2026
NHT

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PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Specialist Mathematics Examination 2

Question and Answer Book

VCE (NHT) Examination – Friday 22 May 2026

- Reading time is **15 minutes**: 2.00 pm to 2.15 pm
- Writing time is **2 hours**: 2.15 pm to 4.15 pm

Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 24 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

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Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1

Consider the following statement.

'If the temperature is 35 °C or more, then the students do not play sport.'

The contrapositive of this statement is

A. if the temperature is 36 °C, then the students do not play sport.

B. if the students do not play sport, then the temperature is 35 °C or more.

C. if the temperature is less than 35 °C, then the students play sport.

D. if the students play sport, then the temperature is less than 35 °C.

Question 2

A function with domain $x \in [0, \infty)$ has the following rule.

$$f(x) = \begin{cases} \dfrac{(\sqrt{x} - 2)\arctan\left(\dfrac{x}{4}\right)}{x^2 - 3x - 4}, & x \in [0, \infty) \setminus \{4\} \\ p, & x = 4 \end{cases}$$

For the function to be continuous on the interval $[0, \infty)$, the value of p must be

A. $\dfrac{9}{4}$

B. $\dfrac{\pi}{80}$

C. $\dfrac{\pi}{40}$

D. 1

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Question 3

Consider the following algorithm to estimate the area of a certain surface of revolution.

```
define f(x)
  return √(x² + 1)
sum ← 0
left ← 0
while left ≤ 1
  area ← 2π × f((left + (left + 1))/2) × √((f(left) − f(left + 1))² + 1)
  sum ← sum + area
  left ← left + 1
end while
print sum
```

The algorithm above will print a value closest to

A. 7.604

B. 14.662

C. 22.265

D. 24.765

Question 4

For $m > 1$, how many vertical asymptotes does the graph of $f(x) = \dfrac{1}{x^3 - x^2 - m^2x + m^2}$ have?

A. 0

B. 1

C. 2

D. 3

Question 5

Consider the circle given by $|z - 2 - i| = 3$, where $z \in C$.

The maximum value of $|z|$ for those points that lie on this circle is

A. $2 + 2\sqrt{2}$

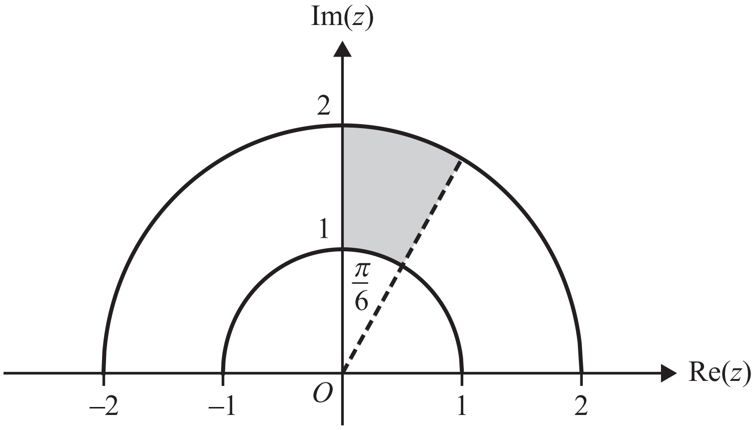
B. $3 + \sqrt{5}$

C. $1 + \sqrt{5}$

D. $3 - \sqrt{5}$

Question 6

Consider the Argand diagram below.



The shaded region is best described by

A. $\{z: 1 < |z| \le 2\} \cap \left\{z: \frac{\pi}{3} \le \arg(z) < \frac{\pi}{2}\right\}$

B. $\{z: 1 < |z| < 2\} \cap \left\{z: \frac{\pi}{6} \le \arg(z) \le \frac{\pi}{3}\right\}$

C. $\{z: 1 \le |z| \le 2\} \cap \left\{z: \frac{\pi}{6} < \arg(z) \le \frac{\pi}{2}\right\}$

D. $\{z: 1 \le |z| \le 2\} \cap \left\{z: \frac{\pi}{3} < \arg(z) \le \frac{\pi}{2}\right\}$

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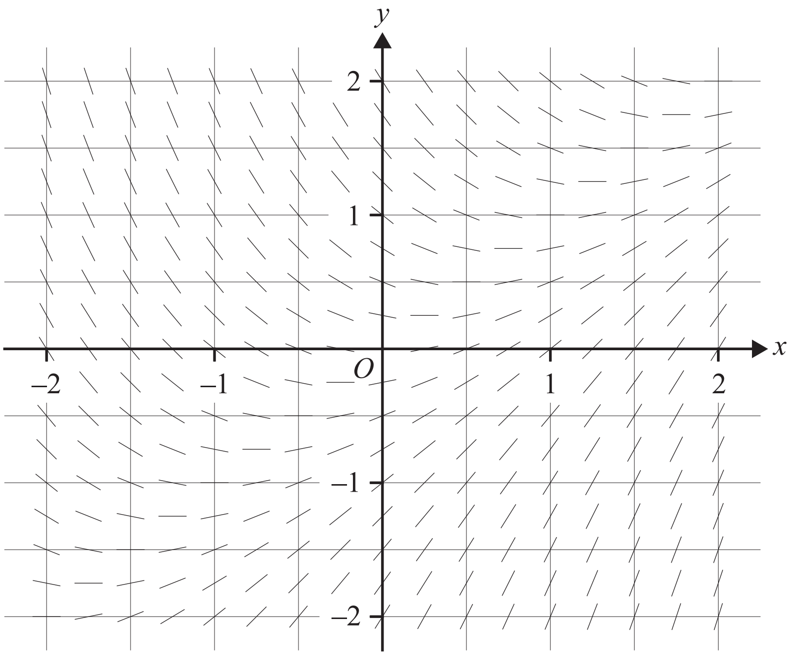
Question 7



The direction field above best represents the differential equation

A. $\frac{dy}{dx} = y - x$

B. $\frac{dy}{dx} = x - y$

C. $\frac{dy}{dx} = \frac{1}{x - y}$

D. $\frac{dy}{dx} = \frac{1}{y - x}$

Question 8

A curve is described parametrically by $x = \sin(t)$, $y = \sec(t)$, where $-\frac{\pi}{2} < t < \frac{\pi}{2}$.

The exact value of t, where $\frac{dy}{dx} = 4\sqrt{3}$, is

A. $\frac{\pi}{6}$

B. 1.047

C. $\frac{\pi}{3}$

D. $\tan^{-1}\left(4\sqrt{3}\right)$

Question 9

Using the substitution $x^3 - 1 = u^2$, $u \geq 0$, the definite integral $\int_1^2 \frac{\sqrt{x^3-1}}{x}\,dx$ can be expressed as

A. $\int_0^{\sqrt{7}} \frac{2}{3}\left(1 - \frac{1}{1+u^2}\right) du$

B. $\int_0^{\sqrt{7}} \frac{1}{3}\left(\frac{u}{1+u^2}\right) du$

C. $\int_1^{2} \frac{3}{2}\left(1 + \frac{1}{1-u^2}\right) du$

D. $\int_1^{2} \frac{2}{3}\left(1 + \frac{1}{1+u^2}\right) du$

Question 10

The curve defined by the parametric equations $x = 2t - \sin(2t)$, $y = 1 - \cos(2t)$ for $0 \leq t \leq \pi$ is rotated about the x-axis to form a surface of revolution.

The area of this surface is given by

A. $16\pi \int_0^{\pi} \cos^3(t)\,dt$

B. $16\pi \int_0^{\pi} \sin^3(t)\,dt$

C. $64\pi \int_0^{\pi} \sin^4(t)\,dt$

D. $64\pi \int_0^{\pi} \cos^4(t)\,dt$

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Question 11

By separation of variables, the differential equation $\dfrac{dy}{dx} = \dfrac{e^{x+y}\sin(2x)}{\sin(x+y) - \sin(x)\cos(y)}$ leads to

A. $\int e^{-y}\sin(y)\,dy = 2\int e^{x}\sin(x)\,dx$

B. $\int e^{-y}\cos(y)\,dy = \int e^{x}\cos(x)\,dx$

C. $\int e^{y}\cos(y)\,dy = 2\int e^{-x}\cos(x)\,dx$

D. $\int e^{y}\sin(y)\,dy = \int e^{-x}\sin(x)\,dx$

Question 12

The growth of a population of emus in a certain region is given by the logistic differential equation $\dfrac{dP}{dt} = 0.02P\left(1 - \dfrac{P}{1000}\right)$, where P models the number of emus after t years.

The initial population of emus is 50.

The time taken, in years, for the number of emus to grow to 500 is given by

A. $\int_{50}^{500} 0.02P\left(1 - \dfrac{P}{1000}\right)dP$

B. $\int_{0}^{500} \dfrac{50}{P} + \dfrac{50}{1000 - P}\,dP$

C. $\int_{0}^{500} 0.02P\left(1 - \dfrac{P}{1000}\right)dP$

D. $\int_{50}^{500} \dfrac{50}{P} + \dfrac{50}{1000 - P}\,dP$

Question 13

A particle moves along a straight line. The position of the particle from a fixed point is given by $x(t) = e^{-t}\sin(t)$ for $0 \le t \le 3\pi$.

The number of times the particle changes direction and the duration of time between successive changes in direction are, respectively,

A. $2, \pi$

B. $2, 2\pi$

C. $3, \pi$

D. $3, 2\pi$

Question 14

A particle moves in a straight line such that its acceleration, a m s^{-2}, is given by $a = 2 + v$, where v m s^{-1} is the velocity of the particle. At the origin, the velocity of the particle is 4 m s^{-1}.

When the velocity of the particle is 6 m s^{-1}, its displacement from the origin, in metres, will be

A. $6 + 2\log_e(4)$

B. $6 - 2\log_e(4)$

C. $2 - 2\log_e\left(\frac{3}{4}\right)$

D. $2 + 2\log_e\left(\frac{3}{4}\right)$

Question 15

Let $\underset{\sim}{u} = a\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$, where $a \in R$, and $\underset{\sim}{v} = 2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$.

If the scalar resolute of $\underset{\sim}{u}$ in the direction of $\underset{\sim}{v}$ is 2, then the value of a is

A. $\frac{2}{5}$

B. $\frac{1}{2}$

C. $\frac{5}{2}$

D. $\frac{17}{2}$

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Question 16

Consider vectors $\underset{\sim}{a} = p\underset{\sim}{i} - \underset{\sim}{j} + 4\underset{\sim}{k}$, $\underset{\sim}{b} = -2\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = -3\underset{\sim}{i} - 4\underset{\sim}{k}$, where p is a positive integer.

Given $\tan\theta = \sqrt{3}$, where θ is the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b} - \underset{\sim}{c}$, the value of p is

A. 4

B. 5

C. 6

D. 7

Question 17

Two objects have velocities given by $\dot{\underset{\sim}{r}}(t) = \underset{\sim}{i} + \cos(t)\underset{\sim}{j} + \sin(t)\underset{\sim}{k}$ and $\dot{\underset{\sim}{s}}(t) = \underset{\sim}{i} + 2\cos(2t)\underset{\sim}{j} - \sin(t)\underset{\sim}{k}$, where components are measured in m s^{-1}.

Given $\underset{\sim}{r}(0) = \underset{\sim}{i}$ and $\underset{\sim}{s}(0) = -\underset{\sim}{k}$, the distance, in metres, between the two objects when $t = 2\pi$ seconds is

A. 1

B. $\sqrt{2}$

C. 2

D. $\sqrt{5}$

Question 18

The angle between the planes given by $-x + \sqrt{2}y - z = 2$ and $-x - \sqrt{2}y + z = 4$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{6}$

Question 19

Which one of the following vector equations describes the line of intersection of the planes given by $x - 2y + z = 3$ and $2x + y - z = 5$?

A. $\underset{\sim}{r} = 2\underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k} + s(\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}),\ s \in R$

B. $\underset{\sim}{r} = 3\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k} + s(\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}),\ s \in R$

C. $\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k} + s(\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}),\ s \in R$

D. $\underset{\sim}{r} = \underset{\sim}{i} - 5\underset{\sim}{j} - 8\underset{\sim}{k} + s(\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}),\ s \in R$

Question 20

Let X and Y be independent normal random variables with $\mathrm{E}(X) = 15$, $\mathrm{E}(Y) = 25$, $\mathrm{Var}(X) = 16$ and $\mathrm{Var}(Y) = 9$.

Let Q be the random variable such that $Q = cX + 2Y$.

Given that $\Pr(Q > 112.2) = 0.1$, the value of c is closest to

A. 1

B. 2

C. 3

D. 4

End of Section A

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Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required for each question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 (11 marks)

Consider the function f with rule $f(x) = \dfrac{x^3 - 2x^2 + 8}{2(4 - x^2)}$.

a. Express $f(x)$ in the form $Ax + B + \dfrac{Cx}{4 - x^2}$, where $A, B, C \in R$. 1 mark

b. State the equations of the straight-line asymptotes of the graph of f. 1 mark

c. Find $f'(x)$. 1 mark

Question 1 continues on the next page.

d. **i.** State the coordinates of any turning points of the graph of f. 1 mark

ii. The graph of f has a stationary point of inflection.

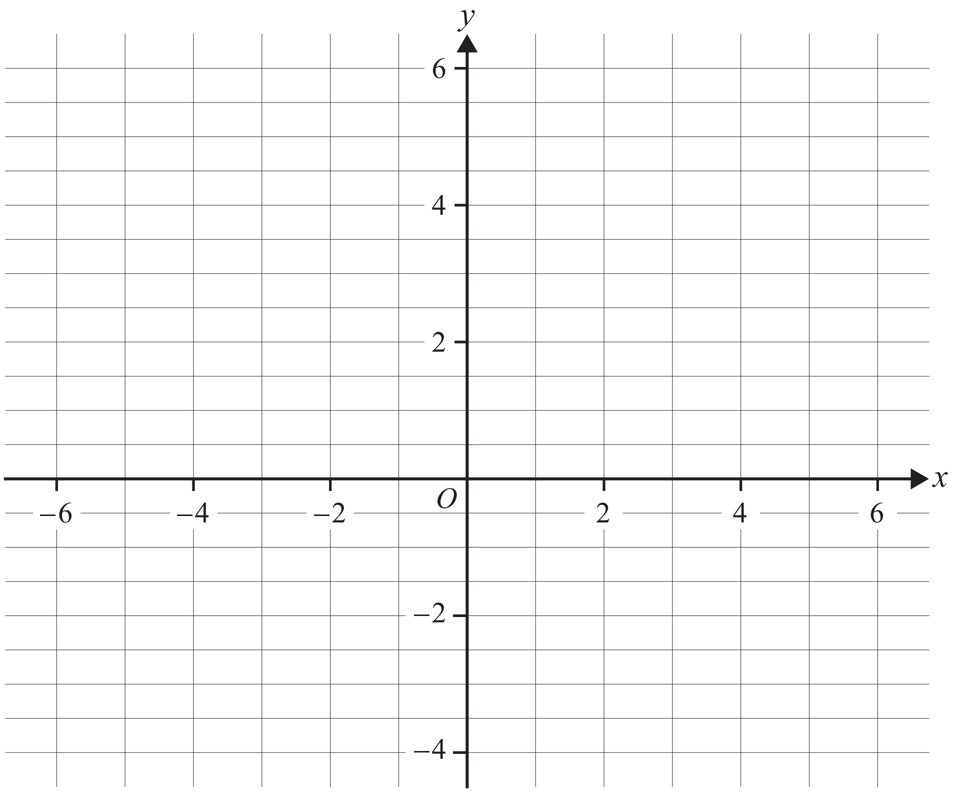
State the coordinates of this point of inflection. 1 mark

e. Sketch the graph of f, including asymptotes, on the axes below.

Label the following features:

- the straight-line asymptotes with their equations
- any turning points and points of inflection, with their coordinates
- any x-intercepts, with values given correct to one decimal place

3 marks



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f. Now consider the function f_k with rule $f_k(x) = \dfrac{x^3 - 2x^2 + 8}{2(k - x^2)}$, where $k \in R$.

For what value(s) of k will the graph of f_k have only one straight-line asymptote? 1 mark

g. Next, consider the function f_b with rule $f_b(x) = \dfrac{x^3 + bx^2 + 8}{2(4 - x^2)}$, where $b \in R$.

i. Find the equation of the oblique asymptote of the graph of f_b. 1 mark

ii. For what value(s) of b will the graph of f_b have two straight-line asymptotes? 1 mark

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Question 2 (9 marks)

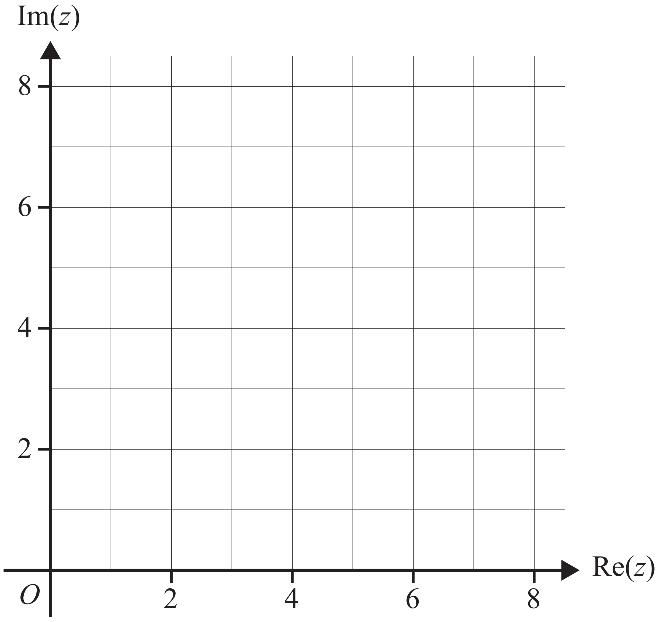
A circle in the complex plane has a centre at $z_0 = 3 + 4i$ and a radius of 2.

For a given value of y, where $y > 4$, the point $v = 4 + iy$ lies on this circle.

Point w is the reflection of point v in the line $\text{Im}(z) = 4$.

a. On the Argand diagram below, sketch the circle and draw the line $\text{Im}(z) = 4$.

Plot and label the points z_0, v and w. 2 marks



b. Find $\text{Im}(v)$. 2 marks

c. Find the area of the **major** segment bounded by an arc of the circle and the chord connecting point v to point w. 2 marks

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d. The tangents to the circle at points v and w intersect at a point, P, on the line $\text{Im}(z) = 4$.

Show that the x-coordinate of P is 7. 1 mark

e. There are two circles that pass through point P that are tangent to the circle $|z - (3 + 4i)| = 2$ and have their centres on the line $\text{Im}(z) = 4$.

Find the equation of each circle in the form $|z - z_c| = R$, where z_c represents the centre of each respective circle. 2 marks

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Question 3 (10 marks)

A curve is defined parametrically by

$\frac{dx}{dt} = 1$, $x(0) = 0$ and $\frac{dy}{dt} = y + 1$, $y(0) = 0$, where $t \geq 0$.

a. **i.** Write down an expression for $\frac{dy}{dx}$. 1 mark

ii. Using integration, show that $y = e^x - 1$. 1 mark

b. The graph of $y = e^x - 1$ where $0 \leq x \leq \log_e(k)$, for constant $k > 1$, is rotated about the x-axis to form a solid of revolution.

i. Given that the volume of the solid of revolution is $\pi \times \log_e(k)$, use calculus to show that k satisfies the equation $k^2 - 4k + 3 = 0$. 2 marks

ii. Hence, find the value of k. 1 mark

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c. **i.** Express the length of the curve given by $y = e^x - 1$ for $0 \le x \le \log_e(5)$ in the form $\int_{t_1}^{t_2} \sqrt{a + e^{bt}}\, dt$. 2 marks

ii. Find the length of the curve given by $y = e^x - 1$ for $0 \le x \le \log_e(5)$.

Give your answer correct to one decimal place. 1 mark

d. The curve given by $y = e^x - 1$ for $0 \le x \le \log_e(5)$ is now rotated about the y-axis to form a surface of revolution.

i. Write down a definite integral that, when evaluated, will give the area of this surface. 1 mark

ii. Find the area of this surface correct to one decimal place. 1 mark

Question 4 (10 marks)

Vectors $\underset{\sim}{D}_1(t) = (2t+1)\underset{\sim}{i} + (3t+1)\underset{\sim}{j} + 5t\,\underset{\sim}{k}$ and $\underset{\sim}{D}_2(t) = 5t\,\underset{\sim}{i} + (10t+2)\,\underset{\sim}{j} + (3t+10)\,\underset{\sim}{k}$ are the respective position vectors of two drones, Drone 1 and Drone 2, relative to an origin O. Displacements are measured in kilometres. Time, $t \geq 0$, is measured in hours.

a. **i.** Find the speed of Drone 1.

Give your answer in kilometres per hour, correct to one decimal place. 1 mark

ii. Find the angle between vector $\underset{\sim}{D}_1(t)$ and vector $\underset{\sim}{D}_2(t)$ when $t = 2$.

Give your answer in degrees, correct to one decimal place. 2 marks

b. Determine the shortest distance between Drone 1 and Drone 2, and the time when this occurs.

Give the distance in kilometres and the time in hours, each correct to three decimal places. 3 marks

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Use the following information to answer part c and part d.

Two other drones, Drone 3 and Drone 4, have respective position vectors $\underset{\sim}{D}_3(t) = t\,\underset{\sim}{i} + \underset{\sim}{j} + t\,\underset{\sim}{k}$ and $\underset{\sim}{D}_4(t) = (t+1)\underset{\sim}{i} + 2\underset{\sim}{j} + (t+1)\underset{\sim}{k}$ for $t \geq 0$, relative to the same origin O. Displacements are measured in kilometres and time is measured in hours.

c. **i.** Find the cross product $\underset{\sim}{D}_3(t) \times \underset{\sim}{D}_4(t)$. 1 mark

ii. Evaluate this cross product when $t = 1$ and explain what this result means in terms of the locations of Drone 3 and Drone 4. 1 mark

d. At time $t = \frac{1}{2}$, Drone 4 begins to malfunction so that its position is given by the new position vector $\underset{\sim}{d}_4(t) = (t+1)\underset{\sim}{i} + \left(\sin(t) + \cos(t)\right)\underset{\sim}{j} + (t+1)\underset{\sim}{k}$.

Using this new position vector, determine the values of t when Drone 3 and Drone 4 have the same speed. 2 marks

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Question 5 (10 marks)

Consider the three points $P_1\left(\frac{4}{3}, \frac{4}{3}, 0\right)$, $P_2\left(\frac{4}{3}, 0, \frac{4}{3}\right)$ and $P_3(0, 2, 2)$.

a. Using a vector method, find the area of the triangle with vertices P_1, P_2 and P_3.

Give your answer in the form $\frac{a\sqrt{b}}{c}$, where $a, b, c \in Z^+$ and $c = a + 1$. 3 marks

b. The points P_1, P_2 and P_3 lie in the plane Π_1.

Find the Cartesian equation of this plane. 2 marks

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Use the following information to answer part c and part d.

Now consider the plane Π_2 with equation $2x - y + z = 6$.

c. **i.** Show that the plane Π_3 given by $\dfrac{d - 1.5z}{3} = x - \dfrac{1}{2}y$, where $d \in R$, is parallel to the plane Π_2. 1 mark

ii. Determine the values of d so that the minimum distance between plane Π_2 and plane Π_3 is $2\sqrt{6}$. 2 marks

d. Consider the plane Π_2, and the plane Π_4 given by $p^2x - (p + 2)y - 5z = 1$ for $p \in R$.

Find the possible values of p for these two planes to be perpendicular. 2 marks

Question 6 (10 marks)

In preparation for an upcoming competition, a snowboarder undertakes many descents down a competition slope.

The times taken for the snowboarder to descend the slope may be assumed to be normally distributed with a mean of 200 seconds and a standard deviation of 5 seconds.

The time taken for any one descent is independent of the time taken for any other descent.

a. Find the probability that the times taken for two descents differ by less than 5 s.

Give your answer correct to four decimal places. 2 marks

b. Find the probability that the average time of four descents differs by less than 5 s from the mean of 200 s.

Give your answer correct to four decimal places. 2 marks

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Use the following information to answer parts c–e.

To try to improve descent times, the snowboarder applies a new Teflon-based wax to the snowboard. To check the effectiveness of the new wax, the snowboarder makes 25 descents of the slope and finds that the mean time taken for the 25 descents is 198.5 s.

To assess the significance of this result of 198.5 s, a one-sided statistical test at the 5% level of significance is performed. Assume that the previous standard deviation of 5 s still applies.

c. Write down suitable null and alternative hypotheses for the test. 1 mark

d. **i.** Determine the p value for the test.

Give your answer correct to four decimal places. 1 mark

ii. Does the application of the new wax produce a significant improvement in descent times at the 5% level of significance? Justify your answer in terms of the p value. 1 mark

e. The wax manufacturer claims that descent times will be reduced by at least 2% with the application of the new wax.

Assuming the mean descent time of 200 s is in fact reduced by 2%, find the probability that the snowboarder, on the basis of the 25 independent descents, will conclude that the wax is **ineffective**. Assume a standard deviation of 5 s.

Give your answer correct to two decimal places. 3 marks

End of Question and Answer Book

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Specialist Mathematics Examination 2

2026 Formula Sheet

You may keep this Formula Sheet.

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Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(\theta)$	$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \text{Arg}(z) \leq \pi$	$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$ $\mathrm{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\mathrm{E}(X_1) + a_2\mathrm{E}(X_2) + \ldots + a_n\mathrm{E}(X_n)$	
	$\mathrm{Var}(aX_1 + b) = a^2\mathrm{Var}(X_1)$ $\mathrm{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\mathrm{Var}(X_1) + a_2^2\mathrm{Var}(X_2) + \ldots + a_n^2\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\mathrm{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}}, \bar{x} + z\dfrac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$E(\bar{X}) = \mu$
	variance	$\mathrm{Var}(\bar{X}) = \dfrac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = n x^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c,\ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = a e^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\cot(ax)\right) = -a\,\mathrm{cosec}^2(ax)$	$\int \mathrm{cosec}^2(ax)dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}\left(\sec(ax)\right) = a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\,dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}\left(\mathrm{cosec}(ax)\right) = -a\,\mathrm{cosec}(ax)\cot(ax)$	$\int \mathrm{cosec}(ax)\cot(ax)\,dx = -\frac{1}{a}\mathrm{cosec}(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n\,dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\quad n \neq -1$
	$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = u\,v - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x,\ y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f\left(x_n, y_n\right)$
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \frac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \qquad y(t) = y_1 + y_2t \qquad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{r}(s, t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t,\ y(s, t) = y_0 + y_1s + y_2t,\ z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \text{cosec}^2(x)$
$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x + y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x - y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1 - \tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}\left(1 - \cos(2ax)\right)$	$\cos^2(ax) = \dfrac{1}{2}\left(1 + \cos(2ax)\right)$

End of Formula Sheet

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