VCE Mathematical Methods Unit 3

Sample application task: Investigating some polynomial functions

The application task is to be of 4–6 hours’ duration over a period of 1–2 weeks.

Introduction

A context such as the following could be used to develop an application task that investigates graphs of polynomial functions of the form$ f:R\rightarrow R, f\left(x\right)=m(x-a)^{s}(x-b)$, the key features of these graphs, and the number of solutions to equations of the form$ f\left(x\right)=p, where p\in R.$

Component 1

*Introduction of the context through specific cases or examples. Students should*

1. Consider the function$ f:R\rightarrow R, f\left(x\right)=\left(x-1\right)^{2}\left(x-2\right)$. Sketch the graph of $y=f(x)$, and clearly indicate all key features. Find the values of *x* for which $f\left(x\right)=p$ has one, two or three solutions,
where *p* is a real number.
2. State the transformations required to map the graph of $y=f(x)$ onto the graph of $y=f\left(\frac{x}{n}\right)+k$.
If there is a turning point at (2, 3), find all possible values of *n* and *k*. Sketch the corresponding graphs.
3. State the transformations required to map the graph of $y=f(x)$ onto the graph of
$y=Af\left(x-h\right)$. If there is a turning point at (–1, 4), find all possible values of *A* and *h*. Sketch the corresponding graphs.
4. The graph of $y=f(x)$ is mapped onto graph of $y=Af\left(n\left(x-h\right)\right)+k$. Discuss how the values of
*A*, *n*, *h* and *k* change the graph of the original function under various transformations.

Component 2

*Consideration of general features of the context. Students should*

1. Now consider the function $f:R\rightarrow R,f\left(x\right)=m\left(x-a\right)^{2}\left(x-b\right)$ where $m,a,b\in R.$
Investigate the graphs of $y=f(x)$ for combinations and ranges of values of the parameters *a*, *b* and *m*.
2. In each of the cases in step a., find the values of $p\in R $for which $f\left(x\right)=p$ has one, two or three solutions.
3. State the transformations required to map the graph of $y=f(x)$ onto the graph of
$y=Af\left(nx\right)+k$, where *A,* *n* and *k* $\in R$. Investigate how *A,* *n* and *k,* and *a, b* and *m* relate to the location and nature of the stationary points of the graph of $y=Af\left(nx\right)+k$.

Component 3

*Variation or further specification of assumption or conditions involved in the context to focus on a particular feature or aspect related to the context. Students should*

1. Consider the function $f:R\rightarrow R, f\left(x\right)=m\left(x-a\right)^{s}\left(x-b\right)$, where $m,a,b\in R $and $s\in N.$ Investigate the graphs of $y=f(x)$ for cases where $a<b,m>0$ and $s\in N.$ What generalisations can be made?
2. Let $f:R\rightarrow R, f\left(x\right)=\left(x-a\right)^{s}\left(x-b\right)$ where $a,b\in R,$ $a<b$ and $s\in N$. Find the values of *p* for which $f\left(x\right)=p$ has zero, one, two or three solutions when *s* = 1, 2, 3, 4 and 5. What generalisations can be made?

Areas of study

The following content from the areas of study is addressed through this task.

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| **Area of study** | **Content dot points** |
| Functions, relations and graphs | 1, 3, 4, 5 |
| Algebra, number and structure | 1, 4, 5 |
| Calculus | 3, 4, 5 |
| Data analysis, probability and statistics | – |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Outcome** | **Key knowledge dot points** | **Key skills dot points** |
| 1 | 1, 2, 3, 9, 10, 11 | 1, 2, 6, 9, 10, 11, 12 |
| 2 | 1, 2, 3, 5 | 1, 2, 4, 5, 6, 7 |
| 3 | 1, 2, 3, 4, 5, 8 | 1, 2, 3, 4, 5, 6, 7, 9, 11, 12 |