VCE Specialist Mathematics 2023-2027 Units 3&4

Suggested approaches to implementing the 2023-2027 study design

Proof by Induction

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The VCAA respectfully acknowledges the Traditional Owners of Country throughout Victoria and pays respect to the ongoing living cultures of First Peoples.

VCE Mathematics On-demand Videos

To support the implementation of the 2023-2027 study design for Mathematics, we have developed a series of short on-demand videos outlining approaches that teachers may wish to utilise in the classroom.

The information presented in these on-demand videos has been developed by current VCE teachers, in conjunction with the VCAA, and offer suggestions for ways schools could approach the implementation of the 2023-2027 VCE Mathematics study design.

General Outline

Brief outline of the revised material discussed in this on demand presentation

Proofs

1. Proof by Induction – examples shown of three types of questions

Teaching considerations

Dominoes analogy

Template

Three fundamental steps:

- Prove for a starting value (often n = 1)
- Assume true for $n = k$
- Prove true for $n = k + 1$ ** must use assumption*

Teaching example 1 – equality

Prove by mathematical induction that $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=1-\frac{1}{2^n},\ n\in N.$

Suggested approach

Prove for $n = 1$

L.S. = $\frac{1}{2}$ R.S. = $1-\frac{1}{2^1}=\frac{1}{2}$

Assume true for $n = k$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}=1-\frac{1}{2^k}$$

Suggested approach continued

Prove true for *n = k + 1* *

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}+\frac{1}{2^{k+1}}=1-\frac{1}{2^{k+1}}$$

$$\begin{aligned}\text{L.S.}&=1-\frac{1}{2^k}+\frac{1}{2^{k+1}}\\&=1-\frac{2}{2^{k+1}}+\frac{1}{2^{k+1}}\\&=1-\frac{1}{2^{k+1}}\\&=\text{R.S.}\end{aligned}$$

Teaching example 2 – inequality

a. Consider the inequality $2^n > n^2$ for $n \geq n_0, n \in N$.

Show that, for the inequality statement to be true, $n_0 = 5$.

b. Prove by mathematical induction that $2^n > n^2$ for $n \geq 5, n \in N$.

Suggested approach

a. We construct a table

n	1	2	3	4	5	6
2^n	2	4	8	16	32	64
n^2	1	4	9	16	25	36

So $n_0 = 5.$

Suggested approach continued

b. True for $n = 5.$

Assume true for $n = k$

$$2^k > k^2$$

Suggested approach continued

Prove true for *n = k + 1 **

$$2^{k+1} > (k+1)^2$$

For convenience, we expand the right side first

$$\text{R.S.} = k^2 + 2k + 1$$

Suggested approach continued

$$
\begin{aligned}
\text{L.S.} &= 2\times 2^k \\
&> 2k^2 && \text{using assumption} \\
&= k^2 + k^2 && \text{to match R.S.} \\
&\geq k^2 + 5k \quad \text{as } k \geq 5 \\
&= k^2 + 2k + 3k && \text{to match R.S.} \\
&\geq k^2 + 2k + 15 \quad \text{as } k \geq 5 \\
&> k^2 + 2k + 1 \\
&= \text{R.S.}
\end{aligned}
$$

Teaching example 3 – divisibility

Prove by mathematical induction that the number $9^n - 5^n$ is divisible is by 4 for all $n \in N$.

Suggested approach

Prove for $n = 1$

L.S. = $9 - 5 = 4$

Assume true for $n = k$

$9^k - 5^k = 4a, a \in N$

Suggested approach continued

Prove true for *n = k + 1 **

$$9^{k+1} - 5^{k+1} = 4b, b \in N$$

$$\begin{aligned}\text{L.S.} &= 9\times 9^k - 5\times 5^k \\ &= 5\left(9^k - 5^k\right) + 4\times 9^k \qquad \text{split/factorise to use assumption} \\ &= 5\left(4a\right) + 4\times 9^k \\ &= 4(5a + 9^k) \\ &= 4b \\ &= \text{R.S.}\end{aligned}$$

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