VCE Specialist Mathematics   
Units 3 and 4

Sample application task – graphs of rational functions with cubic polynomial numerators and denominators

Introduction

A context such as the following can be used to investigate key features of the graphs of some rational functions of a real variable of low degree, where the denominator function is a cubic polynomial function with integer coefficients and the numerator function is at most a cubic polynomial functions with integer coefficients.

Rational functions can be used to model various situations where ratios, asymptotic behaviour, and a wider range of shapes than graphs of polynomial functions are involved, such as [thermal expansion of metal](http://www.itl.nist.gov/div898/handbook/pmd/section6/pmd64.htm) and [sensor models](https://pdfs.semanticscholar.org/d922/fb2da58ac700d743cd383bc2b3bbf96a5664.pdf) and photogrammetry in geomatics to represent the transformation between an image space and an object space.

For each of the following functions the behaviour and range of shapes of their graphs is to be investigated. The maximal domain and corresponding range should be identified, as well as key features such as axis intercepts, asymptotes, stationary points, points of inflection and symmetry, and the shape of the graph over its natural domain, using the first and second derivative for analysis as applicable.

The location of key features should be determined with respect to different combinations of the coefficients that specify the numerator and denominator polynomial functions, and the different types of graphs identified and classified.

Part 1

Investigate the nature of graphs of reciprocal cubic functions of the form:

 where *p* is non-zero.

Part 2

Investigate the nature of graphs of rational functions of the form:

 where *b* and *p* are non-zero

Part 3

Investigate the nature of graphs of rational functions of the form:

 where *a* and *p* are non-zero

Areas of study

The following content from the areas of study is addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Area of study** | **Topic** | **Content dot point** |
| Functions, relations and graphs | Functions, relations and graphs | 1,2,3 |
| Calculus | Differential and integral calculus | 2, 3 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 3, 6, 7,8 | 2, 6, 7 |
| **2** | 1, 2, 3, 4, 5 | 1, 2, 3, 5, 6 |
| **3** | 1, 2, 3, 5, 6, 7 | 1, 2, 3, 4, 5, 6, 8, 9, 10, 11 |