VCE Specialist Mathematics   
Units 3 and 4

Sample learning activity – Arc lengths of sections of curves

Introduction

Length is one of the basic forms of measurement. This learning activity investigates arc lengths of sections of curves defined by the graph of a function.

Consider a continuous function that satisfies the following conditions: it is non-negativeon the interval [0,1], it has the value 0 at *x* = 0 and *x* = 1; and the area bounded by the graph of the function and the *x*-axis over the interval [0, 1] is one square unit.

For each of the following functions draw the corresponding graphs.

Part 1

1. Let *f*(*x*) = *ax* for 0 ≤ *x* ≤ and *f*(*x*) = –*ax* + *a* for ≤ *x* ≤ 1, where *a* is a non-zero real constant. Find the value of *a* so that *f* satisfies the conditions and find the length of the graph of *y* = *f*(*x*).
2. Repeat this analysis for *f*(*x*) = *ax*(*x* –1), 0≤ *x* ≤ 1.

Part 2

1. Show that if *f* is any continuous function that satisfies the conditions then  
   so does the function *g*(*x*) = (*f*(*x*) + *f*(1–*x*)).
2. Determine the rule for *g*(*x*) if *f* is a quadratic polynomial function satisfying the conditions.
3. Determine the rule for *g*(*x*) if *f* is a cubic polynomial function satisfying the conditions.

Part 3

1. Let *f*(*x*) = *a* sin *nx*. Find the values of *a* and *n* so that *f* satisfies the conditions, and determine the corresponding length of the arc from (0, 0) to (1, 0).
2. Repeat this analysis for *f*(*x*) = *ax* arccos (*x*) and a quartic polynomial function.

Areas of study

The following content from the areas of study is addressed through this learning activity.

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| **Area of study** | **Topic** | **Content dot point** |
| Calculus | Differential calculus and integral calculus | 4, 5, 6 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 3, 7 | 2, 7, 8 |
| **2** | 1, 2, 6 | 1, 2, 3 |
| **3** | 1, 2, 3, 4, 6 | 1, 2, 3, 4, 5, 6, 8, 9 |