

SUPERVISOR TO ATTACH
PROCESSING LABEL HERE

--	--	--	--	--	--	--	--	--

Write your **student number** in the boxes above.

Letter

Specialist Mathematics Examination 1

Question and Answer Book

VCE Examination – Friday 7 November 2025

- Reading time is **15 minutes**: 9.00 am to 9.15 am
- Writing time is **1 hour**: 9.15 am to 10.15 am

Materials supplied

- Question and Answer Book of 12 pages
- Formula Sheet

Students are **not** permitted to bring any technology (calculators or software) or notes of any kind into the examination room.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents	pages
9 questions (40 marks)	2–11

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required for each question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude $g \text{ m s}^{-2}$, where $g = 9.8$

Question 1 (4 marks)

Consider the curve with equation $xe^{-2y} + y^2e^x = 8e^4$.

Find the equation of the tangent to the curve at the point $(4, -2)$.

Question 3 (5 marks)

A particle starts from rest at a fixed point O and travels in a straight line.

The velocity, $v \text{ m s}^{-1}$, of the particle at time t seconds has equation $v(t) = \frac{t}{\sqrt{t^2 + k}}$,
where k is a positive constant and $t \geq 0$.

- a.** Use integration to show that the displacement, x metres, of the particle relative to O is given by $x(t) = \sqrt{t^2 + k} - \sqrt{k}$.

1 mark

- b.** Find the initial acceleration, in terms of k , of the particle in m s^{-2} .

2 marks

- c.** Another particle starts at O at the same time as the first particle and follows the same path.

Its position relative to O is described by the equation $s(t) = t$.

Three seconds after leaving O the second particle is 1 m ahead of the first particle.

Find the value of k .

2 marks

Question 5 (4 marks)

The position vectors of particles P and Q at time t seconds are given by

$$\underline{r}_P(t) = (t^3 + at^2)\underline{i} - t\underline{j} \text{ and } \underline{r}_Q(t) = (bt + 2t)\underline{i} + (2t^2 + ct + t)\underline{j},$$

where $t \geq 0$ and $a, b, c \in R$.

The particles collide when $t = 1$.

- a. Show that for collision to occur when $t = 1$, the value of c is -4 .

1 mark

When the particles collide, their velocities are at right angles to each other.

- b. Find the two possible values of a for collision to occur when $t = 1$.

2 marks

- c. When the particles collide at $t = 1$, the magnitudes of their accelerations are equal.

Find the values of a and b for collision to occur when $t = 1$.

1 mark

Question 6 (4 marks)

Find the volume of the solid of revolution formed when the area between the curve

$$y = \sqrt{\frac{\arctan(x)}{1+x^2}}$$
 and the x -axis from $x = 1$ to $x = \sqrt{3}$ is rotated about the x -axis.

Give your answer in the form $\frac{a\pi^b}{c}$, where $a, b, c \in \mathbb{Z}^+$.

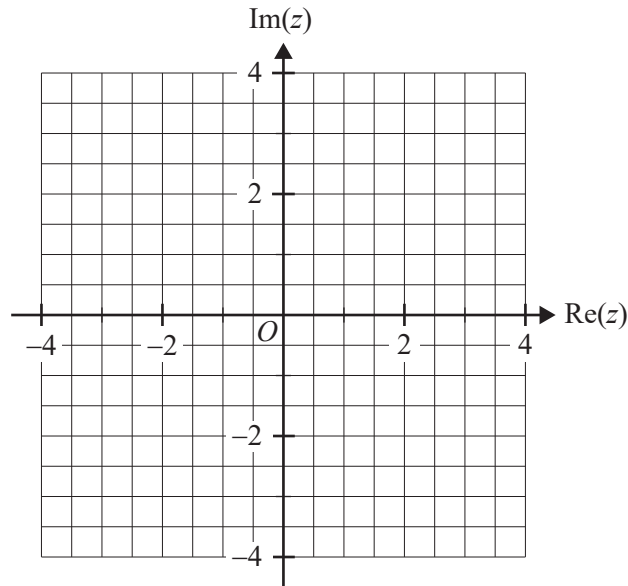
Question 8 (5 marks)

Consider the function with rule $f(z) = z^4 + 6z^2 + 25$, where $z \in C$.

a. Consider $z_1 = 1 + 2i$.

Plot and label z_1 and \bar{z}_1 on the Argand plane below.

1 mark



b. Given that $1 + 2i$ is a solution of $f(z) = 0$, find a quadratic factor of $f(z)$.

2 marks

c. Hence, find all remaining solutions of $f(z) = 0$.

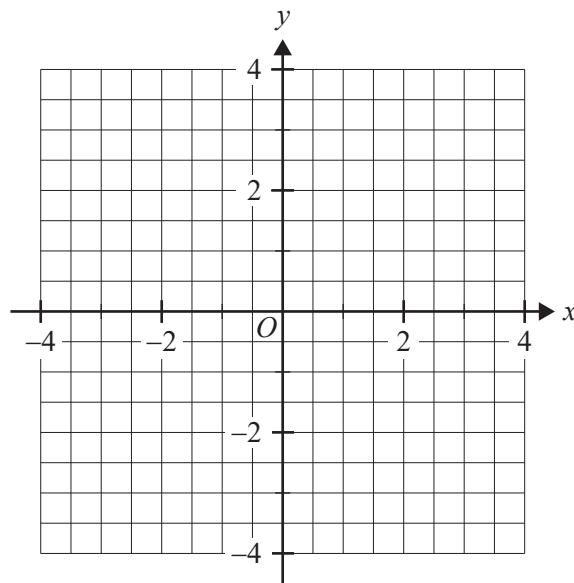
2 marks

Do not write in this area.


c. Sketch the graph of $y = f(x)$ on the axes below.

Label the asymptotes with their equations.

3 marks



© Victorian Curriculum and Assessment Authority 2025



Specialist Mathematics Examination 1

2025 Formula Sheet

You may keep this Formula Sheet.

Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab \cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$	$ z = \sqrt{x^2 + y^2} = r$	
$-\pi < \operatorname{Arg}(z) \leq \pi$	$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables X_1, X_2, \dots, X_n	$E(aX_1 + b) = a E(X_1) + b$ $E(a_1X_1 + a_2X_2 + \dots + a_nX_n)$ $= a_1E(X_1) + a_2E(X_2) + \dots + a_nE(X_n)$	
	$\text{Var}(aX_1 + b) = a^2\text{Var}(X_1)$ $\text{Var}(a_1X_1 + a_2X_2 + \dots + a_nX_n)$ $= a_1^2\text{Var}(X_1) + a_2^2\text{Var}(X_2) + \dots + a_n^2\text{Var}(X_n)$	
for independent identically distributed variables X_1, X_2, \dots, X_n	$E(X_1 + X_2 + \dots + X_n) = n\mu$	
	$\text{Var}(X_1 + X_2 + \dots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$	
distribution of sample mean \bar{X}	mean	$E(\bar{X}) = \mu$
	variance	$\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$	$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$
$\frac{d}{dx}(\cot(ax)) = -a \operatorname{cosec}^2(ax)$	$\int \operatorname{cosec}^2(ax) dx = -\frac{1}{a} \cot(ax) + c$
$\frac{d}{dx}(\sec(ax)) = a \sec(ax) \tan(ax)$	$\int \sec(ax) \tan(ax) dx = \frac{1}{a} \sec(ax) + c$
$\frac{d}{dx}(\operatorname{cosec}(ax)) = -a \operatorname{cosec}(ax) \cot(ax)$	$\int \operatorname{cosec}(ax) \cot(ax) dx = -\frac{1}{a} \operatorname{cosec}(ax) + c$
$\frac{d}{dx}(\sin^{-1}(ax)) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\cos^{-1}(ax)) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}(\tan^{-1}(ax)) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
	$\int \frac{1}{ax+b} dx = \frac{1}{a} \log_e ax+b + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$
Euler's method	<p>If $\frac{dy}{dx} = f(x, y)$, $x_0 = a$ and $y_0 = b$,</p> <p>then $x_{n+1} = x_n + h$ and</p> <p>$y_{n+1} = y_n + h \times f(x_n, y_n)$</p>
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
surface area Cartesian about x -axis	$\int_{x_1}^{x_2} 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$
surface area Cartesian about y -axis	$\int_{y_1}^{y_2} 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$
surface area parametric about x -axis	$\int_{t_1}^{t_2} 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
surface area parametric about y -axis	$\int_{t_1}^{t_2} 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2} at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underline{r}(t) = x(t)\underline{i} + y(t)\underline{j} + z(t)\underline{k}$	$ \underline{r}(t) = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underline{r}}(t) = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j} + \frac{dz}{dt}\underline{k}$
for $\underline{r}_1 = x_1\underline{i} + y_1\underline{j} + z_1\underline{k}$ and $\underline{r}_2 = x_2\underline{i} + y_2\underline{j} + z_2\underline{k}$	vector scalar product $\underline{r}_1 \cdot \underline{r}_2 = \underline{r}_1 \underline{r}_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underline{r}_1 \times \underline{r}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underline{i} + (x_2z_1 - x_1z_2)\underline{j} + (x_1y_2 - x_2y_1)\underline{k}$
vector equation of a line	$\underline{r}(t) = \underline{r}_1 + t\underline{r}_2 = (x_1 + x_2t)\underline{i} + (y_1 + y_2t)\underline{j} + (z_1 + z_2t)\underline{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \quad y(t) = y_1 + y_2t \quad z(t) = z_1 + z_2t$
vector equation of a plane	$\underline{r}(s, t) = \underline{r}_0 + s\underline{r}_1 + t\underline{r}_2$ $= (x_0 + x_1s + x_2t)\underline{i} + (y_0 + y_1s + y_2t)\underline{j} + (z_0 + z_1s + z_2t)\underline{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t, \quad y(s, t) = y_0 + y_1s + y_2t, \quad z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x + y) = \sin(x) \cos(y) + \cos(x) \sin(y)$	$\sin(x - y) = \sin(x) \cos(y) - \cos(x) \sin(y)$
$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y)$	$\cos(x - y) = \cos(x) \cos(y) + \sin(x) \sin(y)$
$\tan(x + y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$	$\tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$
$\sin(2x) = 2 \sin(x) \cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$	$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \frac{1}{2}(1 - \cos(2ax))$	$\cos^2(ax) = \frac{1}{2}(1 + \cos(2ax))$

© Victorian Curriculum and Assessment Authority 2025