VCE Mathematical Methods (CAS)

Written examination 1 from 2010 (inclusive)

From 2010 the Mathematical Methods (CAS) examination 1 will be based on content from the areas of study for **Mathematical Methods (CAS) 2006–2012**.

For the period 2006–2009 the common examination 1 for Mathematical Methods and Mathematical Methods (CAS) was based on content from the areas of study for Mathematical Methods in relation to Outcome 1. The content for Outcome 1 of Mathematical Methods (CAS) includes, for example, the use of matrices for transformations of the plane, Markov chains and the representation of systems of simultaneous linear equations.

The following collection of sample questions is taken from the content for Mathematical Methods (CAS) which is supplementary to the content of the former Mathematical Methods study 2006–2009. These questions are also supplementary to the sample examination material for examination 1 currently on the website.

There is no change to Mathematical Methods (CAS) examination 2.

Schools are reminded that Mathematical Methods (CAS) is the only Mathematical Methods study now available.

Question	**Area of study, related content from study design, notes**
Question 1 Let $f: R \to R,\ f(x) = x^3 + (k+1)x^2 + kx$. Solve $f(x) = 0$ for x. 3 marks	**Algebra** • solution of literal equations
Question 2 For the simultaneous linear equations $ax + 3y = 0$ $2x + (a+1)y = 0.$ where a is a real constant, find the value(s) of a for which there are infinitely many solutions. 3 marks	**Algebra** • solution of systems of simultaneous linear equations, including consideration of cases where no solution or an infinite number of possible solutions exist
Question 3 Find the equation of the image of $y = \frac{1}{x}$ under the transformation defined by the matrix $\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$ and describe a sequence of transformations that maps the graph of $y = \frac{1}{x}$ onto the graph of its image. 3 marks	**Functions and graphs** • transformation from $y = f(x)$ to $y = Af(n(x+b)) + c$ **Algebra** • application of matrices to transformations of points in the plane (assumed knowledge and skills from Units 1 and 2)

April 2010

Question 4 State the subset of R for which the graph of the function $f(x) = x^4 - x^2$ is strictly decreasing. 3 marks **Note:** A function f is said to be strictly decreasing on a given set if for all a and b in the set $a < b$ implies $f(a) > f(b)$. In this question this means the answer is $(-\infty, -\frac{\sqrt{2}}{2}] \cup [0, \frac{\sqrt{2}}{2}]$ or an equivalent.	**Calculus** • application of differentiation to curve sketching and identification of key features of curves, identification of intervals over which a function is constant, stationary, strictly increasing or strictly decreasing
Question 5 Write down a formula that generates all real solutions of the equation $\sin(x) + \cos(x) = 0$. 3 marks	**Algebra** • general solutions of equations such as $\cos(x) + \cos(3x) = \frac{1}{2}$, $x \in R$ and the specification of exact solutions or numerical solutions, as appropriate, within a restricted domain
Question 6 For the simultaneous linear equations $mx + 12y = 12$ $3x + my = m$ find the value(s) of m for which the equations have **i.** a unique solution **ii.** infinitely many solutions. 4 marks	**Algebra** • solution of systems of simultaneous linear equations, including consideration of cases where no solution or an infinite number of possible solutions exist
Question 7 Sharelle is the goal shooter for her netball team and during matches has many attempts at scoring a goal. Assume that the success of an attempt to score a goal depends only on the success or otherwise of her previous attempt at scoring a goal. If an attempt at scoring a goal in a match is successful, then the probability that her next attempt at scoring a goal in the match is successful is 0.84. However, if an attempt at scoring a goal in a match is unsuccessful, then the probability that her next attempt at scoring a goal in the match is successful is 0.64. In the long term, what percentage of her attempts at scoring a goal are successful? 3 marks	**Probabilty** • Bernoulli trials and two state Markov chains, including the length of run in a sequence, steady values for a Markov chain (familiarity with the use of transition matrices to compute values of a Markov chain will be assumed)
Question 8 Show that the graph of $h(x) = \frac{x^n}{e^x}$, where n is a positive integer, has a local maximum at $x = n$. 3 marks	**Study design statement** • identification of local maximum/ minimum values over an interval

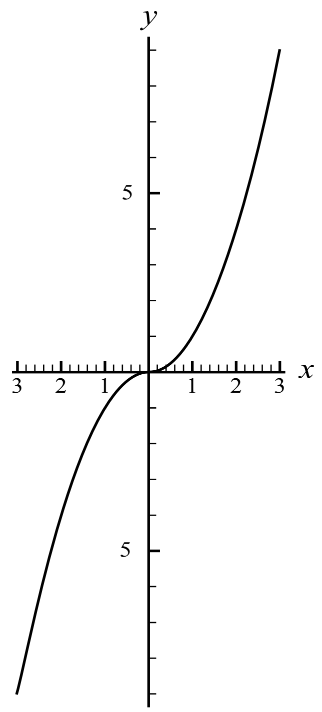
Question 9 Let $g: R \to R$, $g(x) = x^2$. Show that $g(u + v) + g(u - v) = 2(g(u) + g(v))$. 2 marks	**Algebra** • the relationship of $f(x \pm y)$, $f(xy)$ and $f(xy)$ to values of $f(x)$ and $f(y)$ for different functions f
Question 10 For the functions $f: R \to R$, $f(x) = e^x + e^{-x}$ and $g: R \to R$, $g(x) = e^x - e^{-x}$ show that **i.** $[f(x)]^2 = f(2x) + 2$ **ii.** $f(x)g(x) = g(2x)$ **iii.** $[f(x)]^2 - [g(x)]^2 = 4$. 3 marks	**Study design statement** • the relationship of $f(x \pm y)$, $f(xy)$ and $f(xy)$ to values of $f(x)$ and $f(y)$ for different functions f
Question 11 Find the average value of $y = e^x$ over the interval [0, 2]. 2 marks	**Calculus** • application of integration . . . average value of a function

Question 12

Part of the graph of the hybrid function

$$f(x)=\begin{cases}-x^2, x\le 0\\ x^2, \text{otherwise}\end{cases}$$

where x is a real number is shown below.



a. Draw the graph of the derivative function f' on the same set of axes.

b. Write down a rule for the derivative function.

4 marks

Note: The sgn function is defined by

$$\operatorname{sgn}(x)=\begin{cases}1 \text{ where } x>0\\ 0 \text{ where } x=0\\ -1 \text{ where } x<0.\end{cases}$$

The hybrid function f is differentiable at $x = 0$ with $f'(0) = 0$, so f' can either be specified as a hybrid function, or alternatively using the sgn function, as indicated in the answer, in which case $f'(x) = 2x \operatorname{sgn}(x)$.

Functions and graphs

- applications of simple combinations of the above functions (including simple hybrid functions)

Calculus

- deducing the graph of the derivative function from the graph of a function
- application of differentiation to curve sketching and identification of key features of curves, identification of intervals over which a function is constant, stationary . . .

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Question 13

The speed v, in metres per second, of an object moving in a straight line is given as a function of time t, in seconds, where

$$v(t) = \frac{24}{t+1} \text{ where } t \geq 0.$$

a. State the initial speed of the object.

b. Find the values of t for which the speed is **less than** 2 metres per second.

c. Find the distance travelled by the object in the first 10 seconds.

3 marks

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Calculus

- application of integration to problems involving . . . distance travelled in a straight line . . . and finding a function from a known rate of change
- application of anti-differentiation to problems involving straight-line motion, including calculation of distance travelled (assumed knowledge and skills from Units 1 and 2)

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