2025 NHT

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Mathematical Methods Examination 1

Question and Answer Book

VCE (NHT) Examination – Tuesday 27 May 2025

- Reading time is **15 minutes**: 10.30 am to 10.45 am
- Writing time is **1 hour**: 10.45 am to 11.45 am

Materials supplied

- Question and Answer Book of 16 pages
- Formula Sheet

Instructions

Students are **not** permitted to bring any technology (calculators or software), or notes of any kind, into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

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Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \frac{\log_e(x)}{x^3}$.

Find and simplify $\frac{dy}{dx}$. 2 marks

b. Let $f(x) = \sin(\pi e^{3x})$.

Evaluate $f'(0)$. 2 marks

Question 2 (3 marks)

Let

$$g : R \to R, g(x) = ax^3 + bx^2 + c \text{ where } a, b, c \in R$$

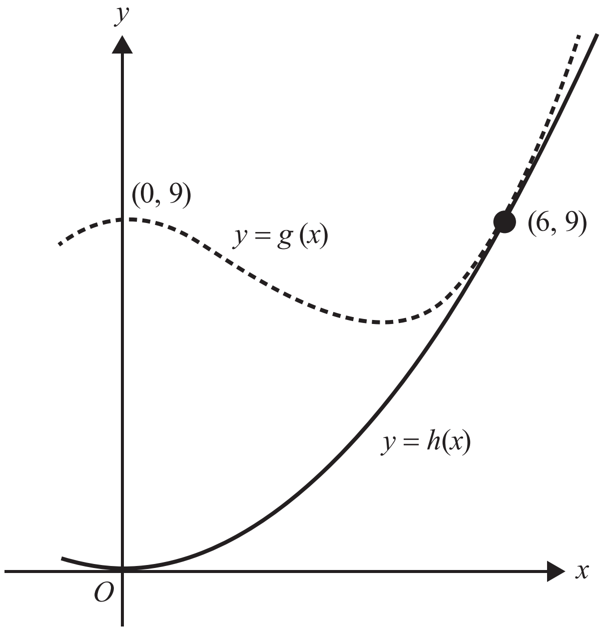
and

$$h : R \to R, h(x) = \frac{x^2}{4}$$

The graph of $y = g(x)$ passes through the point (0, 9).

The graphs of $y = g(x)$ and $y = h(x)$ pass through the point (6, 9) and have the same gradient at this point.

The graphs of $y = g(x)$ and $y = h(x)$ are shown below.



Find the values of a, b and c.

Question 3 (7 marks)

Let $f : D \rightarrow R,\ f(x) = \frac{1}{2}\tan(x) + \frac{1}{2}$, where D is the maximal domain.

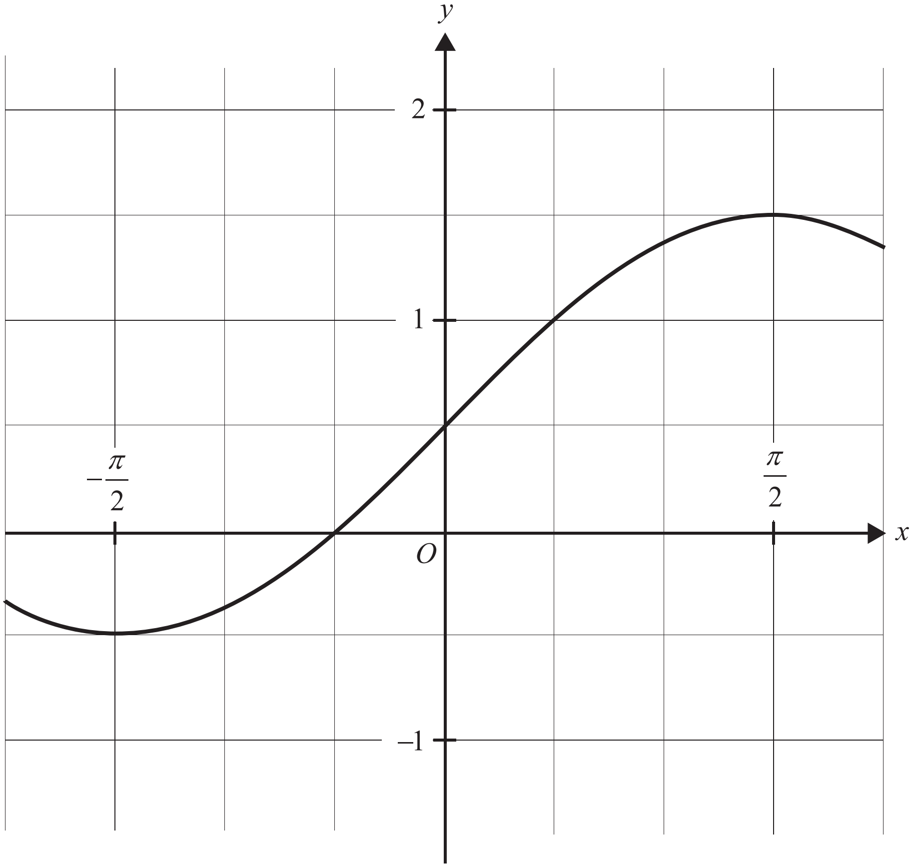
a. Find the general solution for $f(x) = 0$. 2 marks

b. Let $g : R \rightarrow R,\ g(x) = \sin(x) + \frac{1}{2}$.

i. Find all values of x such that $f(x) = g(x)$ for $x \in (-\pi, \pi)$. 2 marks

ii. The graph of $y = g(x)$ is shown below.

On the set of axes below, sketch the graph of $y = f(x)$ over the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, labelling all points where $f(x) = g(x)$ with their coordinates and all asymptotes with their equations. 3 marks



Question 4 (5 marks)

An orchard contains a large number of apples. One third of the apples are bruised.

a. In a random sample of four apples, find the probability that exactly three are bruised. 2 marks

b. Let $\hat{P}$ be the random variable representing the proportion of bruised apples in random samples of 10 apples from the orchard.

Find $\Pr\left(\hat{P} \le \frac{1}{10}\right)$.

Give your answer in the form $\frac{2^m}{3^n}$, where m and n are positive integers. 3 marks

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Question 5 (4 marks)

Let $f: R \to R, f(x) = e^{2x}$ and $g: R \to R, g(x) = e^{x} + 2$.

a. Find the x-value of the point of intersection of the graphs of $y = f(x)$ and $y = g(x)$. 2 marks

b. Find the x-value where the graphs of $y = f(x)$ and $y = g(x-1)$ have equal gradients. 2 marks

Question 6 (4 marks)

A high jumper has up to three attempts to jump over a bar at a specific height.

Once one attempt is successful, the high jumper does not make another attempt at this height.

At this height, let

- A be the event that the high jumper's first attempt is successful
- B be the event that the high jumper's second attempt is successful
- C be the event that the high jumper's third attempt is successful.

$\Pr(A) = 0.6$ and $\Pr(B|A') = 0.5$

a. Let N be a discrete random variable representing the number of attempts the high jumper makes at this height.

i. Complete the table below for the probability distribution of N. 1 mark

n	1	2	3
$\Pr(N = n)$	0.6		

ii. Find E(N), the mean number of attempts the high jumper makes at this height. 1 mark

b. The high jumper was not successful on the first attempt at this height.

Find $\Pr(C|B')$, given that the high jumper has a 0.7 probability of making a successful jump on either their second or third attempt. 2 marks

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Examination continues on the next page.

Question 7 (7 marks)

Let $f: R \to R, f(x) = x^2 + bx - 6$, where $b \in R$.

a. Find the values of b for which the two solutions of $f(x) = 0$ have a difference of 5. 2 marks

b. Let $b < 0$.

i. Explain why an inverse function does not exist when f is restricted to the domain $(0, \infty)$. 1 mark

ii. Find a rule for the inverse function of f when f is restricted to the domain $(-\infty, 0)$. 2 marks

c. Let g be a function with maximal domain and the rule $g(x) = \log_e(-x - 2)$.

Find the value of b for which the maximal domain of the composite function $g \circ f$ is $(-2, 2)$. 2 marks

Question 8 (6 marks)

Let $f: [0, \infty) \to R$, $f(x) = \dfrac{x}{2\sqrt{x+k}}$, where k is a positive real number.

a. Consider the particular case where $k = 6$.

i. Show that $\dfrac{d}{dx}\left(x\sqrt{x+6}\right) = \sqrt{x+6} + \dfrac{x}{2\sqrt{x+6}}$. 1 mark

ii. Hence, or otherwise, evaluate the definite integral $\int_0^6 f(x)\,dx$. 3 marks

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b. Find $\int_0^k f(x)\,dx$ in the form $\frac{a-\sqrt{a}}{b}k\sqrt{k}$ where a and b are positive integers. 2 marks

End of examination questions

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Mathematical Methods Examination 1

2025 Formula Sheet

You may keep this Formula Sheet.

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Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \; n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, \; x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f'\left(x_n\right)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Formula Sheet