Welcome to Mathematical Methods, a series of videos helping you develop a modelling or problem-solving task for Unit 4. This is the task that involves functions, algebra and calculus.

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This PowerPoint and the accompany set of videos will outline a process for developing a modelling or problem-solving task, and illustrate how this can be done using a sample task. In fact, for this particular task, I've actually written a theoretical one, just as a little bit of a difference from a real-life task. T

his video also includes information on the purpose, nature, and structure of the task and indicates how related assessments schemes can be devised. The purpose of the modelling or problem-solving task is for students to model and solve a problem or a set of related problems in some depth. Now that word depth is important because in the application task we tend to talk about width. Where we explore in great width what a scenario is, the modelling or problem-solving tasks looks at one particular issue in depth.

And this involves functions and graphs, algebra and calculus. And the content it will involve is combined functions and their graphs, equations and systems of equations, differentiation, anti-differentiation, and integration. And this can be applied to a theoretical or a real-life modelling or problem-solving task. And it can involve stationary points, max and min, distance and area, and graphical models for a real-life scenario. But as I said, this particular task will be a theoretical task.

Mathematical modelling is a process of using mathematical structures and techniques to represent and describe the real world in a simple and concise way. And it allows us to investigate particular features and characteristics and to make predictions related to what we've already looked at.

So it's a cyclic nature, modelling and problem-solving. We look at a problem. We generate the issues, questions, conjectures, new problems may arise, or we vary and reformulate extend or generalise what we've already worked at. So depending on your class, we can in fact go through the cycle several times, reconsidering new things that have been thrown up as we go along through the problem.

A good framework for modelling and problem-solving is in the IMMC, the International Mathematical Modelling Challenge material, where we describe the real-world problem. We specify it, we formulate it, we solve the maths. We interpret, we evaluate the model, and we report the solution. So if you want to look at that link, there are some very good examples of how we can develop a modelling or a problem-solving task.

This is a scheme which has been used for a long time. The idea is we start on the top left. We look at either a real world or a theoretical context. In this case, this one will be theoretical and I've done this on purpose so it's in contrast to the real word context of the application task for Unit 3. So we look at the context, we develop mathematical model or we formulate the problem. We apply the model using problem-solving strategies and techniques. We interpret the results we get, and then we look at it and refine the model and then perhaps go around the circle again. We refine it, we look at it again. We develop maths and we see what mathematics arises.

This particular modelling or problem-solving tasks that VCAA requires is to be of two to three hours duration over a continuous period of one week. The structure of the modelling or problem-solving task is that we introduce the context or the situation, we develop the model by generalisation using parameters. We increase complexity. We consider variation or special cases. And hope this task does all of those things. And we use a model for analysis or problem-solving strategies for finding solutions. Then we evaluate the model. We look at it and we say, are the limitations and what we've looked at? We refine the problem-solving approach, and we interpret solutions.

The four key aspects are, choosing the context. And this is the first aspect that I've talked about before, where mathematicians will often look at something and say, that's a very interesting context I wonder what the mathematics is involved in that. Then choosing the context then we identify the questions of interest, knowing that in a task like this, it's only two to three hours. We can't identify every question of interest that we think of. We relate these questions and relevant concepts to skills and processes that the students can then use whatever depth of skills you require of them or that they come up with themselves. And then we devise an assessment scheme. So in this case, in this set of videos, a three-part theoretical sample task will be developed.

So what we're looking at here is we described the context. We explain how the context was chosen. We identify questions of interest. We identify relevant information and sources, and we give the task a title. The context that I was interested in is rectangular hyperbole. And what happens to the asymptotes and discontinuities? Well I looked at the theoretical context involving rectangular hyperbolas. I looked at the basic one of xy equals one. And what happens when we transform that to x minus h, y minus k where h and k will be positive constants. And that's a translation, and then equals m, which will be a constant of dilation. And then we look at the transformation of linear on linear, which gives us a hyperbola and the graphs that it gives us.

So the title of this task is 'Investigation of the graph of xy=1 and related transformations'. So I started off with looking at pairs of graph, such as y equals two x plus one on two x plus three. And then y is two x plus three on two x plus one. You will notice that all I've done there as I've flipped the numerator and the denominator, you will also notice that the coefficients of the X are the same. So I've started off simple because obviously if the coefficients are different, it's going to be a different case. And then we generalise by considering the graph of y equals ax plus b on ax plus c. Again, noticing that the coefficients of x are the same.

There are lots of things on the internet that you can look at. There's a couple of links there. The maths is fun, talks about the geometry of a hyperbola. There's a YouTube clip about one on x being a reciprocal function, which is in fact a hyperbola.

The questions of interest are, and we won't cover all of these, but the questions of interest are stationary points. Are there any in a hyperbola? Parameters, varying values of those parameters discontinuities, axis intercepts, and asymptotes. And I'm looking at the hyperbola of the form x minus h, y minus k equals m, for some constant m. And the question is where is the centre? And the centre is actually at h, k. So I can say the centre was sort of speaking marks saying it's not actually the centre of the graph. It's actually where the asymptotes cross. So we often call this the centre of the graph. Then where are the asymptotes? Now this hyperbola is seen in some sundials. So if you want to move on to a real life context, you can do this exploring sundials.

Well, I'll finish off this section by looking at a graph. And here I'm looking at instead of x, y equals one, I'm looking at the graph of x minus two, y minus one equals two, which means the asymptote moves to the right by two units. That's the vertical asymptote. The horizontal asymptote moves up by one unit and the graph is dilated by a factor of two. So we could call the centre of that asymptote the point 2, 1. So that's the end of this first introductory section.

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