VCE Specialist Mathematics
Units 1 and 2

Mathematical investigation 3: Rotating conic sections around any given point

Formulation

In this course you have studied circles, ellipses, parabolas and hyperbolae. Their equations all could be expressed in the form *ax*2 + *by*2 +2*gx* + 2*fy* + *c* = 0 where *a, b, g, f* and *c* are real constants. For the basic cases for ellipses and circles where the *x* and *y* axes are the axes of symmetry, equations are *ax*2 + *by*2 = *e*.

We can use the general rotation matrix to obtain the formula for ellipses, hyperbolas and parabolas and plot them after a rotation.

Exploration

Rotate ellipses and hyperbolae and find their Cartesian equations and use a suitable graphing package to illustrate.

The graph shown here is that of $ x^{2}+\frac{y^{2}}{4}=1$ rotated by π in an anticlockwise direction:



The simplified formula is 5*x*2 + 6*xy* + 5*y*2 = 8.

If the rotation from the original conic was θ we have

*x*´ = *x* cos θ – *y* sin θ and *y*´ = *x* sin θ +*y* cos θ

There are now many questions which can be considered using your knowledge of matrices, transformations and trigonometric identities.

Conclusions

Discuss your findings for a specific example and the variations you have tried.