Example assessment task
Level 8 – Linear relations

Overview

Linear functions are used to model situations where there is a constant rate of change over a specified set of input values (the domain of the function).

They can be represented algebraically in the form *y* = *ax* + *b* or *f*(*x*) = *ax* + *b*, where the corresponding graph is straight line with gradient *a* and vertical axis intercept (0, *b*).

A table of values for a linear function has a constant difference of *a* between output values of *y,* for consecutive integer input values of *x*.

In a modelling context, the gradient corresponds to the constant rate of change, and the vertical axis intercept corresponds to the initial, or starting, value.

This example assessment task requires students to apply linear models and solve related problems. This includes:

* interpreting the gradient, *a*, and *y*-axis intercept, *b*, in context
* evaluating a function and solving linear equations
* finding integer solutions to linear relations of the form *ax* + *by* = *c*, where *a*, *b* and *c* are integers
* making and testing conjectures using algorithms and digital tools.

Curriculum connection (Victorian Curriculum Mathematics Version 2.0)

|  |  |
| --- | --- |
| Level 8 achievement standard (linked sentences)  | Level 8 content description |
| Students use mathematical modelling to solve problems using linear relations, interpreting and reviewing the model in context.They make and test conjectures involving linear relations by developing algorithms and using digital tools. | use mathematical modelling to solve applied problems involving linear relations, including financial contexts involving profit and loss; formulate problems with linear functions, and choose a representation; interpret and communicate solutions in terms of the context, and review the appropriateness of the modelVC2M8A03 |

Equipment and duration

The example assessment task is designed to be completed individually over 1 to 2 lessons of 50-minute duration. Students can use a scientific calculator, graph paper and digital tools with table and graphing functionality.

Teachers may wish to present the assessment task as a digital file so that tables and graphical outputs generated using digital tools can be embedded as part of student responses.

Assessment task, with teacher notes

Question 1 – Straight line motion

Linear functions can be used to model motion at constant speed.

Consider a person moving along a straight path from a given starting point (the origin).

To start with, they move at a speed of 1.5 m/s for 2 minutes, and then increase their speed to 2.5 m/s for 1 minute, after which they decrease their speed to 1 m/s for one minute and then stop and rest for 30 seconds.

1. Graph the person’s distance from the origin, in metres, as a function of time, in seconds.
2. Find the total distance travelled by the person from when they start until they stop and rest.
3. The person now moves back to the starting point at a constant speed of 4 m/s. Include this on the graph.
4. How long does the person take until they are back at the starting point?

**Teacher notes**

This part of the assessment task addresses the aspect of the achievement standard ‘students use mathematical modelling to solve problems using linear relations, interpreting and reviewing the model in context.’

Students can demonstrate their ability to suitably scale and draw graphs using line segments on graph paper, and interpret these in context.

Students could also be asked to identify and discuss assumptions made in using this model, such as instantaneous starts, transitions and stops.

Extensions to the task could include:

interpreting motion from a given graph

exploring other movement scenarios for one person, for example, pacing forwards and backwards between 2 points at a constant speed (assuming there is no delay in turning around)

exploring other movement scenarios where there are 2 people involved, with rests and extending over a longer timeframe.

Question 2 – Maximum heart rate formulas

Maximum heart rate formulas are used to develop suitable heart rate ranges for exercise.

These formulas provide models based on linear functions that give an estimated maximum heart rate.

**Model 1**

A simple commonly used model is that the maximum heart rate = 220 − age.

1. What maximum heart rate does this model estimate for you?
2. Construct a table of values using this model for ages 10–70 years.
3. Graph this model, interpret the constant rate of change, and describe what the model shows.
4. For what age does this model estimate a maximum heart rate of 170?

**Model 2**

An alternative model is that the maximum heart rate = 208 − 0.7 × age.

1. What maximum heart rate does this model estimate for you?
2. Construct a table of values using this model for ages 10–70 years.
3. Graph this model, interpret the constant rate of change, and describe what the model shows.
4. For what age does this model estimate a maximum heart rate of 170?

Comparing the models:

1. Graph both models together.
2. At what age do both models estimate the same maximum heart rate?
3. Describe the difference in estimated maximum heart rates between the 2 models over the age range 10–70 years.

**Teacher notes**

This part of the task addresses the aspect of the achievement standard ‘students use mathematical modelling to solve problems using linear relations, interpreting and reviewing the model in context’.

The context provides an opportunity for cross-curriculum connection with the Health and Physical Education learning area.

While individual calculations for a specific age can be carried using a scientific calculator, the use of digital tools such as a spreadsheet, Desmos, Wolfram Alpha or similar will be useful for this part of the task, to assist in the representation and analysis of the models using functionalities that include creating tables of values, using graphs and/or solving equations.

The graphing parts of questions can be completed either on graph paper or using digital tools as specified by the teacher.

Equations can be solved using tables of values, graphically or algebraically, for example, solving
220 − *a* = 208 − 0.7*a* where *a* is the variable that represents age in years.

Question 3 – Integer solutions of a linear relation

Consider the linear relation 2*x* + 3*y* = 48.

1. Find the value of *y* when *x* = 0 and the value of *x* when *y* = 0.
2. Graph the relation.
3. Find all positive integer values of (*x*, *y*) for which the relation is true.
4. If *x* or *y* can be positive or negative integers, find several other pairs of values (*x*, *y*) for which the relation is true.
5. Describe a rule (algorithm) that will enable you to find any pair of values (*x*, *y*) for which the relation
is true, and explain why the rule (algorithm) works.
6. Explore how the answers for **a.** to **e.** change when the rule of the linear relation is 2*x* − 3*y* = 48.

**Teacher notes**

The use of a scientific calculator or digital tools such as a spreadsheet, Desmos, Wolfram Alpha or similar will be useful for this part of the task, which addresses the aspect of the achievement standard ‘they make and test conjectures involving linear relations by developing algorithms and using digital tools’.

Initially students will likely proceed by trial and error to identify and verify particular values for (*x*, *y*) by inspection.

They can then use digital tools with tables of values and/or graphs to explore how these might be systematically extended, by conjecturing and developing a rule (algorithm) for generating solutions for a specific relation.

The linear relation *ax* + *by* = *c* where *a*, *b* and *c* are integers has integer solutions for (*x*, *y*) when the highest common factor (greatest common divisor) of *a* and *b* is also a factor of *c*.

While students are not necessarily expected to formulate their rule (algorithm) using algebraic expressions, the solutions to 2*x* + 3*y* = 48 can be generated as follows: choose any integer *n*, then *x* = 24 − 3*n* and
*y* = 2*n*.

Work on this question can be scaffolded by considering a sequence of simpler relations, for example:

*x* + *y* = 12

2*x* + *y* = 12

*x* + 3*y* = 12

2*x* + 3*y* = 12

This part of the assessment task involves:

identifying and verifying integer solutions to linear relations of the form *ax* + *by* = *c*, where *a*, *b* and *c* are integers

using tables of values and linear graphs to represent and analyse these relations

forming and testing conjectures and developing algorithms.

**► A student version of the assessment task has been included in the following pages.**

Assessment task: Linear relations

Question 1 – Straight line motion

Linear functions can be used to model motion at constant speed.

Consider a person moving along a straight path from a given starting point (the origin).

To start with, they move at a speed of 1.5 m/s for 2 minutes, and then increase their speed to 2.5 m/s for 1 minute, after which they decrease their speed to 1 m/s for one minute and then stop and rest for 30 seconds.

1. Graph the person’s distance from the origin, in metres, as a function of time, in seconds.
2. Find the total distance travelled by the person from when they start until they stop and rest.
3. The person now moves back to the starting point at a constant speed of 4 m/s. Include this on the graph.
4. How long does the person take until they are back at the starting point?

Question 2 – Maximum heart rate formulas

Maximum heart rate formulas are used to develop suitable heart rate ranges for exercise.

These formulas provide models based on linear functions that give an estimated maximum heart rate.

**Model 1**

A simple commonly used model is that the maximum heart rate = 220 − age.

1. What maximum heart rate does this model estimate for you?
2. Construct a table of values using this model for ages 10–70 years.
3. Graph this model, interpret the constant rate of change, and describe what the model shows.
4. For what age does this model estimate a maximum heart rate of 170?

**Model 2**

An alternative model is that the maximum heart rate = 208 − 0.7 × age.

1. What maximum heart rate does this model estimate for you?
2. Construct a table of values using this model for ages 10–70 years.
3. Graph this model, interpret the constant rate of change, and describe what the model shows.
4. For what age does this model estimate a maximum heart rate of 170?

Comparing the models:

1. Graph both models together.
2. At what age do both models estimate the same maximum heart rate?
3. Describe the difference in estimated maximum heart rates between the 2 models over the age range 10–70 years.

Question 3 – Integer solutions of a linear relation

Consider the linear relation 2*x* + 3*y* = 48.

1. Find the value of *y* when *x* = 0 and the value of *x* when *y* = 0.
2. Graph the relation.
3. Find all positive integer values of (*x*, *y*) for which the relation is true.
4. If *x* or *y* can be positive or negative integers, find several other pairs of values (*x*, *y*) for which the relation is true.
5. Describe a rule (algorithm) that will enable you to find any pair of values (*x*, *y*) for which the relation
is true, and explain why the rule (algorithm) works.
6. Explore how the answers for **a.** to **e.** change when the rule of the linear relation is 2*x* − 3*y* = 48.