VCE Specialist Mathematics Unit 1

Mathematical investigation: Linear recurrence relations

Formulation

You will have seen in this course how linear recurrence relations can be used to accurately model financial situations.

There are situations where we can use interrelated linear recurrence relations to investigate such things as competitions in markets or growth of two populations. For example, two firms could sell potato chips in a city and both have some natural growth, but we consider what happens to the sales of one as the other increases. You could undertake this for two species of animals where one preys on the other but there is a fine balance to make sure that both species can survive

Exploration

Use spreadsheets or coding to explore what happens. For example, for the potato chip company scenario.

Let *xn*be the sales per month of the Kryspy company and *yn*be the sales per month of the Curly company. Consider the following scenario:

*x*1 = 40 000 and *y*1= 25 000 and

*x*n + 1 = 40000 + 0.05 *x*n –0.06*yn*

*y*n + 1 = 40000 + 0.2 *y*n –0. 03*xn*

The questions are now clear:

* Do the sales stabilise? (the answer for this one is: ‘yes, quickly’)
* What are the stable sales for both?
* How close do the sales of Curly come to the sales of Kryspy?
* Adjust the coefficients to explore further for different scenarios. Can you get more or less equal sales?
* Can you develop a rule for the *nth* term?

This is just an example.

* Try it with a population of eagles and mice and construct different scenarios.
* Try it with more firms and different conditions, that is with 3 or 4.

Conclusions

Discuss your findings for a specific example and the variations you have tried.

Discuss different scenarios and other applications.

Discuss what the weaknesses of your model are.

Discuss what could be done to improve your model.

Discuss other contexts in which these techniques may be used and suggest the recursion equations for these.

Areas of study

The following content from the areas of study is addressed through this task.

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| **Area of study** | **Topic** | **Content dot point** |
| Discrete mathematics | Sequences and series | 3, 4 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| **1** | 4, 14, 15, 16 | 4, 14 |
| **2** | 1, 2, 3, 4 | 1, 2, 3 ,4, 5 |
| **3** | 1, 2, 3, 4, 5, 6 | 1, 2, 3, 5, 7, 10, 11, 12 |