VCE Specialist Mathematics 2023-2027 Units 3&4

Suggested approaches to implementing the 2023-2027 study design

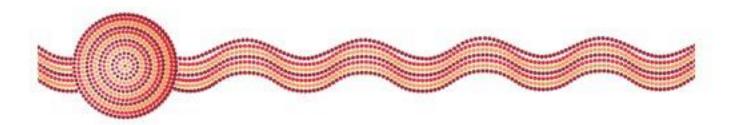
Proof by Induction





Acknowledgement of Country

The VCAA respectfully acknowledges the Traditional Owners of Country throughout Victoria and pays respect to the ongoing living cultures of First Peoples.







VCE Mathematics On-demand Videos

To support the implementation of the 2023-2027 study design for Mathematics, we have developed a series of short on-demand videos outlining approaches that teachers may wish to utilise in the classroom.

The information presented in these on-demand videos has been developed by current VCE teachers, in conjunction with the VCAA, and offer suggestions for ways schools could approach the implementation of the 2023-2027 VCE Mathematics study design.





General Outline

Brief outline of the revised material discussed in this on demand presentation

Proofs

1. Proof by Induction – examples shown of three types of questions



Teaching considerations

Dominoes analogy

Template

Three fundamental steps:

- Prove for a starting value (often n = 1)
- Assume true for n = k
- Prove true for n = k + 1 * must use assumption



Teaching example 1 – equality

Prove by mathematical induction that $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}, n \in \mathbb{N}$.



Suggested approach

Prove for
$$n = 1$$

L.S. = $\frac{1}{2}$ R.S. = $1 - \frac{1}{2^1} = \frac{1}{2}$

Assume true for n = k

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^k} = 1 - \frac{1}{2^k}$$



Prove true for n = k + 1*

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}$$

$$L.S. = 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{2}{2^{k+1}} + \frac{1}{2^{k+1}}$$

$$= 1 - \frac{1}{2^{k+1}}$$

$$= R.S.$$



Teaching example 2 – inequality

- **a.** Consider the inequality $2^n > n^2$ for $n \ge n_0$, $n \in N$. Show that, for the inequality statement to be true, $n_0 = 5$.
- **b.** Prove by mathematical induction that $2^n > n^2$ for $n \ge 5$, $n \in N$.



Suggested approach

a. We construct a table

$$n$$
 1
 2
 3
 4
 5
 6

 2^n
 2
 4
 8
 16
 32
 64

 n^2
 1
 4
 9
 16
 25
 36

So
$$n_0 = 5$$
.



b. True for n = 5.

Assume true for n = k

$$2^k > k^2$$



Prove true for n = k + 1*

$$2^{k+1} > (k+1)^2$$

For convenience, we expand the right side first

$$R.S. = k^2 + 2k + 1$$



L.S. =
$$2 \times 2^{k}$$

 $> 2k^{2}$
 $= k^{2} + k^{2}$
 $\ge k^{2} + 5k$ as $k \ge 5$
 $= k^{2} + 2k + 3k$
 $\ge k^{2} + 2k + 15$ as $k \ge 5$
 $> k^{2} + 2k + 1$
 $= R.S.$

using assumption

to match R.S.

to match R.S.



Teaching example 3 – divisibility

Prove by mathematical induction that the number $9^n - 5^n$ is divisible is by 4 for all $n \in \mathbb{N}$.



Suggested approach

Prove for n = 1

L.S. =
$$9-5=4$$

Assume true for n = k

$$9^k - 5^k = 4a, a \in N$$



Prove true for n = k + 1*

$$9^{k+1} - 5^{k+1} = 4b, b \in N$$

$$L.S. = 9 \times 9^{k} - 5 \times 5^{k}$$

$$= 5(9^{k} - 5^{k}) + 4 \times 9^{k}$$

$$= 5(4a) + 4 \times 9^{k}$$

$$= 4(5a + 9^{k})$$

$$=4(5a+9^k)$$

=4b

$$= R.S.$$

split/factorise to use assumption



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