2025
NHT

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Write your **student number** in the boxes above. **Letter**

Mathematical Methods
Examination 2

Question and Answer Book

VCE (NHT) Examination – Wednesday 28 May 2025

- Reading time is **15 minutes**: 10.30 am to 10.45 am
- Writing time is **2 hours:** 10.45 am to 12.45 pm

Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 24 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

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Section A (20 questions, 20 marks)	2–9
Section B (5 questions, 60 marks)	10–23

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Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

If $x - a$ is a factor of $3x^3 - 5x^2 - ax$, where $a \in R\backslash\{0\}$, then the value of a is

A. -2

B. $-\frac{4}{3}$

C. $\frac{4}{3}$

D. 2

Question 2

Suppose that $f: R \to R$ and $g: R \to R$ are functions such that $f'(x) = g'(x) - 2$, $f(0) = 1$ and $g(0) = 2$. Then

A. $g(x) = f'(x) + 2$

B. $g(x) = f(x) + 2x + 1$

C. $g(x) = 3$

D. $g(x) = f(x) + 2$

Question 3

If $\int_a^b f(x)\,dx = 3$, then $\int_b^a \left(2f(x) + 3\right)dx$ is equal to

A. $3(b - a + 2)$

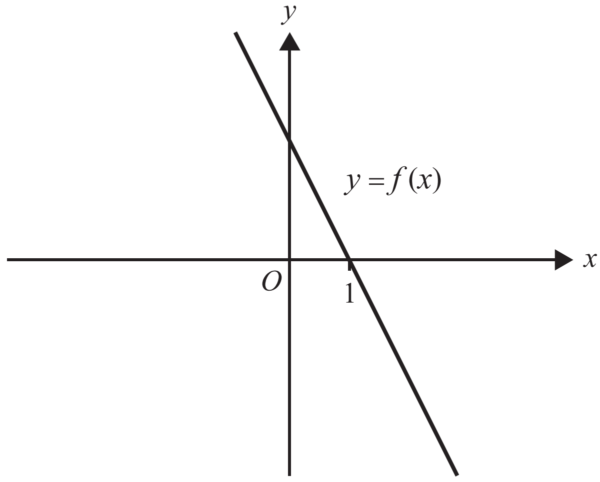
B. $3(b - a - 2)$

C. $3(a - b + 2)$

D. $3(a - b - 2)$

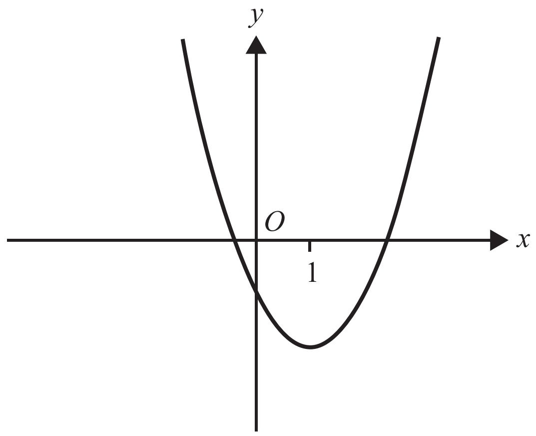
Question 4

The graph of $y = f(x)$ is shown below.

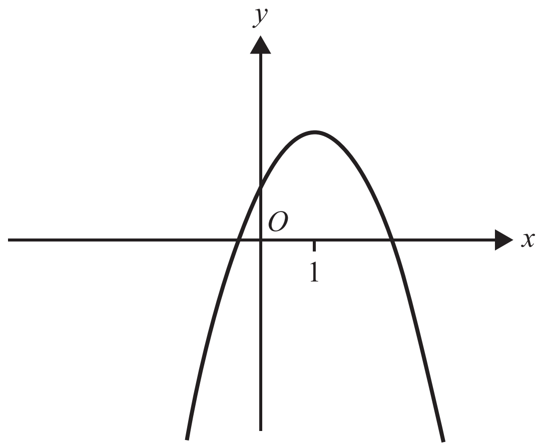


Which option shows the graph of $y = F(x)$, where F is an anti-derivative of f?

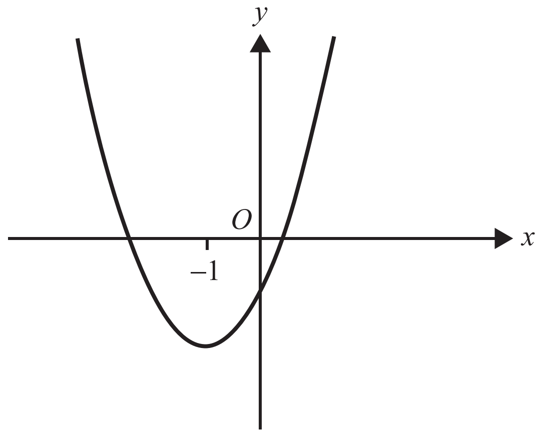
A.



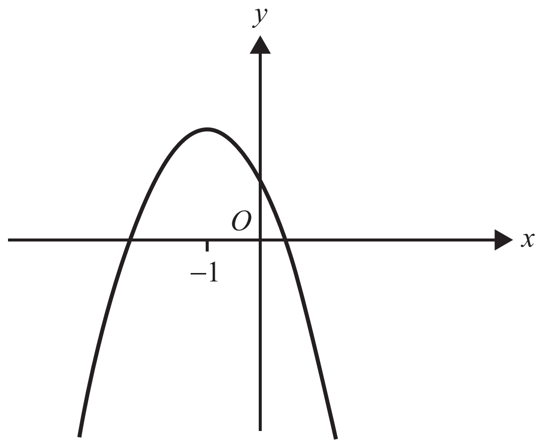
B.



C.



D.



Question 5

The scores of a national standardised test are approximately normally distributed with mean of 120 and standard deviation of 15. Andrew scored 117.9 and Chloe's score is 1.2 standard deviations above the mean. The probability that a randomly selected participant achieved a score between Andrew and Chloe is closest to

A. 0.33

B. 0.39

C. 0.44

D. 0.87

Question 6

A biased coin has a probability of 0.4 of showing heads. The coin is flipped 10 times.

The probability of the coin showing heads at most twice is closest to

A. 0.12

B. 0.13

C. 0.16

D. 0.17

Question 7

Let $g : [-1.1, 0] \to R$, $g(x) = x^5 - 3x^4 + 2x^3 - 6x + 2$.

The range of g, correct to two decimal places, is

A. $[0.00, 2.00]$

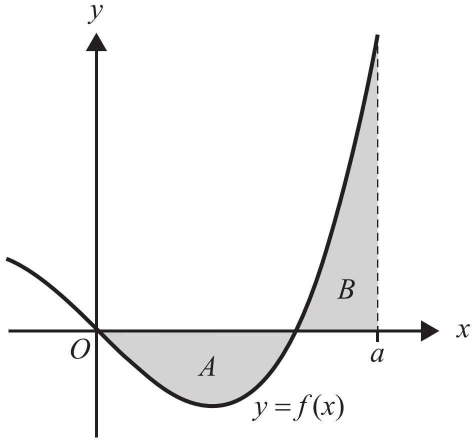
B. $[-0.06, 4.71]$

C. $[0.00, 4.71]$

D. $[-0.06, 2.00]$

Question 8

The graph of $y = f(x)$ is shown below, where $f: R \to R, f(x) = x^3 - x$.



The area of the region marked A is the same as the area of the region marked B.

Given that $a > 0$, the value of a is

A. $\frac{1}{4}$

B. 1

C. $\sqrt{2}$

D. $\sqrt{3}$

Question 9

Given that the graph of $y = g(x)$ has a local minimum at $(2, 1)$, the graph of $y = g(-2x)$ must have

A. a local minimum at $(-1, 1)$.

B. a local minimum at $(-4, 1)$.

C. a local maximum at $(-4, 1)$.

D. a local maximum at $(2, -2)$.

Question 10

Let $f(x) = e^{2x+3}$, $g(x) = x^2 + 2x - 3$, and $h(x) = g\left(f(x)\right)$.

Then $h'(x)$ is equal to

A. $4f(x)\left(f(x)+1\right)$

B. $g'\left(f(x)\right)$

C. $16f(2x)f(x)$

D. $4g(x) + 4f(x)$

Question 11

Consider the functions $f : (0, \infty) \to R$, $f(x) = \log_e(x)$ and $g : R \to R$, $g(x) = \sin(x) + 2$.

The range of the function $f(g(x))$ is

A. $(0, \infty)$

B. $[0, 1]$

C. $[0, \log_e(3)]$

D. $[1, 3]$

Question 12

A game consists of rolling a fair six-sided die. Points are scored according to the following table.

Number rolled	Points
1 or 2	6
3, 4 or 5	–3
6	9

Let X be the random variable representing the number of points scored per roll.

The mean E(X) is

A. 2

B. 3

C. 4

D. 5

Question 13

Let $f(x) = e^{2x+4}$, where $x \in \left(0, \frac{1}{2}\right)$.

The inverse of f is

A. $f^{-1}(x) = \frac{1}{2}\log_e(x) - 2$, where $x \in (0, \infty)$

B. $f^{-1}(x) = 2\log_e(x) + 4$, where $x \in (0, \infty)$

C. $f^{-1}(x) = \frac{1}{2}\log_e(x) - 4$, where $x \in (e^4, e^5)$

D. $f^{-1}(x) = \frac{1}{2}\log_e(x) - 2$, where $x \in (e^4, e^5)$

Question 14

Consider the functions $f(x) = \log_e(x^2 + 2x - 3)$ and $g(x) = \sqrt{2x+7}$, each defined over its maximal domain.

The maximal domain of the product function $h(x) = f(x)\,g(x)$ is

A. $(-\infty, -3) \cup (1, \infty)$

B. $\left(1, \frac{7}{2}\right)$

C. $\left[-\frac{7}{2}, \infty\right)$

D. $\left[-\frac{7}{2}, -3\right) \cup (1, \infty)$

Question 15

The function g is differentiable over the domain R and satisfies $g(2) = g(4) = a$ and $g'(2) = g'(4) = b$.

Given that $p(x) = g(x)\,g(2x)$, then $p'(2)$ must be equal to

A. ab

B. $2ab$

C. $3ab$

D. $a^2 + b^2$

Question 16

Let X be a normal random variable that satisfies $\Pr(X > 6) = \Pr(X < 8) = \Pr(Z < 0.5)$, where $Z \sim \mathrm{N}(0, 1)$ is the standard normal random variable.

The standard deviation of X is

A. 0.5

B. 1

C. 2

D. 4

Question 17

In a set of 10 products, six products are defective. Two products are randomly selected, without replacement, from the set of 10 products. It is known that at most one of the two selected products is defective.

What is the probability that exactly one of the two selected products is defective?

A. $\frac{4}{5}$

B. $\frac{2}{3}$

C. $\frac{8}{13}$

D. $\frac{8}{15}$

Question 18

Consider the algorithm below, which can be used to find an approximate solution to the equation $\log_e(x) - \frac{x}{3} = 0$.

```
define f(x)
    return loge(x) - x/3
a ← 4
b ← 6
for i from 1 to 3
    c ←(a + b)/2
    if -0.01 < f(c) < 0.01 then
        print c
    else if f(a) × f(c) < 0 then
        b ← c
    else
        a ← c
    end if
end for
```

Which of the following numbers would be printed by this algorithm?

A. 4.54

B. 4.5

C. 1.9

D. 1.86

N H T

Do not write in this area.

Question 19

Let B and C be discrete random variables whose values are obtained from two rolls of a fair six-sided die.

Let B be the number obtained on the first roll and C be the number obtained on the second roll.

Find the probability that the quadratic equation $x^2 + Bx + C = 0$ has **at least one** real solution.

A. $\frac{19}{36}$

B. $\frac{1}{2}$

C. $\frac{17}{36}$

D. $\frac{1}{18}$

Question 20

In the xy-plane, there is a system of two linear equations with a unique solution and another system of two linear equations with infinitely many solutions.

These four linear equations are combined into a new system of four linear equations.

All possibilities for the number of solutions of this new system will be

A. 0 or 1

B. 1 or infinitely many

C. 1

D. 0, 1 or infinitely many

End of Section A

Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (12 marks)

Consider the function $g : R \rightarrow R, g(x) = (x^2 - 1)(x + 1)^2$.

a. Find the x-value of the stationary point of inflection of the graph of $y = g(x)$. 1 mark

b. Find the coordinates of the local minimum of the graph of $y = g(x)$. 1 mark

c. Find the average rate of change of $g(x)$ between $x = 0$ and $x = 2$. 1 mark

d. Find the equations of the tangent lines to the graph of $y = g(x)$ that have a gradient of -2. 2 marks

e. Describe a sequence of two transformations that maps the graph of $y = g(x)$ to the graph of $y = g(1 - x)$. 2 marks

1.

2.

N H T

f. Let a and b be positive real numbers, such that the graph of $y = a \times g(x) + b$ has a local minimum on the x-axis.

Express b in terms of a. 1 mark

g. For each real number k, let g_k be the function $g_k : R \to R$, $g_k(x) = (x^2 - k)(x + k)^2$.

i. Find all values of k for which the graph of $y = g_k(x)$ has exactly one x-intercept. 1 mark

ii. Find all values of k for which the graph of $y = g_k(x)$ has exactly one stationary point. 3 marks

Question 2 (12 marks)

Consider the function $f: D \to R$, $f(x) = 4e^{3\log_e(x)}$, where D is the maximal domain of f.

a. Find D. 1 mark

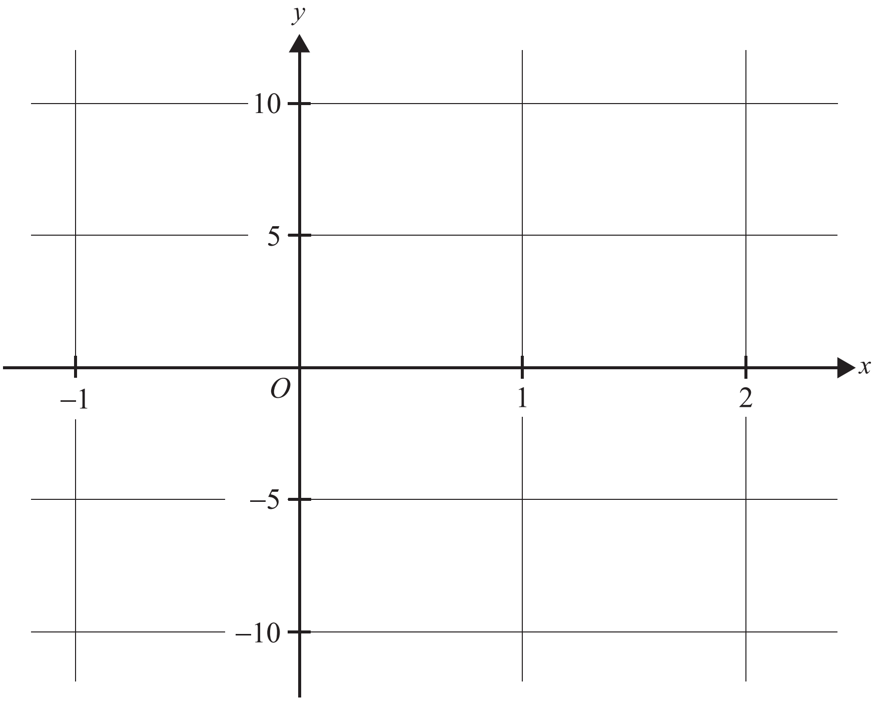
b. Show that f can be expressed in the form $f: D \to R$, $f(x) = 4x^3$. 1 mark

c. Find the area bounded by the graph of $y = f(x)$, the x-axis and the lines $x = 1$ and $x = 2$. 1 mark

d. The function g is the anti-derivative of f that passes through the point $(1, -5)$.

i. Sketch the function g on the axes below, over the domain D.

Label the coordinates of the endpoint, the x-intercept and the points on the graph where $x = 1$ and $x = 2$. 3 marks



ii. Let $a > 0.5$

The area bounded by the graph of $y = f(x)$, the x-axis and the lines $x = 0.5$ and $x = a$ can be expressed in the form $g(a) + b$.

Find the value of b. 2 marks

e. Consider the function $h : (1.5, \infty) \to R$, $h(x) = f(2x - 3) - f(x)$.

i. Find a quadratic expression for $h'(x)$. 1 mark

ii. Hence, or otherwise, find the coordinates of the point on the graph of $y = h(x)$ where the **gradient** is a minimum. 2 marks

iii. Over the interval $(1.5, p]$, the **gradient** of h is strictly decreasing.

Find the largest possible value of p. 1 mark

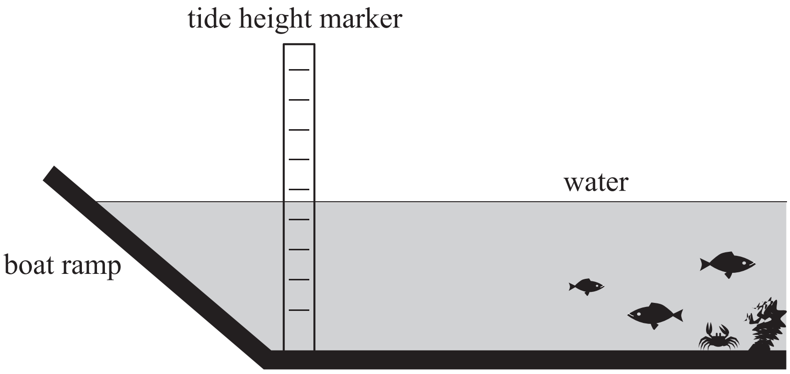
Question 3 (9 marks)

Let $h(t) = 2\sin\left(\frac{4\pi t}{25}\right) + \frac{1}{5}\sin(3\pi t) + 3$, where $t \geq 0$.

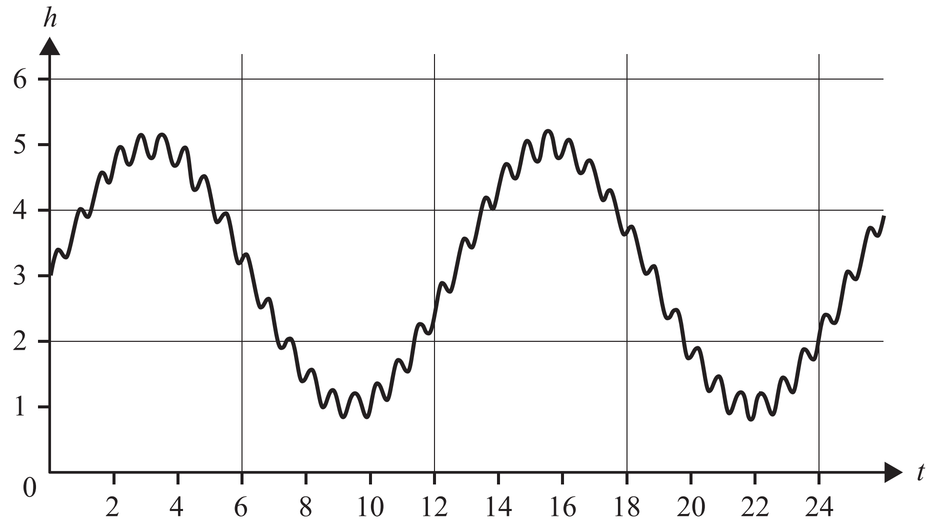
The function h models the water height, in metres, on a tide height marker at the base of a boat ramp.

The variable t represents time in hours after midnight on one particular Monday morning.

For example, $t = 6$ represents 6.00 am on this Monday morning.



Part of the graph of h is shown below.



a. Find the water height at 6.00 am on this Monday morning.

Give your answer in metres, correct to two decimal places. 1 mark

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b. Find the average height of water between 6.00 am and 6.00 pm on this Monday.

Give your answer in metres, correct to two decimal places. 2 marks

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c. A boat requires a water height of more than 1.4 metres to be safely launched off the ramp.

Between 6.00 am and 6.00 pm on this Monday, find the total amount of time when it is safe to launch this boat.

Give your answer in hours, correct to two decimal places. 2 marks

d. Find the absolute maximum height of the water on this Monday after 6.00 am, and the time when it occurs.

Give the height in metres, correct to two decimal places, and the time correct to the nearest minute. 2 marks

e. Show that $h(25k) = 3$ for all positive integers k. 1 mark

f. Find the period of the function h. 1 mark

Question 4 (13 marks)

A claim is a request to an insurance company for payment after an incident.

Company A offers car insurance. Let X be a random variable representing the claim amount, in thousands of dollars, for each claim to insurance company A.

The probability density function of X is given by

$$f(x) = \begin{cases} \dfrac{8}{9x^4} & x \geq \dfrac{2}{3} \\ 0 & \text{otherwise} \end{cases}$$

a. Find the mean claim amount, $E(X)$, in thousands of dollars. 1 mark

b. A claim of more than \$2000 (that is, $X > 2$) is classified as large.

i. Find the probability that a claim is classified as large. 1 mark

ii. Given that a claim is classified as large, find the probability that it is more than \$4000. 2 marks

Do not write in this area.

iii. In a random sample of 10 claims, find the probability that more than one claim is classified as large.

Give your answer correct to three decimal places. 2 marks

iv. A household has two cars insured with company A.

For each car, there is a 20% chance that a claim will be made next year, independent of the other car. The claim amounts are also independent.

At most one claim can be made per car next year.

Find the probability that next year in this household there is **at least one** claim classified as large (that is, $X > 2$).

Give your answer correct to four decimal places. 2 marks

Question 4 continues on the next page.

c. Let T_A be a random variable that approximates the time taken, in days, for company A to settle a claim.

Suppose T_A follows a normal distribution with a mean of 32 days and a standard deviation of 7 days.

i. Find $\Pr(T_A < 45)$.

Give your answer correct to three decimal places. 1 mark

ii. Company A settles 99.5% of claims within k days.

Find the value of k.

Give your answer correct to the nearest whole number. 1 mark

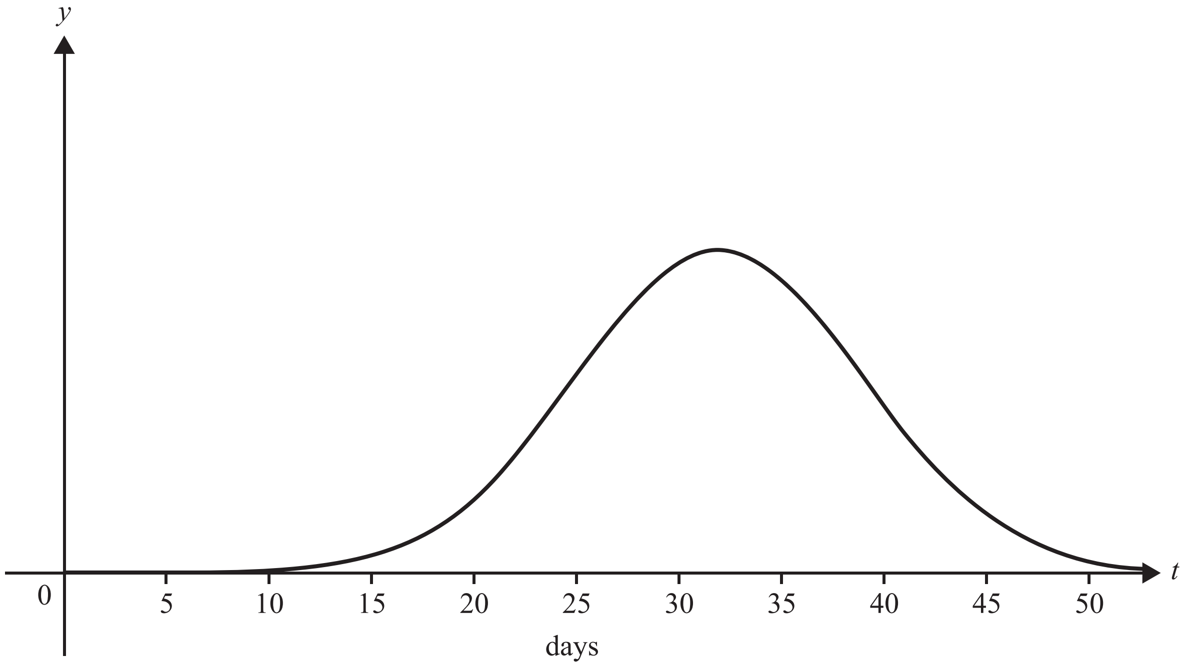
iii. Company B also offers car insurance. In a random sample of 80 claims to company B, 76 were paid.

Find the approximate 95% confidence interval for the proportion of claims paid by company B.

Give your answer correct to three decimal places. 1 mark

iv. Let T_B be a random variable that approximates the time taken, in days, for company B to settle a claim.

Suppose T_B follows a normal distribution with a mean of 25 days and a standard deviation of 5 days.



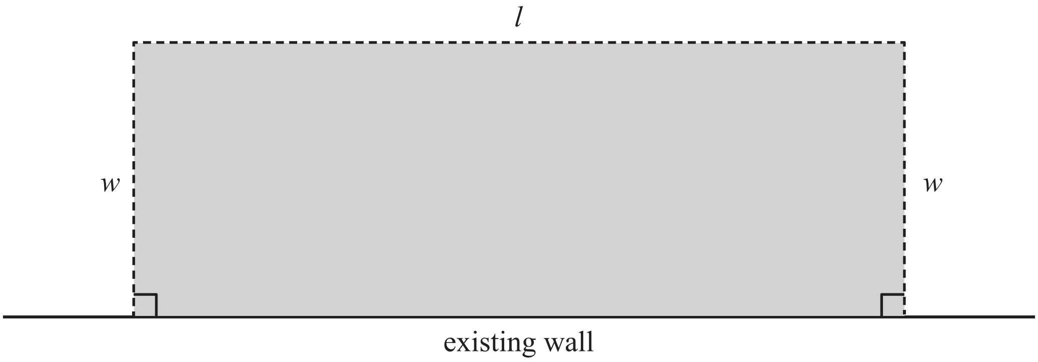
The probability density function for $T_A \sim \mathrm{N}(32, 7^2)$ is shown on the axes above.

Sketch the probability density function for $T_B \sim \mathrm{N}(25, 5^2)$ on the same set of axes. 2 marks

Question 5 (14 marks)

David is planning to build a vegetable garden in his backyard, using an existing wall for one side and up to 12 metres of timber for the remaining sides. Assume that David's backyard is modelled by a flat plane.

a. David considers building a rectangular vegetable garden as shown below, with length l metres and width w metres.



i. David uses 12 metres of timber for the remaining three sides.

Show that the area of the rectangular vegetable garden, in square metres, can be expressed as $12w - 2w^2$. 1 mark

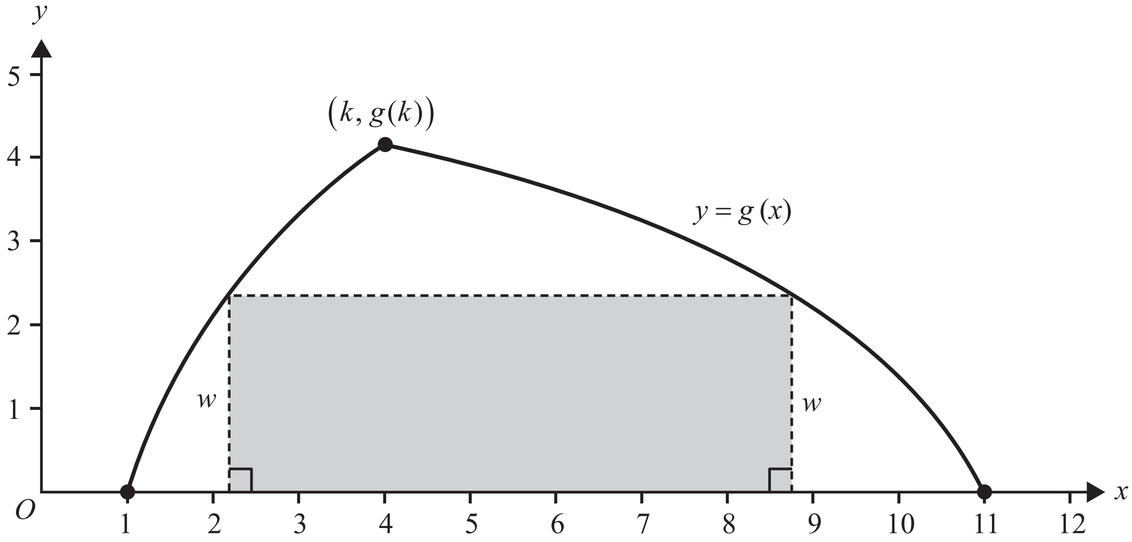
ii. Find the maximum possible area of the rectangular vegetable garden.

Give your answer in square metres. 2 marks

b. David realises that his new vegetable garden must fit within the boundary created by his garden path.

Let the existing wall be defined by the x-axis and the boundary of the garden path be defined by the continuous function g below.

$$g(x)=\begin{cases}3\log_e(x) & 1\le x\le k\\ 2\log_e(12-x) & k<x\le 11\end{cases}$$



Both x and y have units of metres.

David considers a rectangular vegetable garden, with two corners on the existing wall and two corners on the boundary of the garden path. Let w metres be the width of the rectangular vegetable garden, perpendicular to the existing wall, and $A(w)$ square metres be the area of the rectangular vegetable garden.

i. Given that g is continuous, find the value of k. 2 marks

Question 5 continues on the next page.

ii. Show that $A(w) = w\left(12 - e^{\frac{w}{2}} - e^{\frac{w}{3}}\right)$. 2 marks

iii. Hence, find the value of w that gives the maximum area of this rectangular vegetable garden, and state the maximum of $A(w)$.

Give your answers correct to two decimal places. 2 marks

iv. Can David build the rectangular vegetable garden of maximum area, as described in **part b.iii**, with less than 12 metres of timber?

Justify your answer. 1 mark

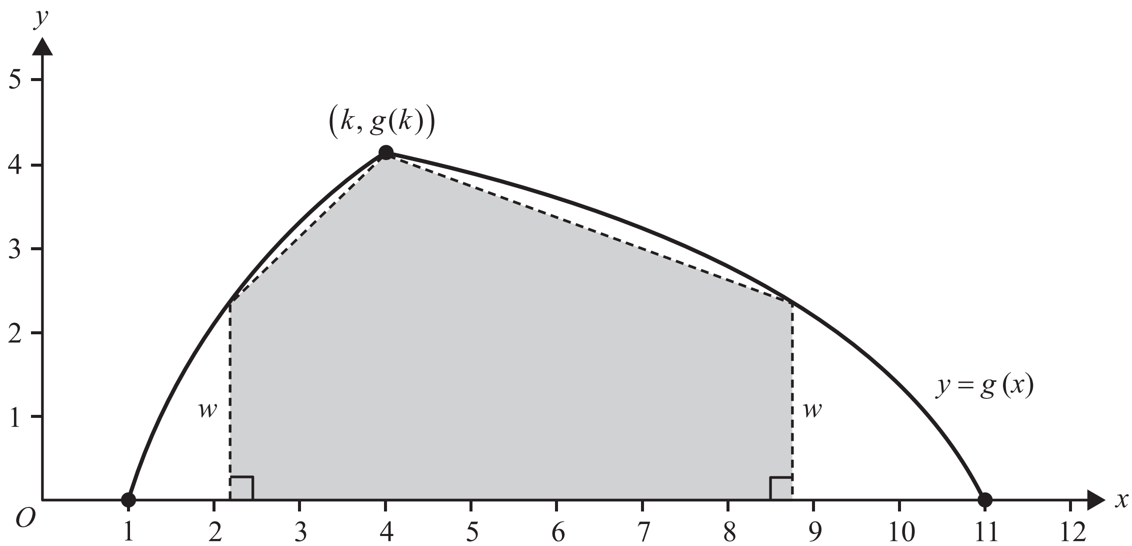
Do not write in this area.

c. Finally, David considers a pentagonal vegetable garden, using the existing wall for one side and his 12 metres of timber for the remaining four sides.

He requires two sides of equal length w metres from the existing wall to the garden path $y = g(x)$.

Both of these sides are perpendicular to the existing wall. The remaining two sides will meet at the point $(k, g(k))$.

The function g and the value k are those defined in **part b**.



Find the maximum possible area for this pentagonal vegetable garden that David can build with 12 metres of timber or less.

Give your answer in square metres, correct to two decimal places. 4 marks

End of Question and Answer Book

2025 NHT

Mathematical Methods Examination 2

2025 Formula Sheet

You may keep this Formula Sheet.

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Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f\left(x_n\right)}{f'\left(x_n\right)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Formula Sheet