement, the one which has to be proven.

The next step is to work through algebraically and prove that the contradictory statement or the assumption is false. When we prove that the contradictory statement is false, it follows that the original statement must be true. The following example is a straightforward question which we have to prove by contradiction so we need to prove that if n is odd then n cubed plus one is even as with any proof by contradiction the first thing that we do is we assume that the opposite is true. In other words, if any is odd, then n cubed plus one is odd. Now, any odd number can be written as two k plus one where k is an integer, so if we double it we get an even number at one to that we must get an odd number.

So therefore using the statement above our assumption n cubed plus one can be written as two k plus one and as n is odd, that can be written as n equals two p plus one, where p is any integer. Now the right hand side is two k plus one which is an odd number. The left hand side one to replace n is two p plus one all cubed plus one. If we then expand that, we get in the brackets eight p cubed plus twelve p squared plus six p plus one, and we've got the plus one outside the bracket. Adding those together, we get a plus two on the end and it is clear that two is a common factor. Therefore, the left hand side is an even number as the left hand side is even, and the right hand side is odd, we have a contradiction, they cannot be equal.

Hence, if n is odd, n cubed plus one must be even and we have proven by contradiction what we set out to do. This is a slightly more difficult proof by contradiction in that we have to employ some techniques which we haven't yet encountered so we want to prove that root three plus root five is greater than root eleven. As usual, we start off by assuming the opposite is true so the opposite of root three plus root five is greater than root eleven is root three plus root five is less than or equal to root eleven. Now to resolve this, we look at the fact that both sides are clearly positive, so that the left side is less than or equal to the right side, implies that the left side squared will be less than or equal to the right side squared.

A cautionary note, this method can only be utilised if we know that both sides are positive which is the case here. If either side is negative this technique will not work, so don't use it. So we square both sides and we get eight plus two. root fifteen, it's less than four equal to eleven and simplifying we get to root fifteen is less than equal to three. At this point we have a few different approaches which we can use. We can once again square both sides and this would give sixty is less than or equal to nine, which is clearly a contradiction. An alternative method is to use the fact that we know that root fifteen is greater than root nine, which is three. So root fifteen is greater than three, and we can conclude from that the two root fifteen is greater than three which is obviously a contradiction.

In either case, we can conclude that the assumption was incorrect and therefore that the original statement, root three plus root five is greater than root eleven is proven. In our last example, we're going to prove by contradiction that log base e of five is irrational. Now, note that that's often written as ln of five . As usual, we start off by assuming that the opposite is true. That is that log base e of five is a rational number.

Now what that means is that log base e of five would be equal to a over b where a can be any integer, and b is a natural number and the fractions in simplest form. The reason that we stipulate that b is a natural number is to avoid a situation where there's any possibility of getting a zero on the denominator which would mean that the number would be undefined. We now translate from log form into exponential form, giving us e to the power of a over b is equal to five. We then use our index laws. We raise both sides to the power of b, giving us e to the power of a is equal of five to the power of b. It's clear at this point that the right hand side must be divisible by five but the left hand side is not. So that leads us to the contradiction meaning that our original statement log base of five is an irrational number.

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