VCE Specialist Mathematics Unit 2

Mathematical investigation: vector proofs

Formulation

In this investigation vector proofs are to be developed with respect to points of concurrence of line segments associated with the sides and/or angles of a triangle.

Vectors provide the tools necessary to complete the proofs.

For each of the following, use geometry software or pencil on paper using geometric instruments to construct a range of illustrative diagrams. These constructions are examples of algorithms.

Discuss the necessary constructions with each part and complete detailed proofs.

Exploration

Part 1

1. Prove that the altitudes of a triangle are concurrent (this point of concurrence is called the *orthocentre* of the triangle).
2. Is the orthocentre always in the interior of a triangle? Explain.
3. Prove that the perpendicular bisectors of a triangle are concurrent (this point of concurrence is called the *circumcentre* of the triangle). Use this to draw the corresponding *circumcircle* for several triangles.

Part 2

1. Prove that the medians of a triangle are concurrent (this point of concurrence is called the*centroid*).
2. Describe several of the properties of the centroid.
3. Prove that if the vertices of the triangle have position vectors ****respectively,then the position vector of the centroid is **.**

Part 3

1. Describe Euler’s line, and illustrate it for several triangles.
2. For a triangle with vertices at points *A*, *B* and *C*, prove that If *O* is the circumcentre of the triangle and *H* is the orthocentre of the triangle then



Conclusions

This investigation results in specific results. These should be illustrated with suitable constructions applied to different types of triangles and commenting on the position of key points and lines.

Areas of study

The following content from the areas of study is addressed through this learning activity.

|  |  |  |
| --- | --- | --- |
| **Area of study** | **Topic** | **Content dot point** |
| Space and measurement | Vectors in the plane | 1, 2, 3, 4, 5, 6, 7 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

|  |  |  |
| --- | --- | --- |
| **Outcome** | **Key knowledge dot point** | **Key skill dot point** |
| 1 | 11, 12, 13 | 8, 9, 11 |
| 2 | 1, 2, 3, 4, 5 | 1, 2, 3, 4, 5, 6 |
| 3 | 1, 2, 6 | 1, 2, 3, 8, 9 |