VCE Specialist Mathematics
Units 3 and 4

Sample modelling or problem-solving task – Geese

Introduction

This modelling or problem-solving task involves modelling a population of geese using differential equations.

Part 1

The number of geese living in a particular area has been closely monitored over a number of years. The population over the first 12 years of observation, starting in 1948, is shown in the table. The graph of the data for this period is also shown.

200

400

600

800

1000

1200

1400

1600

1800

0

1

2

3

4

5

6

7

8

9

10

11

12

Year

Population size

|  |  |
| --- | --- |
| 1948 | 300 |
| 1949 |  |
| 1950 | 500 |
| 1951 |  |
| 1952 |  |
| 1953 |  |
| 1954 |  |
| 1955 | 900 |
| 1956 |  |
| 1957 | 1150 |
| 1958 | 1350 |
| 1959 | 1650 |

Assume that during these years the birthrate and deathrate of the population is proportional to population size and there are no other influencing factors.

1. Set up a differential equation to model this situation, and find the general solution of the differential equation to give the population size *P*, *t* years after 1948. Find a particular solution which models the data.
2. Use this to give the number of geese in 1956, a year when the census was not taken.

The actual population of geese in 1974 was 5200. What does the model predict for this year?

1. Use exponential regression fucntionality of technology to obtain constants *P*o and *b* such that the population *P* at time *t* years after 1948 is given by *P* = *P*o*ebt*. These values are determined by a statistical technique. Compare the two models.

Part 2

The population growth patterns changed over the next decade. The extended graph is as shown below.

500

1000

1500

2000

2500

3000

3500

4000

4500

0

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

21

22

23

24

25

Population size

Year

The initial model does not work for this extended set of data. A commonly used form of differential equation to model this type of population growth is:

= , where *k* and *Pe*are constants and *t* is the time in years after 1959.

1. Solve this differential equation assuming that when *t* = 0, *P* = 2069 and *Pe* = 3545, *k* = 0.833.
Write the solution in the form
2. Find in terms of *t*.
3. Sketch the graphs of *P* against *t*, against *P*, and against *t*.
4. For what population is the rate of change, , a maximum? In which year is the rate a maximum? Use this model to predict the population in 1985.

Part 3

1. Form a hybrid function using the results from **Part 1** and **Part 2** that describes the growth of the population from 1948 to 1971. Plot the graph of this hybrid function.
2. By 1994 the population had grown considerably. The authorities allowed geese to be shot at a rate
of *S* geese per year. A differential equation to describe this is:

= – *S*,where *Pe* and *k* are constants.

1. If the population is to become stable, a restriction on the number of geese shot per year is necessary. Find the maximum number of geese which can be shot per year in terms of *Pe* and *k*.
(Hint: Consider = 0 and use the discriminant related to the resulting quadratic equation.)
2. For *Pe*= 16000 and *k* = 0.1, find the maximum number of geese which can be shot to enable a stable population to be reached. Find the size of the stable population corresponding to this value of *S*.
3. Solve the differential equation = – *S* with *Pe*= 16000 and *k* = 0.1,
and *P* = 13000 when *t* = 0 (1994) and *S* =300.

Areas of study

The following content from the areas of study is addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Area of study** | **Topic** | **Content dot point** |
| Calculus | Differential equations | 1, 2, 3, 4 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 6, 7, 8, 9 | 2, 6, 7, 9 |
| **2** | 1, 2, 3, 4, 5 | 1, 2, 3, 5, 6 |
| **3** | 1, 2, 3, 5, 6, 7 | 1, 2, 3, 4, 5, 6, 8, 9, 10, 11 |