VCE Mathematical Methods Unit 2

Sample investigation: Simulating motion along a straight line

The investigation is to be conducted over a period of about one week.

Introduction

A context such as the following could be used to develop a mathematical investigation that applies the three components of formulation, exploration and communication to simulate and analyse the motion of an object along a straight line. The position of the object, $y$, in metres, with respect to a fixed origin at time $0 \leq t \leq n$, in seconds, is modelled by a polynomial function $y=f\left(t\right)$, using a set of points $(t,f\left(t\right))$ generated at one second intervals over a period of $n$ seconds, where $n \in N$.

Formulation

*Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues, or problems of interest.*

In this task, formulation is related to the selection and variety of position-time functions and their parameters, representation of average and instantaneous speed and acceleration, continuity and differentiability, and the description representation, implementation of the simulation, and questions for analysis.

Exploration

*Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures, or hypotheses, using mathematical concepts, skills, and processes, including the use of technology and application of computational thinking.*

In this task, exploration is related to the analysis of position, speed and acceleration (including sign and direction) of related examples for selected polynomial functions and variations in their defining parameters. It also relates to the application of the simulation using technology, visual description and interpretation in context, the simulated motion, corresponding position-time graphs, and the systematic analysis of results, including the use of calculus.

Communication

*Summary, presentation, and interpretation of the findings from the mathematical investigation and related applications*

In this task, communication is related to the summary of results for the position-time functions involved (constant, linear, piecewise linear, quadratic and cubic), discussion and interpretation of aspects of the motion, such as constant or varying position, speed and acceleration, stationary points (including turning points), and distance travelled over an interval. This will also involve consideration and interpretation of the simulated motion in relation to the corresponding position-time graph.

Part 1

1. Consider position-time functions of the form $f\left(t\right)=at+b$ where $a$, $b$ are real constants. Vary the values of $a$ and $b$, plot the corresponding position-time graphs for a set of points $(t,f\left(t\right))$ over a suitable domain. Interpret the motion of the object with reference to initial position, direction, speed, acceleration and the values of $a$ and $b$. Calculate the corresponding distance travelled by the object for one of these functions.
2. Repeat step a. for several piecewise linear functions, which include different combinations of motion away from and towards the origin, where and when the object is at rest, and change from one constant speed to another. Calculate the corresponding distance travelled by the object for one of these functions.
3. Consider position-time functions of the form $f\left(t\right)=at^{2}+bt+c$ where $t\geq 0$ and $a \ne 0$, $b$, $c $are real constants. Vary the values of $a$, $b$ and $c$, plot the corresponding position-time graphs for a set of points $(t,f\left(t\right))$ over a suitable domain. Interpret the motion of the object with reference to initial position, direction of motion, speed, acceleration and the values of $a$, $b$ and $c$. Calculate the corresponding distance travelled by the object for one of these functions.
4. Repeat step *c*. for a cubic polynomial position-time graph that has two stationary points. Calculate the corresponding distance travelled by the object for this function.

Part 2

1. For each of the position-time function types from Part 1, use technology to animate a plot of the set of points $(0,f\left(t\right))$ for the object along a vertical line, with a suitable scale, where the animation increases $t $from 0 to $n$ in steps of 1.
2. Interpret the graph of the animated function with respect to the corresponding graphs from Part 1.

Areas of study

The following content from the areas of study is addressed through this task.

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| **Unit 2** |
| **Area of study** | **Content dot points** |
| Functions and graphs | 1, 2  |
| Algebra (Units 1 and 2) | 1, 2, 4 |
| Calculus | 1, 3, 4, 5, 6 |
| Probability and statistics | – |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Outcome** | **Key knowledge dot points** | **Key skill dot points** |
| 1 | 8, 9, 10, 11 | 8, 9, 10, 11 |
| 2 | 1, 2, 4, 5 | 1, 2, 3, 5, 6 |
| 3 | 1, 2, 3, 4, 5, 7, 8 | 1, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13 |