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PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Mathematical Methods Examination 1

Question and Answer Book

VCE Examination – Wednesday 5 November 2025

- Reading time is **15 minutes**: 9.00 am to 9.15 am
- Writing time is **1 hour**: 9.15 am to 10.15 am

Materials supplied

- Question and Answer Book of 16 pages
- Formula Sheet

Instructions

Students are **not** permitted to bring any technology (calculators or software), or notes of any kind, into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

	pages
9 questions (40 marks)	2–13

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Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (3 marks)

a. Let $y = x^2 \cos(x)$.

Find $\frac{dy}{dx}$. 1 mark

b. Let $f(x) = 6\sqrt{x+1} + 5$.

Find the gradient of the tangent to $y = f(x)$ at $x = 8$. 2 marks

Question 2 (2 marks)

Let $g(x)$ be a function defined for $x > -\frac{3}{2}$ so that $g'(x) = \frac{1}{2x+3}$ and $g(1) = 0$.

Find $g(x)$.

Question 3 (6 marks)

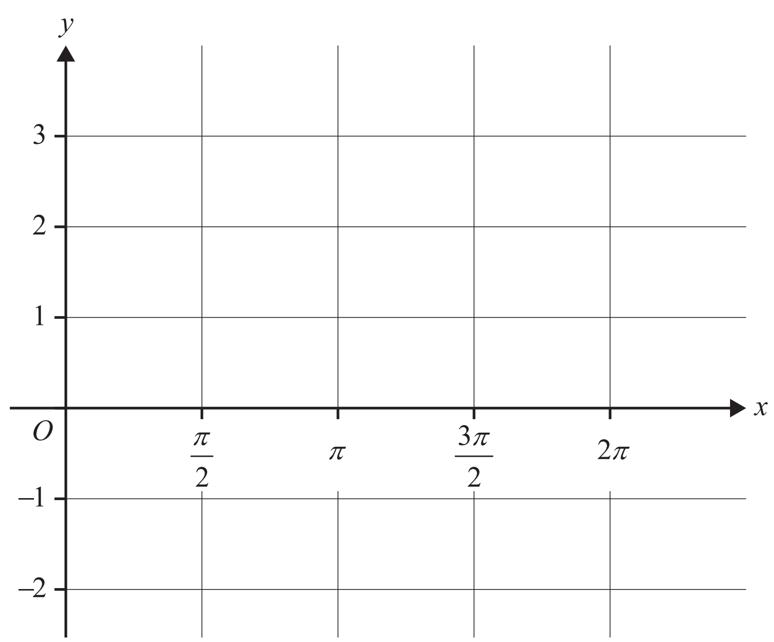
Let $f : [0, 2\pi] \rightarrow R,\ f(x) = 2\cos(2x) + 1$.

a. State the range of f. 1 mark

b. Solve $f(x) = 0$ for x. 3 marks

c. Sketch the graph of $y = f(x)$ for $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ on the axes below.

Label the endpoints with their coordinates. 2 marks



Question 4 (4 marks)

The probability distribution for the discrete random variable X is given in the table below, where k is a positive real number.

x	0	1	2	3
$\Pr(X = x)$	$\frac{4}{k}$	$\frac{2k}{75}$	$\frac{k}{75}$	$\frac{2}{k}$

a. Show that $k = 10$ or $k = 15$. 2 marks

b. Let $k = 15$.

i. Find $\Pr(X > 1)$. 1 mark

ii. Find $\mathrm{E}(X)$. 1 mark

Question 5 (4 marks)

a. Solve $e^{2x} - 8e^{x} + 7 = 0$ for x. 2 marks

b. Let $g(x) = e^{2x} - 8e^{x} + 7$, where $x \in R$.

The function $g(x)$ has exactly one stationary point, a local minimum.

Find the largest value of a such that when g is restricted to the domain $(-\infty, a]$ it has an inverse function. 2 marks

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Question 6 (3 marks)

Consider the binomial random variable $X \sim \text{Bi}\left(6, \frac{1}{4}\right)$.

a. Find $\text{var}(X)$. 1 mark

b. Determine $\Pr(X \geq 5)$.

Give your answer in the form $\frac{a}{2^b}$, where $a, b \in Z$. 2 marks

Question 7 (6 marks)

Let $f: R \to R,\ f(x) = x^3 - x^2 - 16x - 20$.

a. Verify that $x = 5$ is a solution of $f(x) = 0$. 1 mark

b. Express $f(x)$ in the form $(x+d)^2(x-5)$, where $d \in R$. 2 marks

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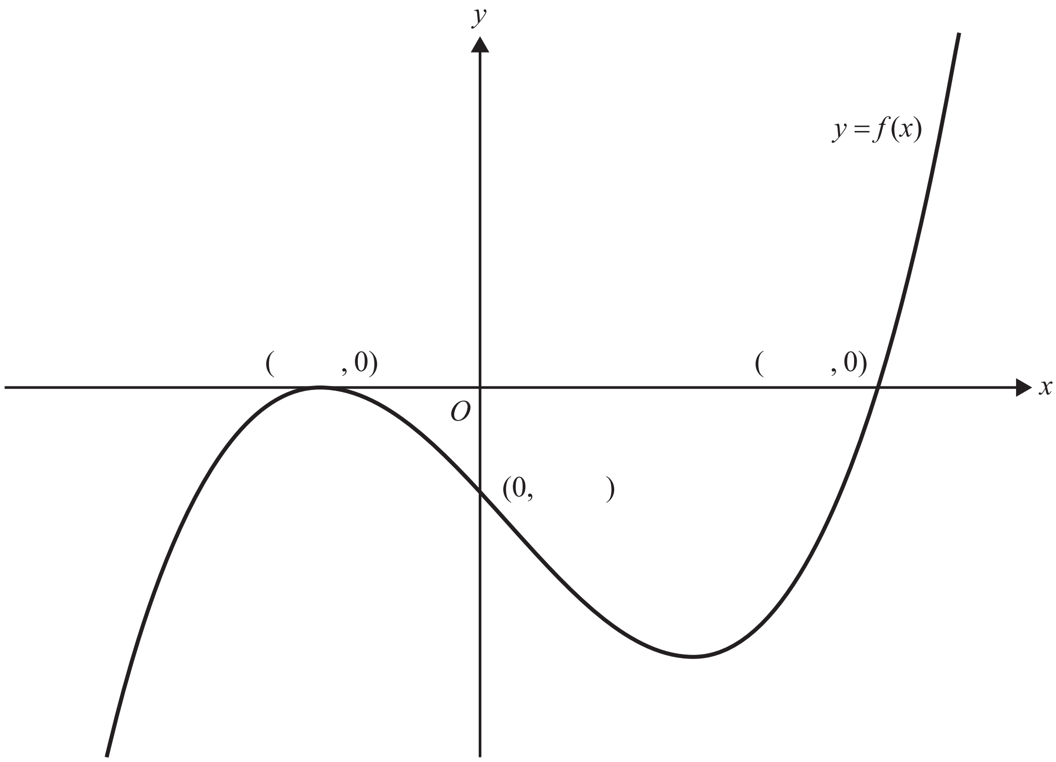
c. Consider the graph of $y = f(x)$, as shown below.

Complete the coordinate pairs of all axial intercepts of $y = f(x)$. 1 mark



d. Let $g : R \to R,\ g(x) = x + 2$.

i. State the coordinates of the stationary point of inflection for the graph of $y = f(x)g(x)$. 1 mark

ii. Write down the values of x for which $f(x)g(x) \geq 0$. 1 mark

Question 8 (5 marks)

Consider

$$f(x)=\begin{cases}\frac{3}{8}(4-3x) & 0\le x\le \frac{4}{3}\\ 0 & \text{otherwise}\end{cases}$$

a. The continuous random variable X has probability density function $f(x)$.

Find k such that $\Pr(X>k)=\frac{9}{16}$. 3 marks

b. The function $h(x)$ is a transformation of $f(x)$ such that

$$h(x)=mf(x)+n$$

where m and n are real numbers.

Find $\int_0^{\frac{4}{3}} h(x)\,dx$ in terms of m and n. 2 marks

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Examination continues on the next page.

Question 9 (7 marks)

Consider the functions

$$f: R \setminus \{1\} \to R,\ f(x) = \frac{w^2}{(x-1)^2}$$

and

$$g: R \to R,\ g(x) = (x-w)^2$$

where $w \in R$.

a. If $w = -3$, find the four solutions to $f(x) = g(x)$. 3 marks

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b. Consider the case where $w > 0$.

i. Find, in terms of w, the coordinates of the minimum point of the graph of $y = (x-1)(x-w)$. 2 marks

ii. Hence, or otherwise, find the positive values of w for which $f(x) = g(x)$ has exactly three solutions. 2 marks

End of examination questions

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Mathematical Methods
Examination 1

2025 Formula Sheet

You may keep this Formula Sheet.

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Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left((ax+b)^n\right) = an\left(ax+b\right)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c,\ x > 0$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n)\right]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		**Mean**	**Variance**
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
binomial	$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Formula Sheet