VCE Mathematical Methods
Units 3 and 4

Areas of Study 1, 2 and 3

Example of learning activity: A product function

Introduction

This learning activity considers a family of product functions based on power and exponential functions, and explores key features and the behaviour of their graphs.

Part 1

Consider the function $f:R\rightarrow R, f\left(x\right)=x e^{-x}$.

1. Draw the graph of *y* = *x* and *y* = *e–x* on the same set of axes, and show how the graph of *f* could be obtained from these graphs.
2. Describe the behaviour of the graph of the function as $x\rightarrow -\infty $ and as $x\rightarrow \infty $.
3. Find the coordinates of any points of intersection between the graph of *y* = *e-x* and the graph of *f*.
4. Find the derivative of *f* and hence identify the location and nature of any stationary points and the location of any points of inflection.
5. Show that *y* = *x* is tangent to the graph of *f* at the origin.

Part 2

Consider the family of functions $f\_{n}:R\rightarrow R, f\left(x\right)=x^{n} e^{-x}$, where *n* is a positive integer. The function in step a. is the case for *n* = 1.

1. Draw graphs of $f\_{n}$ for several values of *n*, and identify the nature and location of any stationary points, and the location of any points of inflection.
2. Find the coordinates of any points of intersection between the graph of *y* = *e*–*x* and the graph of $f\_{n}$.
3. Describe the behaviour of the graph of the function as $x\rightarrow -\infty $ and as $x\rightarrow \infty $.
4. State the intervals over which $f\_{n}$ is strictly increasing, and the intervals over which it is strictly decreasing.
5. Summarise the key features of the graphs of these functions in terms of *n*.

Part 3

Let *g* be a differentiable function with domain *R*. Carry out similar analysis for $h\left(x\right)=g(x)e^{-x}$.

Areas of study

The following content from the areas of study is addressed through this learning activity.

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| **Units 3 and 4** |
| **Area of study** | **Content dot point(s)** |
| Functions, relations and graphs  | 2, 5 |
| Algebra, number and structure  | 5 |
| Calculus | 4, 5 |
| Data analysis, probability and statistics | – |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Units 3 and 4** |
| **Outcome** | **Key knowledge dot points** | **Key skills dot point(s)** |
| 1 | 1, 4, 7, 10 | 1 |
| 2 | 1, 2, 5 | 1, 2 |
| 3 | 3, 4, 5, 8 | 3, 5, 7 |