Teaching and learning unit: 3.1.2 Addition, subtraction and inverse

Mathematics, Level 3

# Overview, including Victorian Curriculum F–10 links

| **Description of the teaching and learning unit** | **Cohort considerations (in relation to this teaching and learning unit)** |
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| This unit covers the connection between addition and subtraction and their inverse relationship as operations. It has been designed to develop a conceptual understanding of the processes for partitioning, rearranging and regrouping (renaming) two- and three-digit numbers and applies these to assist in calculations. The mathematics learning is sequenced as follows:* part-whole strategy applying place value (adding and subtracting multiples of ten, within 100 and then beyond 100)
* place-value based strategy to add without renaming tens and ones
* place-value based strategy to subtract without renaming tens and ones
* place-value based strategy to add with renaming tens and ones
* place-value based strategy to subtract with renaming tens and ones
* recognise and explain the connection between addition and subtraction as inverse operations.

The unit is planned as 11 lessons of 60 minutes duration over a period of approximately 2–3 weeks.  | Relevant student data may include teacher judgements, prior units of learning in place value or additive tasks, and any pre-assessments undertaken. When differentiating for learners, cognitive and affective considerations will be relevant, including for students who have an individual education plan (IEP) and associated goals. Students working below Level 3 can address partitioning two-digit numbers into tens and ones using physical and virtual materials and be supported to add and subtract working with numbers to 20 (Level 1), before adding and subtracting one- and two-digit numbers (Level 2).Students working beyond Level 3 can be encouraged to use estimation to check the reasonableness of their calculations, and to develop efficient mental and written strategies (Level 4).Common misconceptions associated with addition and subtraction often arise from an underdeveloped understanding of the place value system. These misconceptions may be demonstrated when students are unsure of the value of 0s between other digits in a number, or have difficulty renaming into hundreds, tens and ones leading to errors in calculation. |

## Content description links and associated achievement standard links

| **Content description link(s)** | **Achievement standard link(s)** |
| --- | --- |
| add and subtract two- and three-digit numbers using place value to partition, rearrange and regroup numbers to assist in calculations without a calculator VC2M3N04  | They partition, rearrange and regroup two- and three-digit numbers in different ways to assist in calculations. Students extend and use single-digit addition and related subtraction facts and apply additive strategies to model and solve problems involving two- and three-digit numbers. |
| estimate the quantity of objects in collections and make estimates when solving problems to determine the reasonableness of calculations  VC2M3N06  | They make estimates and determine the reasonableness of financial and other calculations.   |
| recognise and explain the connection between addition and subtraction as inverse operations, apply to partition numbers and find unknown values in number sentences  VC2M3A01  | Students find unknown values in number sentences involving addition and subtraction. |

## Continuum of learning

### Achievement standards

| **Level 2** | **Level 3** | **Level 4** |
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| … apply knowledge of place value to partition, rearrange and rename two- and three-digit numbers in terms of their parts …Students describe and continue patterns that increase and decrease additively by a constant amount and identify missing elements in the pattern. | They partition, rearrange and regroup two- and three-digit numbers in different ways to assist in calculations. Students extend and use single-digit addition and related subtraction facts and apply additive strategies to model and solve problems involving two- and three-digit numbers. | … students use their understanding of place value to represent tenths and hundredths in decimal form and to multiply natural numbers by multiples of 10.They use their proficiency with addition, subtraction, multiplication facts for tens (× 10) and related division facts to perform arithmetic operations to add and subtract, and multiply and divide numbers efficiently. |
| They use mathematical modelling to solve practical additive and multiplicative problems, including money transactions, representing the situation and choosing calculation strategies. | They make estimates and determine the reasonableness of financial and other calculations.   | They choose rounding and estimation strategies to determine whether results of calculations are reasonable. |
| – | Students find unknown values in number sentences involving addition and subtraction. | – |

### Content descriptions

| **Level 2** | **Level 3** | **Level 4** |
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| add and subtract one- and two-digit numbers, represent problems using number sentences and solve using part-part-whole reasoning and a variety of calculation strategies VC2M2N04 | add and subtract two- and three-digit numbers using place value to partition, rearrange and regroup numbers to assist in calculations without a calculator VC2M3N04  | develop efficient mental and written strategies and use appropriate digital tools for solving problems involving addition and subtraction, and multiplication and division where there is no remainder VC2M4N06 |
| – | estimate the quantity of objects in collections and make estimates when solving problems to determine the reasonableness of calculations  VC2M3N06  | choose and use estimation and rounding to check and explain the reasonableness of calculations, including the results of financial transactions VC2M4N07 |
| recognise, describe and create additive patterns that increase or decrease by a constant amount, using numbers, shapes and objects, and identify missing elements in the patternVC2M2A01 | recognise and explain the connection between addition and subtraction as inverse operations, apply to partition numbers and find unknown values in number sentences  VC2M3A01  | find unknown values in numerical equations involving addition and subtraction, using the properties of numbers and operations VC2M4A01 |

## Other curriculum links and notes

<Links to the Victorian Curriculum F–10 Version 2.0 cross-curriculum priorities and capabilities will be added to this example document after these curriculum areas are published.>

# Essential questions

| **Essential questions to foster inquiry, understanding and transfer of learning** |
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| How can addition and subtraction help solve practical problems?How can I use my understanding of place value to add and subtract?Are addition and subtraction related? In what ways are they similar? In what ways are they different?How might I use the properties of odd and even numbers to check the reasonableness of calculations? |

# Assessment and learning sequence details

| **Assessment task name and type** | **Link to achievement standard** | **Moderation**  |
| --- | --- | --- |
| Pre-test (formative): Maths SS L3 Number and Place Value (Digital Assessment Library) | Level 2apply knowledge of place value to partition, rearrange and rename two- and three-digit numbers in terms of their parts; and regroup partitioned numbers to assist in calculationsLevel 3They partition, rearrange and regroup two- and three-digit numbers in different ways to assist in calculations. | NA |
| Week 1, Lesson 2: Activity, Stepping forwards and backwards (formative) | Students find unknown values in number sentences involving addition and subtraction. | NA |
| Week 2, Lessons 9–10: Task, Organising buses (VCAA Level 3 example assessment task – Addition and Subtraction, summative)  | They partition, rearrange and regroup two- and three-digit numbers in different ways to assist in calculations.  | This task may be moderated by the school team or a professional learning community (PLC) with respect to the Level 3 achievement standard, using a representative selection of student work samples  |
| Week 3, Lesson 11: End-of-unit test (summative)  | They partition, rearrange and regroup two- and three-digit numbers in different ways to assist in calculations. They make estimates and determine the reasonableness of financial and other calculations.  Students extend and use single-digit addition and related subtraction facts and apply additive strategies to model and solve problems involving two- and three-digit numbers.Students find unknown values in number sentences involving addition and subtraction. | The results could be moderated across classes by using a marking scheme to identify three levels of demonstrated student knowledge and skills: developing, established and excellent. |

| **Week** | **Lesson**  | **Learning goal (e.g. learning intention and success criteria)** | **Lesson elements**  | **Differentiation and/or individualisation** | **Assessment** | **Resources** |
| --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | We are learning to add two-digit numbers using place value.**Success criterion 1:**I can partition numbers into tens and ones.**Success criterion 2:**I can add two-digit numbers using place value. | This lesson involves using place value to partition numbers to assist in mental and written calculations Warm-up: Establish if students can calculate the following without renaming:123 + 5 = ?41 + 24 = ?202 + 56 = ?Lesson: Prepare an empty number line with an addition problem for all students to see, for example 35 + 46.Progress to using different strategies to solve an open-ended question, such as ‘When I added some odd numbers together the answer was 178. What might the numbers be?’Students show their thinking in two different ways. (e.g. number line, part-part-whole model, vertical algorithm).Reflection: Which strategy did you use to calculate today? Was this effective? How do you know? | Scaffold based on insights gained from pre-test.**Enabling and extending prompts:**Enabling:Which two numbers have been added to make 178 on this number line?A number line that has the starting number blank, then +50, +10, +10 and +5 across the number line, with the end number as 178.Extending:Is it better to use addition or subtraction to solve this problem? Why? |   |  |
| 1 | 2 | **Learning intention:**We are learning to add and subtract using place value.**Success criterion 1:**I can add a two-digit number to a three-digit number and subtract a two-digit number from a three-digit number.**Success criterion 2:**I can use a number line to model thinking involving addition and subtraction of two-digit numbers. | This lesson involves partitioning numbers to support subtraction calculations and explore the inverse relation between addition and subtraction.Warm-up: Partition given numbers and recognise the connection between the addition and subtraction; for example, 96 partitioned could be written and represented in various forms like 60 + 36, 96 – 36 = 60, 96 – 60 = 36 Lesson: Use partitioning to calculate a two-digit number being added to or subtracted from a three-digit number. Explore all 3 rearrangements: 40 + 83 = □, □ – 40 = 83, □ – 83 = 40Use empty number lines for students to model the different arrangements.Students explore different partitioning arrangements and consider which are more efficient.Reflection: Why is it possible to solve this problem using addition or subtraction? | **Enabling and extending prompts:**Enabling:Simplify by adding multiples of ten or use numbers that do not require renaming into tens and ones.Extending:Use a number line to demonstrate the connection between addition and subtraction. | Activity: Stepping forwards and backwards (formative) | Mathematics Curriculum Companion: [Empty number lines](https://fuse.education.vic.gov.au/MCC/CurriculumItem?code=VCMNA153)reSolve: Maths by Inquiry:[Assessing Reasoning: Year 3 Exemplars](https://www.resolve.edu.au/assessing-reasoning-year-3-exemplars?lesson=3795) |
| 1 | 3 | **Learning intention:**We are learning to add two- and three- digit numbers using place value.**Success criterion 1:**I can add two- and three-digit numbers without renaming.**Success criterion 2:**I can add 10 or 100 more to a number. | This lesson involves recognition of patterns generated from the addition or subtraction of multiples of 10.Warm-up: Count back by hundreds, starting at 1200. Stop at 800. Use materials or a visual to support this if needed.Lesson: Students explore addition and subtraction patterns generated when adding hundreds, tens or ones. For example:Here are some addition number sentences:35 + 4635 + 146135 + 146235 + 146What do you notice? What has changed? What has remained the same?What do you think 435 + 146 will be?How about 535 + 246?Base ten materials can be used to support the conceptual understanding and show why the digit in the hundreds or tens place changes while the ones remain unchanged.Reflection:How might we use the patterns generated through addition to assist calculations? | **Enabling and extending prompts:**Enabling:Students add multiples of 10 using base ten materials to model the problem.Extending: Change each addition sum into a subtraction question. Does the pattern still emerge? Explain why or why not.Students generate their own problems for others to solve. |   | reSolve: Maths by Inquiry:[Algebra: Number sequences](https://www.resolve.edu.au/algebra-number-sequences) |
| 1 | 4 | **Learning intention:**We are learning to add using place value.**Success criterion 1:**I can rename or regroup numbers to a near ten or hundred.**Success criterion 2:**I can rename or regroup numbers to balance a number sentence. | This lesson involves partitioning and regrouping numbers to facilitate addition calculationsWarm-up: Play a game or activity where students estimate and round to near tens or hundreds. For example, 328 + 257 is about ...Lesson: Use a task that requires students to regroup numbers to a near ten or hundred to make calculations more efficient. For example: Dino’s four cousins and their dog Fletcher were coming to stay at his house while visiting from Perth. He thought they might like to visit a famous Melbourne attraction, so he bought each person an entry ticket to Luna Park, which cost $49 per ticket, and a ball for $5 for their dog Fletcher. How much did Dino spend on the gifts?Discuss: How does rearranging these numbers make calculations easier?Check for understanding using a different question, for example:498 + 240 = \_\_\_ + \_\_\_\_ | **Enabling and extending prompts:**Enabling:Provide a similar, simpler question involving partitioning and equivalence. For example, 29 + 3 = 30 + ?Extending:What if the tickets were for a week at $147 per ticket, and the ball was $16?How much would Dino have spent on the gifts then? |  |  |
| 1 | 5 | **Learning intention:**We are learning to construct equivalent number sentences using addition and subtraction. **Success criterion 1:**I can create equivalent number sentences involving addition.**Success criterion 2:**I can create equivalent number sentences involving addition and subtraction. | This lesson involves developing equivalence through partitioning and rearranging numbers.Warm-up: Use a balance scale to emphasise the idea of equivalence in relation to the equal sign (=). Show three images of a scale, of which two are unbalanced and one is balanced and ask students to discuss what they notice. What might be on each side? This may or may not be a numerical value.To consolidate Lesson 4’s focus, adapt the task to allow students to continue to develop this conceptually through a similar but different version of the task. For example, empty number sentences can be used to build understanding of equivalence and rearranging numbers.

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Reflection:What did you learn about equivalence today? How will this understanding help you? | **Enabling and extending prompts:** Enabling:Use addition only to balance the equation\_\_ + \_\_\_ = \_\_\_ + \_\_\_True or false:32 + 27 = 72 – 13Extending:Apply the understanding of equivalence to a multiplicative context.\_\_\_ × \_\_\_ = \_\_\_ × \_\_\_\_ |  | NRICH: [Difference](https://nrich.maths.org/927) |
| 2 | 6 | **Learning intention:**We are learning to show the difference between amounts.**Success criterion 1:**I can use addition or subtraction to find the difference between two values.**Success criterion 2:**I can use a number sentence to solve a word problem involving addition and subtraction. | This lesson involves explaining the connection between addition and subtraction, through the concept of difference.Warm-up: Ask students to make two groups, with a given number of more or less people in one group than the other. Emphasise the number of people in each group and the difference between these groups. Lesson: Students apply understanding of addition or subtraction to find the difference between two amounts. Difference problems require students to compare the number of objects in different sets, which are structurally different from part-whole situations, and so can be more challenging for students. For example: Sam and Josh baked 55 cookies. Sam baked 17 more cookies than Josh. How many cookies did Josh bake?Discuss: How does determining the difference explain the connection between addition and subtraction? | **Enabling and extending prompts:**Enabling: Use a coloured bar model to show the relationship between the cookies baked by Sam and Josh.

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A number sentence can be used to provide a different representation of this problem.\_\_\_ + \_\_\_ + 17 = 55Extending:For each cookie Sam made he burnt one too. Does Sam now have more or fewer cookies than Josh? What is the difference now? |  | NZ Maths:[Difference between](https://nzmaths.co.nz/difference-between)NZ Maths:[Difference with sets to 20](https://nzmaths.co.nz/resource/difference-sets-twenty) |
| 2 | 7 | **Learning intention:** We are learning to add using three-digit numbers.**Success criterion 1:**I can add two or more three-digit numbers.**Success criterion 2:**I can apply understanding of place value in a game.**Success criterion 3:**I can use a number sentence to record a sum. | This lesson involves using understanding of place value for three-digit addition.Warm-up: Review place value up to four-digit numbers using a number line, for example 1000. Discuss some suitable scales.Lesson: Students play a game involving three-digit addition, in an attempt to reach a target number (e.g. 1000 as in the ‘Dicey Operations in Line’ supporting resource from NRICH).Questions for students to consider when playing the game:* Is it always a good idea to put the largest digits in the hundreds place? Why or why not?
* Which place did you need to think the most carefully about when placing your numbers and why?
* Are your estimates improving with each game? Why or why not?

Note whether students are renaming when adding three-digit numbers and showing their calculations.Reflection: Where did you choose to place larger digits? When might it be better to place an 8 in the tens or ones column than in the hundreds? | **Enabling and extending prompts:**Enabling: Students add two-digit numbers.Students add to a smaller value (e.g. 100 or 500).Extending:Start with 1000. The closest to 0 wins.Change the structure to \_ \_ \_ + \_ \_ \_ – \_ \_ \_ = \_\_\_\_ |  | NRICH:[Nice or Nasty](https://nrich.maths.org/6605)NRICH (games 1 and 2 focus on addition and subtraction):[Dicey Operations in Line](https://nrich.maths.org/13261)NRICH:[Dicey Addition](https://nrich.maths.org/11863) |
| 2 | 8 | **Learning intention:** We are learning to rename to support addition calculations.**Success criterion 1:**I can add and rename two-digit numbers.**Success criterion 2:**I can use addition to solve a word problem. | This lesson involves creating and solving addition problems involving renaming.Warm-up: Revisit Lesson 2 focus by presenting a number line and asking students to identify the connected calculation. For example, 251 + 823 = 1074 , 1074 – 251 = 823, 1074 – 823 = 251Lesson: Use a context that requires students to problem-solve to find the solution and apply the skills of adding and renaming. The supporting resource ‘Reach 100’ from NRICH could be used to encourage this approach. This task can be varied to remove the requirement to reach 100 at first, then introduced part-way through the lesson once students have had an opportunity to explore the ‘box’.Questions to encourage and support thinking:* How might you rename this number?
* What did you notice about the numbers needed to fill the box? Which numbers are a better choice?

Reflection: Which place did you need to think the most carefully about when placing your numbers and why?  | **Enabling and extending prompts:**Enabling:If the total has to be 100, which four numbers can be used to make a zero in the ones place?Extending:How many ways can you do this? |  | NRICH: [Reach 100](https://nrich.maths.org/1130) |
| 2 | 9–10 | NA | These two lessons involve an extended practical task as summative assessment.Introduce the context to students: planning for upcoming swimming lessons (note this context can be varied to a more relevant context for the learner cohort).Discuss the expectations of completing an assessment task and that this may take two to three lessons to complete.Explain which materials are available to support learning and clarify any questions with students before commencing the task. Remind students that they can use number lines, tables, ten-frames, algorithms etc. to show their thinking about addition and subtraction. | Adapt the context to suit the cohort and make it relevant to the group of learners.  | Organising buses (VCAA Level 3 example assessment task – Addition and subtraction) |  |
| 3 | 11 | NA  | Summative assessment test  |  | End-of-unit test |  |

# Unit reflection

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| **Reflection questions:*** How does the teaching and learning unit provide evidence of student learning and progress?
* Does the teaching and learning unit:
* specify the content descriptions addressed in the unit
* specify the achievement standards addressed in the unit
* include the resources and activities used to develop knowledge and skills
* provide for a range of student abilities
* specify the assessments used to monitor and progress student learning
* provide guidance about the approximate time required for the unit?
* Considering your responses to the questions above and other relevant reflections, how can the teaching and learning unit be improved?
* How will the evidence of student learning from this teaching and learning unit influence the subsequent teaching and learning unit?

**Responses:*** This teaching and learning unit has a balance of formative and summative assessment. In the next unit, given it is half the length and the students managed this ratio well, the team has only one formative and one summative assessment planned. The team will assess if this is appropriate halfway through, just in case there is a need for more formative assessment.
* The unit specifies the three content descriptions and the four achievement standard sections addressed (at Level 3) as well as the continuum of learning for at, above and below.
* It describes the learning activities used to develop mathematical concepts, knowledge, skills and processes and provides links to related resources. The teachers in the team indicated that students were engaged by the variety of tasks. In future units the team wants to focus on maintaining the high variety of tasks due to how it was received.
* The extended practical summative assessment task provided any entry points for students and supported diverse learners. As a team we will look to use a task like this again.
* The timing (11 lessons) was appropriate as it provided adequate time for students to engage with the content and progress without experiencing fatigue. As a team we would not change the length of this unit.
* The unit catered well for a range of student abilities. The pre-testing noted that students were well equipped to approach this unit, and there were planned enabling and extending prompts for each lesson. If there was a greater spread of students next time, we might need to expand this even further.
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