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PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Mathematical Methods
Examination 2

Question and Answer Book

VCE (NHT) Examination – Wednesday 27 May 2026

- Reading time is **15 minutes**: 10.30 am to 10.45 am
- Writing time is **2 hours**: 10.45 am to 12.45 pm

Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 28 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents pages

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Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

In a random sample of 300 builders, 230 had completed an apprenticeship in the last 10 years.

Using this sample, an approximate 95% confidence interval for the population proportion of builders who have completed an apprenticeship in the last 10 years, correct to three decimal places, is

A. (0.727, 0.807)

B. (0.719, 0.815)

C. (0.712, 0.821)

D. (0.704, 0.830)

Question 2

Two dice are rolled at the same time. The algorithm below generates the sample space.

```
for i from 1 to 6
  for j from 1 to 6
     print i, j
  end for
end for
```

The first printed output is 1, 1.

The ninth printed output is

A. 2, 3

B. 3, 2

C. 6, 3

D. 3, 6

Question 3

Assume that the time, in minutes, it takes for a particular student to travel to school is a normal random variable with mean 15 and variance 4.

Let Z be a standard normal random variable.

The probability that it takes the student more than 20 minutes to travel to school is equal to

A. $\Pr(Z < 1.25)$

B. $\Pr(Z < 2.5)$

C. $1 - \Pr(Z < 1.25)$

D. $1 - \Pr(Z < 2.5)$

Question 4

The function $f : D \to R,\ f(x) = \dfrac{4}{x} + 2$ has range $(-2, 1]$.

The domain D is

A. $[-4, -1)$

B. $(-4, -1]$

C. $(-1, 4]$

D. $[-1, 4)$

Question 5

Newton's method can be applied to find an approximate solution to $e^{-x} - 2 = 0$.

If the initial value is $x_0 = a$, where a is a real number, then the value of x_1 is

A. $2e^{a} - 1$

B. $1 - 2e^{a}$

C. $2e^{a} - a - 1$

D. $1 + a - 2e^{a}$

Question 6

The graph of $y = 2^{x-1} + 3$ is reflected in the line $y = x$.

The rule for the image is

A. $y = \log_2(2x - 6)$

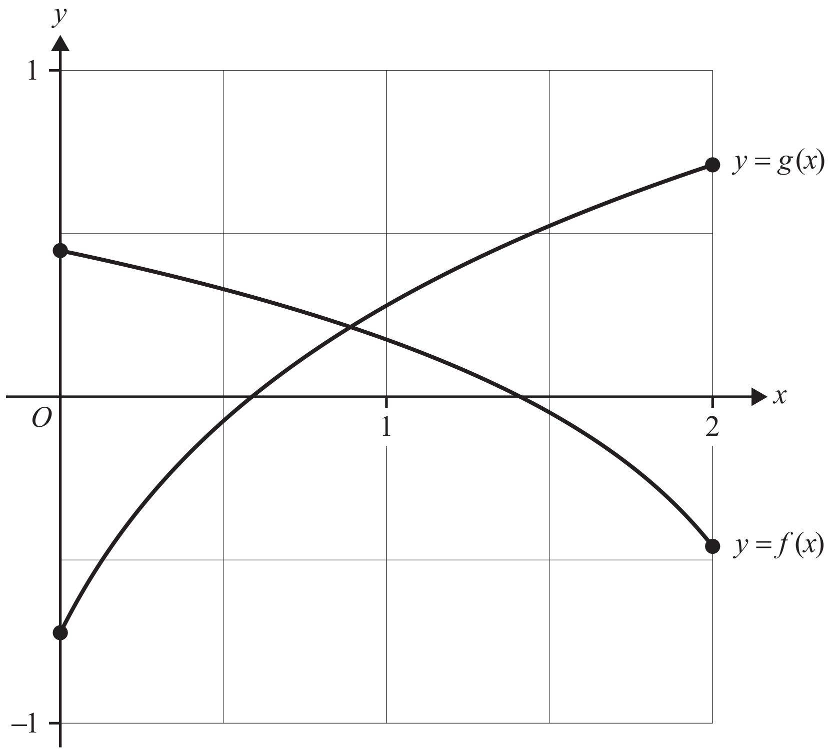
B. $y = \log_2(2x - 3)$

C. $y = \log_2(3 - x) + 1$

D. $y = \log_2(x) - 2$

Question 7

Consider the continuous functions $f : [0, 2] \to R$ and $g : [0, 2] \to R$, whose graphs are drawn to scale below.



There is a value of x for which $f(x) + g(x) = 0$ within the interval

A. $\left[0, \frac{1}{2}\right)$

B. $\left[\frac{1}{2}, 1\right)$

C. $\left[1, \frac{3}{2}\right)$

D. $\left[\frac{3}{2}, 2\right]$

Question 8

The following sequence of steps is an attempt to solve the exponential equation $\left(\frac{3}{4}\right)^x = 3$.

The sequence of steps contains at least one error.

Step 1 Take a logarithm of both sides:

$$\log_2\left(\left(\frac{3}{4}\right)^x\right) = \log_2(3)$$

Step 2 Use a law of logarithms to rewrite the equation:

$$x\log_2\left(\frac{3}{4}\right) = \log_2(3)$$

Step 3 Make x the subject of the equation:

$$x = \frac{\log_2(3)}{\log_2\left(\frac{3}{4}\right)}$$

Step 4 Use a law of logarithms to simplify the fraction:

$$x = \log_2\left(3 - \frac{3}{4}\right)$$

$$= \log_2\left(\frac{9}{4}\right)$$

Step 5 Evaluate the base 2 logarithm using a square root:

$$x = \sqrt{\frac{9}{4}}$$

$$= \frac{3}{2}$$

The **first** step containing an error is

A. step 2

B. step 3

C. step 4

D. step 5

Question 9

A straight line is tangent to the graphs of both $y = x^2 + 4x$ and $y = (x + 5)^2 - 4$.

The gradient of this line is

A. -4

B. 0

C. 4

D. 5

Question 10

Let $X \sim \text{Bi}(n, p)$, where $0.5 < p < 1$.

For a particular value of n, as p increases

A. $\text{E}(X)$ increases and $\text{var}(X)$ increases.

B. $\text{E}(X)$ increases and $\text{var}(X)$ decreases.

C. $\text{E}(X)$ decreases and $\text{var}(X)$ increases.

D. $\text{E}(X)$ decreases and $\text{var}(X)$ decreases.

Question 11

Let

$$f : R \to R, f(x) = x^2 + cx + ab$$
$$g : R \to R, g(x) = (x - a)^2(x - b)$$

where $a, b, c \in R$.

Given that the graphs $y = f(x)$ and $y = g(x)$ have the same x-intercepts, which one of the following must be true for all possible values of a and b?

A. $c = ab$

B. $c = -ab$

C. $c = a + b$

D. $c = -(a + b)$

NHT

Do not write in this area.

Question 12

Consider the 10 functions, all with domain $[0, 2\pi]$ and defined as $f_n(x) = \sin(nx)$, where n is an integer with $1 \le n \le 10$.

$$f_1(x) = \sin(x),\ f_2(x) = \sin(2x),\ f_3(x) = \sin(3x),\ \ldots,\ f_{10}(x) = \sin(10x)$$

How many values of $x \in [0, 2\pi]$ exist such that $f_n(x) = 0$ for all integers n with $1 \le n \le 10$?

A. 2

B. 3

C. 4

D. 5

Question 13

Which set of x-values is **not** a subset of the general solution to $\sin(2x) = \frac{\sqrt{3}}{2}$?

A. $\left\{\frac{-2\pi}{3} + k\pi,\ k \in Z\right\}$

B. $\left\{\frac{\pi}{3} + 2k\pi,\ k \in Z\right\}$

C. $\left\{\frac{5\pi}{6} + k\pi,\ k \in Z\right\}$

D. $\left\{\frac{7\pi}{6} + 2k\pi,\ k \in Z\right\}$

Question 14

Consider the system of two linear equations below, where a and b are real numbers.

Equation 1: $2x - 3y = 1$

Equation 2: $ax + by = 2$

The system of two equations has no real solutions.

If $x = 1$ and $y = 2$ satisfies equation 2, it must be true that

A. $x = 4$ and $y = 4$ satisfies equation 2

B. $x = 4$ and $y = 0$ satisfies equation 2

C. $x = 3$ and $y = 5$ satisfies equation 2

D. $x = 5$ and $y = 3$ satisfies equation 2

Question 15

Consider a continuous function $f : R \to R$, which has an average value of 2 over the interval $[-1, 1]$.

Let $g : R \to R$, $g(x) = mf(nx) + c$, where m, n and c are positive real numbers.

Which one of the following must be true for all possible values of m, n and c?

A. g has an average value of $2m$ over the interval $[-n, n]$

B. g has an average value of $2m$ over the interval $\left[-\frac{1}{n}, \frac{1}{n}\right]$

C. g has an average value of $2m + c$ over the interval $[-n, n]$

D. g has an average value of $2m + c$ over the interval $\left[-\frac{1}{n}, \frac{1}{n}\right]$

Question 16

Let $f : R \to R$, $f(x) = \cos\left(\frac{\pi}{6}x\right)\sin\left(\frac{\pi}{3}x\right)$.

Let $h : R \to R$ be the result of applying the following sequence of transformations to f:

1. a dilation by a factor of $\frac{1}{2}$ from the y-axis
2. a dilation by a factor of 2 from the x-axis
3. a translation of 2 units in the positive direction of the x-axis

The period of h is

A. 3

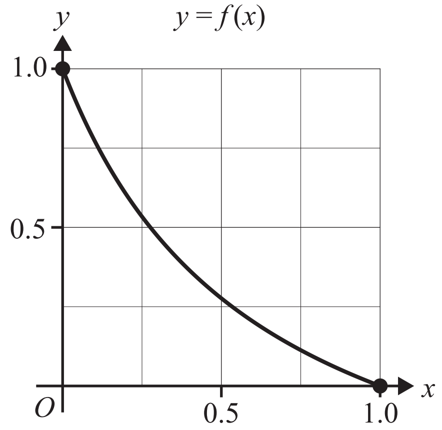
B. 6

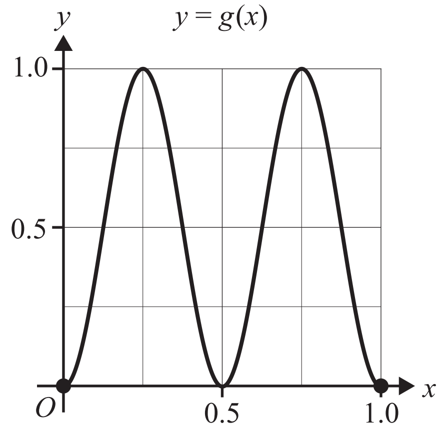
C. 12

D. 24

Question 17

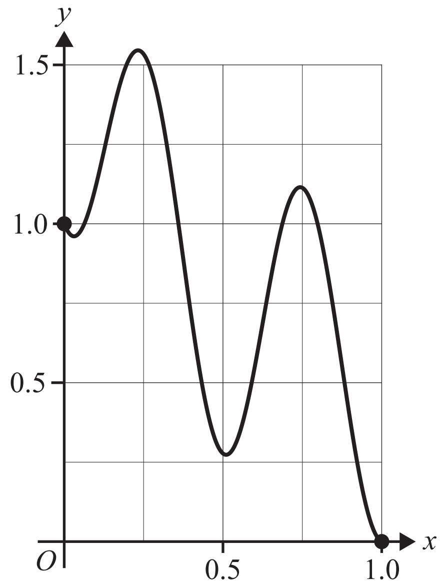
Suppose the graphs of the functions $f : [0, 1] \to R$ and $g : [0, 1] \to R$ are as shown below.



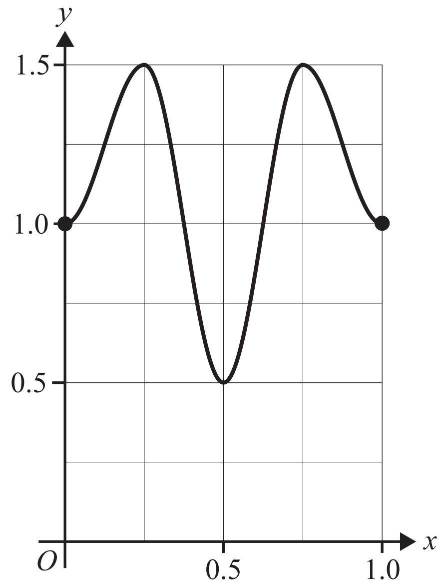


The graph of $y = (f \circ g)(x)$ is best represented by

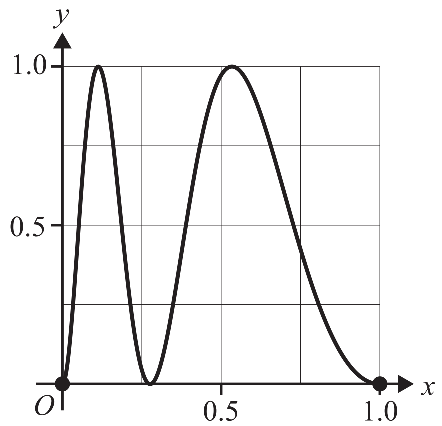
A.



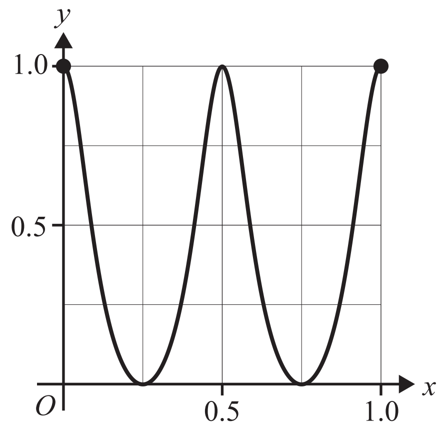
B.



C.

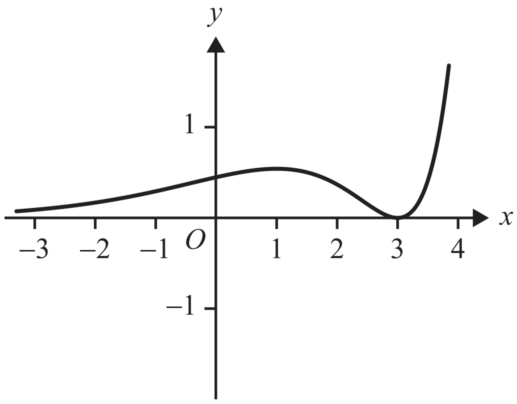


D.



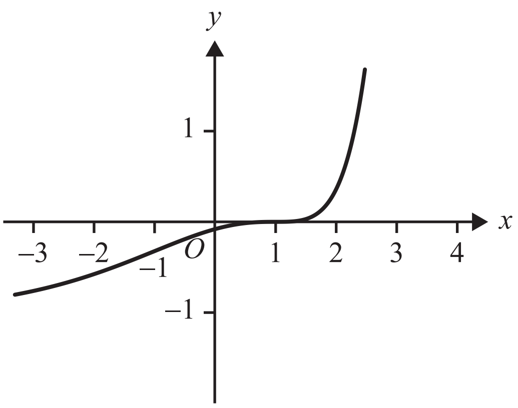
Question 18

The graph of $y = f'(x)$ is shown below.

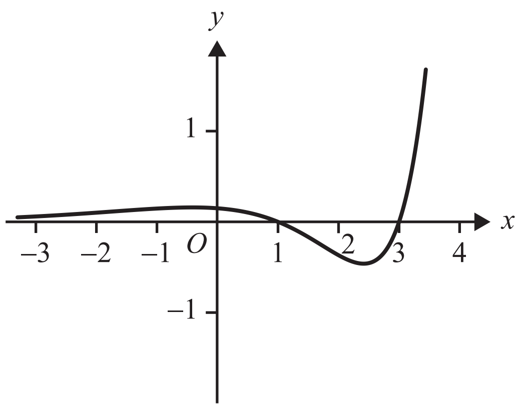


The graph of $y = f(x)$ could be

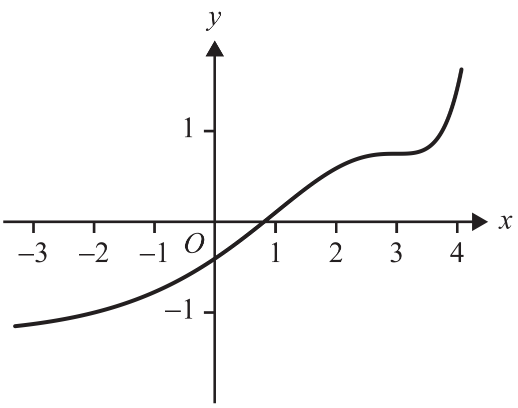
A.



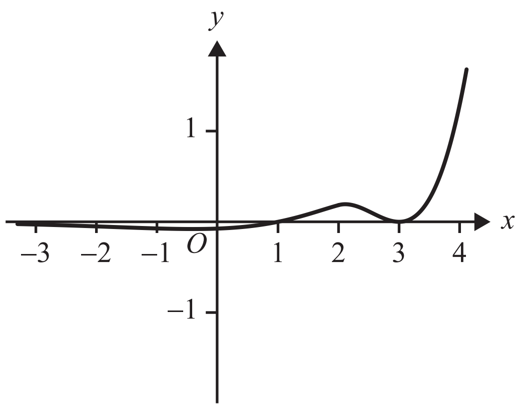
B.



C.



D.



Question 19

A basketball player has three attempts to score from the free throw line. If the player scores at least twice, they win the game.

Assume that for each attempt, the probability that the player will score is p, where $0 < p < 1$. The outcome of each attempt is independent of all others.

Given that the player wins the game, the probability that exactly one of the first two attempts was missed is given by

A. $\dfrac{1-p}{3-2p}$

B. $\dfrac{1-p}{p(3-2p)}$

C. $\dfrac{2(1-p)}{3-2p}$

D. $\dfrac{2(1-p)}{p(3-2p)}$

Question 20

A soup company wishes to design a new can for its soup. The can must be in the shape of a cylinder with a fixed volume V cm^3.

In order to minimise costs, the company chooses the radius and height that minimise the total surface area of the can, which includes the base and top.

The minimum possible total surface area of the can is equal to

A. $\sqrt[3]{\dfrac{V}{2\pi}}$ cm^2

B. $\sqrt[3]{\dfrac{V}{\pi}}$ cm^2

C. $3\left(\sqrt[3]{\pi V^2}\right)$ cm^2

D. $3\left(\sqrt[3]{2\pi V^2}\right)$ cm^2

End of Section A

Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

A glass of water is placed in a refrigerator. The temperature of the water, in degrees Celsius, t hours after it is placed in the refrigerator is given by the function

$$f: [0, \infty) \to R, f(t) = 6 + 27e^{-kt}, \text{ where } k \text{ is a real number.}$$

Two hours after being placed in the refrigerator, the temperature of the water is 9 °C.

a. State the initial temperature of the water, in degrees Celsius. 1 mark

b. State the equation of the asymptote for the graph of $y = f(t)$. 1 mark

c. Find the value of k and, hence, show that $f(t)$ can be expressed in the form $f(t) = 6 + 3^{3-t}$. 2 marks

d. How long after being placed in the refrigerator does it take the water temperature to reach 8.5 °C?

Give your answer in hours, correct to two decimal places. 1 mark

e. Three hours after the water is placed in the refrigerator (that is, when $t = 3$), the refrigerator door is opened. The temperature of the water then increases at a rate of $g'(t) = 14e^{-t}$ degrees Celsius per hour.

Let h be a continuous piecewise function representing the temperature of the water, in degrees Celsius, t hours after the water is placed in the refrigerator.

$$h(t) = \begin{cases} f(t) & 0 \le t \le 3 \\ g(t) & t > 3 \end{cases}$$

i. Find $g(t)$. 2 marks

ii. Find $h'(t)$ for $t \ne 3$ and verify that the function h is not differentiable at $t = 3$. 3 marks

Question 1 continues on the next page.

f. Temperature can also be measured in degrees Fahrenheit.

The temperature of the water in degrees Fahrenheit is given by the equation

$$p(t) = \frac{9}{5}h(t) + 32.$$

i. State a sequence of two transformations, a dilation followed by a translation, that will map the graph of $y = h(t)$ to the graph of $y = p(t)$. 2 marks

1. ______________________________

2. ______________________________

ii. State a sequence of two transformations, a translation followed by a dilation, that will map the graph of $y = h(t)$ to the graph of $y = p(t)$. 1 mark

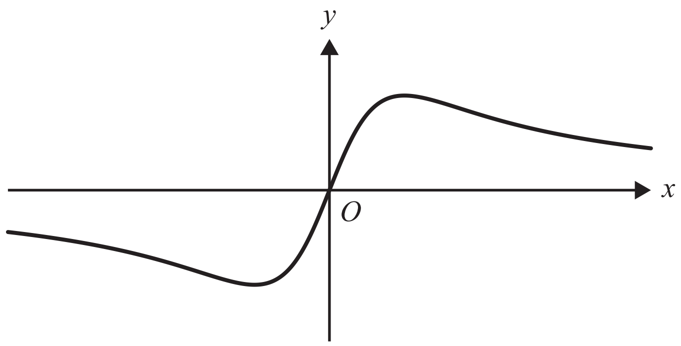
1. ______________________________

2. ______________________________

Question 2 (15 marks)

Let $f: R \to R,\ f(x) = \dfrac{x}{x^2+1}$.

The graph of $y = f(x)$ is shown below.



a. State the coordinates of both stationary points of f. 1 mark

b. Find the area bounded by the graph of $y = f(x)$, the line $x = -1$, the line $x = 1$ and the x-axis. 2 marks

c. Find the equation of the tangent line to the graph of $y = f(x)$ at the origin. 1 mark

Question 2 continues on the next page.

d. Find the equations of the tangent lines to the graph of $y = f(x)$ that have a gradient of $-\frac{1}{8}$. 2 marks

e. Let $a, b \in R$, where $0 < a < b$.

i. Show, using algebra, that the average rate of change of f over the interval $[a, b]$ is given by $\frac{1-ab}{(a^2+1)(b^2+1)}$. 3 marks

ii. Hence, or otherwise, find a in terms of b such that $f(a) = f(b)$. 1 mark

f. Let $g : R \to R,\ g(x) = \dfrac{x}{x^2 + k}$, where $k \in (0, \infty)$.

i. Find the x-values of the stationary points of g in terms of k. 1 mark

ii. Find all values of k such that g is strictly decreasing for $x \geq 1$. 1 mark

iii. Let L be the straight line passing through the stationary points of g.

Find the total area of the regions bounded by the graph of $y = g(x)$ and the line L, between the stationary points of g. 3 marks

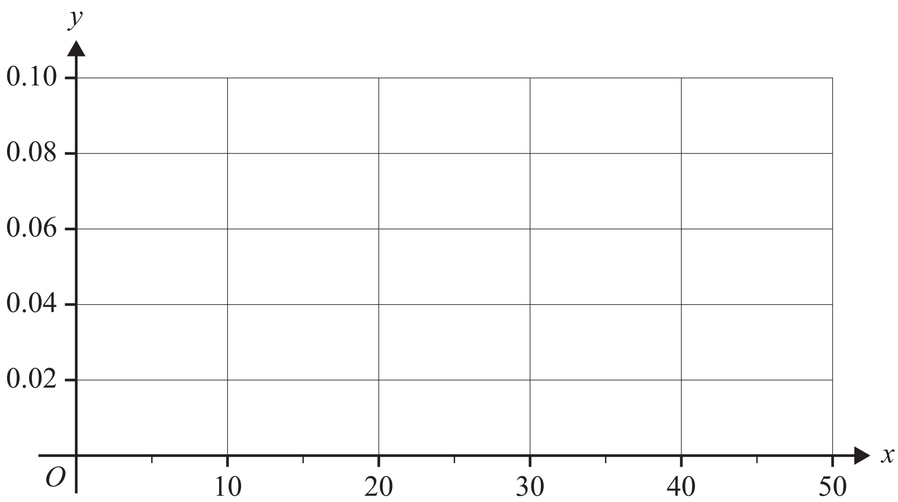
Question 3 (15 marks)

Two hiking apps, app A and app B, provide information about hiking trails.

a. For app A, the distance, in kilometres, of a randomly selected hiking trail can be modelled by a continuous random variable X with probability density function

$$f(x)=\begin{cases}\dfrac{x}{250} & 5\le x\le 15\\[2ex] \dfrac{(x-45)^2}{15\,000} & 15<x\le 45\\[2ex] 0 & \text{otherwise}\end{cases}$$

i. Sketch the graph of $y=f(x)$ for $0\le x\le 50$ on the axes below. 2 marks



ii. Find the mean distance of the hiking trails listed on app A.

Give your answer in kilometres, correct to one decimal place. 1 mark

b. App B contains a large list of hiking trails. The distance, in kilometres, of a randomly selected hiking trail from app B can be modelled by a normal distribution with a mean of 16.5 and a standard deviation of 5.

In app B, the hiking trails that are classified as difficult are the longest 15%.

i. Find the probability that a randomly selected hiking trail from app B is longer than 12 km.

Give your answer correct to three decimal places. 1 mark

ii. Find the minimum distance for a hiking trail from app B to be classified as difficult.

Give your answer in kilometres, correct to one decimal place. 1 mark

iii. A hiker randomly selects 10 hiking trails from app B. Assume that the 10 trails are selected independently.

Find the probability that exactly two hiking trails are classified as difficult.

Give your answer correct to three decimal places. 2 marks

Question 3 continues on the next page.

iv. In a random sample of 10 hiking trails from app B, let $\hat{P}$ be the random variable that represents the proportion of hiking trails that are classified as difficult. Assume that the 10 trails are selected independently.

Find $\Pr\left(\hat{P} > \frac{1}{3}\right)$, correct to three decimal places. 2 marks

v. Consider a random sample of n hiking trails from app B.

Use a normal approximation to find the minimum value of n, such that the probability that 10% or more of the trails are classified as difficult is at least 0.95 2 marks

c. A hiker randomly selects either app A or app B with equal probability. They then randomly select a hiking trail from that app.

The probability distributions for apps A and B are those defined in **part a** and **part b** respectively.

i. Find the probability that the selected trail is longer than 15 km.

Give your answer correct to three decimal places. 2 marks

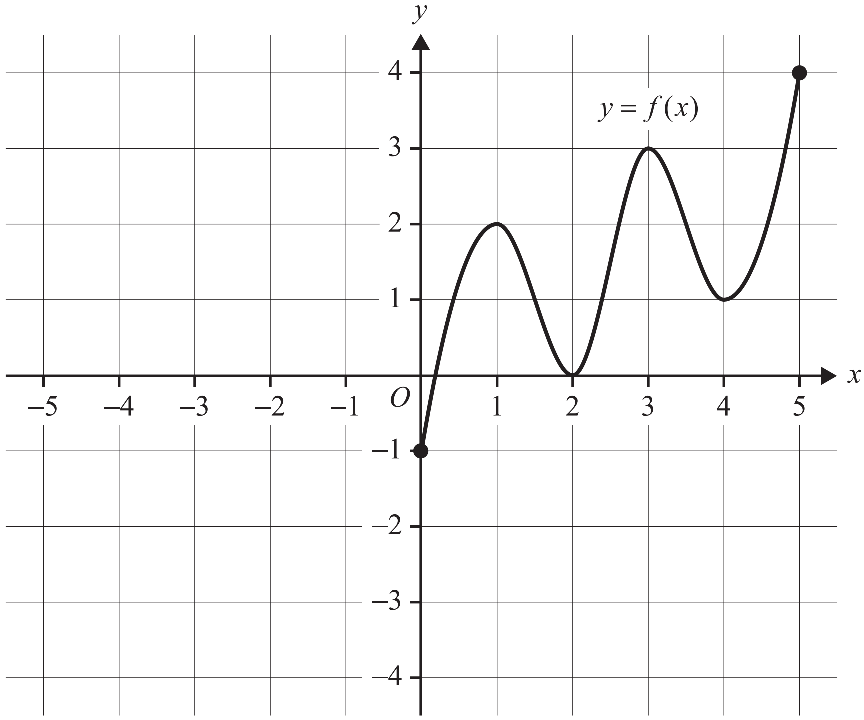
ii. Given that the selected trail is longer than 15 km, find the probability that it was selected from app A.

Give your answer correct to three decimal places. 2 marks

Question 4 (17 marks)

Let f be a differentiable function with domain $[0, 5]$.

The graph of $y = f(x)$ is shown below. It has exactly four stationary points, which have coordinates $(1, 2)$, $(2, 0)$, $(3, 3)$ and $(4, 1)$.



a. State the values of x for which $f'(x) = 0$. 1 mark

b. **i.** Find all real values of k for which $f(x) = k$ has exactly two solutions for x. 1 mark

ii. Find all real values of k for which $f(x) = k$ has exactly three solutions for x. 1 mark

c. **i.** When the graph of $y = f(x)$ is reflected in the x-axis, its image has two local maximum turning points.

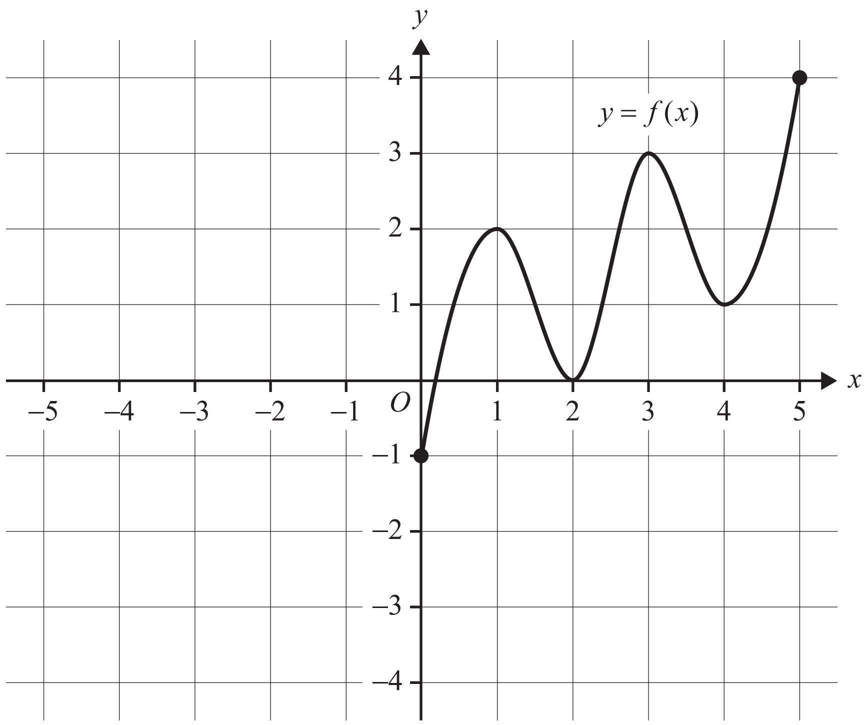
State the coordinates of these two turning points. 1 mark

ii. The function f has the property that when its graph is reflected in the x-axis and then reflected in the y-axis, its image can be expressed in the form $y = f(x + b) + c$, for some $b, c \in R$.

Find the values of b and c. 1 mark

Question 4 continues on the next page.

d. The graph of $y = f(x)$ is shown again below.



Let h_1 be the inverse function of f when f is restricted to have domain $[1, 2]$.

i. State the domain of h_1. 1 mark

ii. Find $h_1(2)$. 1 mark

e. Let h_m be the inverse function of f when f is restricted to have domain $[m, m+1]$ for particular values of $m \in [0, 4]$.

i. Explain why m must be an integer for the inverse function h_m to exist. 1 mark

ii. Find the value(s) of m such that $h_m(1)$ is an integer. 1 mark

f. Let $g: (-0.8, 1.8) \to R$, $g(x) = x^3 - x^2 - x + 3$.

i. Find the range of g and, hence, show that $f\left(g(x)\right)$ is defined for all $x \in (-0.8, 1.8)$. 2 marks

ii. Find the minimum value of $f\left(g(x)\right)$ and the value of x for which the minimum occurs. 2 marks

Question 4 continues on the next page.

iii. Find the x-values of the stationary points of $f\left(g(x)\right)$. 4 marks

End of examination questions

End of examination. There are no more questions.

2026 NHT

Mathematical Methods Examination 2

Formula Sheet

You may keep this Formula Sheet.

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Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left((ax+b)^n\right)=an\left(ax+b\right)^{n-1}$	$\int (ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c,\ n\neq -1$
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$	$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}\,dx=\log_e(x)+c,\ x>0$
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$	$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$	$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$	

product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$	Newton's method	$x_{n+1}=x_n-\frac{f\left(x_n\right)}{f'\left(x_n\right)}$
trapezium rule approximation	$Area\approx\frac{x_n-x_0}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-2})+2f(x_{n-1})+f(x_n)\right]$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A\|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = \mathrm{E}(X)$	variance	$\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$
binomial coefficient	$\dbinom{n}{x} = \dfrac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
binomial	$\Pr(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$\mathrm{E}(\hat{P}) = p$
standard deviation	$\mathrm{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

End of Formula Sheet