VCE General Mathematics Unit 4

Area of Study 2 Matrices sample modelling or problem-solving task: Trophic cascades

The modelling or problem-solving task is to be of 2–3 hours’ duration over a period of one week.

Introduction

[Trophic cascades](https://www.nature.com/scitable/knowledge/library/trophic-cascades-across-diverse-plant-ecosystems-80060347)occur when predators in a food web alter the abundance of their prey to such an extent that they protect the next [trophic level](https://www.sciencedaily.com/terms/trophic_level.htm) down from predation or herbivory.

Contextual information and relevant examples to investigate for the scenario in each of the parts of the task below can be obtained from various web sources, such as the [wolves in Yellowstone Park](http://www.untamedscience.com/biology/ecology/ecology-articles/wolf-reintroduction-yellowstone/).

Part 1

Using a chosen trophic cascade scenario (for example, sea otter), construct a network structure where a combination of around ten carnivores and herbivores are chosen and investigated to find the dominant carnivore or herbivore using a one-step, and then a one and two-step, dominance matrix.

1. Construct the corresponding one-step dominance matrix for your chosen combination and provide a rank order of all listed carnivores and herbivores.
2. Calculate a two-step dominance matrix. Choose three non-zero elements from this matrix and list the particular predator-prey sequence linked to the choices made. Complete the one and two-step dominance ranking to determine if one particular herbivore or carnivore is dominant over all others.

Part 2

Create a transition matrix by using a series of numeric values to represent percentage changes in population values of chosen species due to birth and death rates and immigration and emigration rates.

Couple this with an initial state matrix of populations of species and investigate changes in population over a period of time. Defined populations could be used to trigger the promotion of the next trophic level down from predation or herbivory. Times needed to reach the ‘trigger’ point could be investigated.

1. Construct a transition matrix for several species (for example, killer whale, sea otter and sea urchin) in a trophic cascade, then find the state matrices for the first few years and describe any trends. For example:

 and 

1. Investigate and comment on the populations reaching the same levels. What would happen in the long term if this transition scenario was to continue?
2. Introduce a critical population level for a species (for example, sea urchin) that will trigger the promotion of the next trophic level down from predation or herbivory. Use the transition and initial state matrices to investigate when this will occur.

Part 3

To sustain a longer cycle for the promotion of the next trophic level down, additional species are introduced into the trophic cascade. Each year ‘farmed’ species are transported to the site and introduced.

1. Create a matrix to represent additional numbers of species into the system and investigate the effect this has on the sustainability of the system.
2. If the initial time period to reach the critical trigger point was to double and the transition and initial state matrices kept their initial values, investigate the number of species that would need to be ‘farmed’ and introduced each year.

Areas of study

The following content from the areas of study is addressed through this task.

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| **Unit 4** | | |
| **Area of study** | **Topics** | **Content dot points** |
| Discrete mathematics | Matrices and their applications  Transition matrices | 1, 3, 5  1, 2, 3, 4 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Unit 4** | | |
| **Outcome** | **Key knowledge dot point** | **Key skills dot point** |
| 1 | 1, 3, 4 | 1, 2, 3, 4 |
| 2 | 1, 2, 3, 4 | 2, 3, 4 |
| 3 | 1, 2, 4, 5, 7 | 1, 2, 3, 4, 9, 10, 11, 12 |