VCE Mathematical Methods Unit 1

Sample investigation: Implementing the bisection algorithm with polynomial functions

The investigation is to be conducted over a period of about one week.

Introduction

A context such as the following could be used to develop a mathematical investigation that applies the three investigation components of formulation, exploration and communication to connect several aspects of the mathematics of polynomial functions. These include remainder, factor and rational root theorems. The application of the numerical method of the bisection algorithm for approximating irrational real roots of these functions.

The [bisection algorithm](https://www.codesansar.com/numerical-methods/bisection-method-algorithm.htm) is a bracketing method and [examples of implementation](https://atcm.mathandtech.org/EP2013/regular_papers/3612013_20267.pdf) can be found online.

Formulation

*Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues, or problems of interest.*

In this task, formulation is related to the statement of theorems, selection and variety of functions, how the description will be represented and implemented as an algorithm, and questions for analysis.

Exploration

*Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures, or hypotheses, using mathematical concepts, skills, and processes, including the use of technology and application of computational thinking.*

In this task, exploration is related to application of theorems and analysis of related examples for selected polynomial functions, the application of the algorithm using technology, and the systematic analysis of results, including consideration of accuracy.

Communication

*Summary, presentation, and interpretation of the findings from the mathematical investigation and related applications*

In this task, communication is related to the summary of results for the functions involved, and discussion of the terminating conditions for the algorithm with respect to the accuracy of the approximate solutions for roots.

Part 1

1. State the remainder, factor, and rational root theorems, and give an example of their application to each of a quadratic, cubic and quartic polynomial function, for cases where a rational root exists, and for cases where a rational root does not exist.
2. Use a suitable selection of graphs to show that a quadratic function can have 0, 1 or 2 real roots, a cubic polynomial function can have 1, 2 or 3 roots, and a quartic polynomial function can have 0, 1, 2, 3 or 4 real roots.
3. Explain why a cubic polynomial function must have at least one real root.

Part 2

1. Explain how the bisection algorithm works and show how it is implemented by the technology that is being used.
2. Apply the bisection algorithm to determine point estimates for the non-rational real roots of a quadratic function, a cubic function, and a quartic function.
3. Investigate the effect of the terminating condition on the accuracy of the approximation with respect to the number of iterations required.

Areas of study

The following content from the areas of study is addressed through this task.

|  |  |
| --- | --- |
| **Area of study** | **Content dot point(s)** |
| Functions and graphs | 4 |
| Algebra | 1, 2, 6, 7 |
| Calculus |  |
| Probability and statistics | – |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| 1 | 1, 10, 12 | 7, 10, 11, 12, 13 |
| 2 | 1, 2, 3, 4, 5 | 2, 3, 5, 6 |
| 3 | 1, 2, 5, 6, 7, 8 | 1, 3, 4, 5, 7, 9, 10, 11, 12, 13 |