

# 2017 VCE Mathematical Methods 2 (NHT) examination report

## Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

## Section A – Multiple-choice questions

Question	Answer
1	A
2	E
3	C
4	D
5	C
6	B
7	A
8	D
9	C
10	A
11	C
12	B
13	B
14	E
15	E
16	B
17	C
18	D
19	B
20	D

## Section B

### Question 1a.

$T(t) = 19 + 6\sin\left(\frac{\pi}{12}(t-8)\right)$ , range of function is  $[-6+19, 6+19] = [13, 25]$ , minimum temperature is

13 °C, maximum temperature is 25 °C

### Question 1b.

$T(6) = 16$  °C

### Question 1c.

$T(8) = 19$  °C

**Question 1d.**Solve  $T(t) \geq 19$  °C,  $8 \leq t \leq 20$ ,  $20 - 8 = 12$  hours**Question 1e.**

Average rate of change =  $\frac{T(12) - T(8)}{12 - 8} = \frac{3\sqrt{3}}{4}$  °C/hr

**Question 1fi.**

$$T'(t) = \frac{\pi}{2} \cos\left(\frac{\pi}{12}(t-8)\right) \text{ or } T'(t) = -\frac{\pi}{2} \cos\left(\frac{\pi}{12}t + \frac{\pi}{3}\right), \text{ or equivalent}$$

**Question 1fii.**

Find the minimum of the derivative, decreasing most rapidly at 8.00 pm or 20 hours.

**Question 2a.** $f: R \rightarrow R$ , where  $f(x) = (x-2)^2(x-5)$ ,  $f'(x) = 3(x-4)(x-2)$ , or equivalent**Question 2b.**Solve  $f'(x) < 0$ ,  $2 < x < 4$ **Question 2ci.**

$$f(1) = -4, f(5) = 0, \frac{f(5) - f(1)}{5 - 1} = 1$$

**Question 2cii.**Midpoint  $\left(\frac{5+1}{2}, \frac{-4+0}{2}\right) = (3, -2)$ ,  $f(3) = -2$  hence midpoint lies on the graph of  $y = f(x)$ **Question 2ciii.**

Solve  $f'(x) = 1$ ,  $x = \frac{9+2\sqrt{3}}{3}$  or  $x = \frac{9-2\sqrt{3}}{3}$

**Question 2d.**

$$g: R \rightarrow R, \text{ where } g(x) = (x-2)^2(x-a), g'(x) = 0, x = 2 \text{ or } x = \frac{2(a+1)}{3},$$

$$p = \frac{2}{3}, g\left(\frac{2(a+1)}{3}\right) = -\frac{4}{27}(a-2)^3, q = -\frac{4}{27}$$

**Question 2e.** $g'(a) = (a-2)^2$ ,  $(a-2)^2 \geq 0$ , when  $a = 2$ ,  $g'(x) = 0$ , gradient of the tangent is positive for  $a \in R \setminus \{2\}$ **Question 2fi.**

$$g'(x) = (a-2)^2, \left(\frac{8-a}{3}, -\frac{4}{27}(a-2)^3\right)$$

**Question 2fii.**

$$Q\left(\frac{2(a+1)}{3}, -\frac{4}{27}(a-2)^3\right) \text{ and } \left(\frac{8-a}{3}, -\frac{4}{27}(a-2)^3\right),$$

$$\text{distance} = \sqrt{\left(-\frac{4}{27}(a-2)^3 - \left(-\frac{4}{27}(a-2)^3\right)\right)^2 + \left(\frac{8-a}{3} - \frac{2(a+1)}{3}\right)^2} = a-2$$

**Question 3a.**

$$X_A \sim N\left(11, \left(\frac{1}{4}\right)^2\right), \Pr(X_A > 10.5) = 0.977, \text{ correct to three decimal places}$$

**Question 3b.**

$$E(X_B) = \int_0^{12} xf(x)dx = 7.75 \text{ hours, correct to two decimal places}$$

**Question 3c.**

$$\text{sd}(X_B) = \sqrt{\int_0^{12} x^2 f(x)dx - \left(\int_0^{12} xf(x)dx\right)^2} = 2.31 \text{ hours, correct to two decimal places}$$

**Question 3d.**

$$\Pr(X_B > 10.5) = \int_{10.5}^{12} f(x)dx = 0.1134, \text{ correct to four decimal places}$$

**Question 3e.**

$$\Pr(\text{boxes mislabelled}) = \Pr(A \cap (X_A < 10.5)) + \Pr(B \cap (X_B > 10.5))$$

$$= 0.5 \times 0.0228 + 0.5 \times 0.1134$$

$$= 0.068, \text{ correct to three decimal places}$$

**Question 3f.**

$$\Pr(B|\text{mislabelled}) = \frac{\Pr(B \cap \text{mislabelled})}{\Pr(\text{mislabelled})} = \frac{0.5 \times 0.1134}{0.0681} = 0.833, \text{ correct to three decimal places}$$

**Question 3g.**

$$X_1 \sim \text{Bi}(26, 0.05) \text{ or } 1 - 0.95^{26}, \Pr(X_1 \geq 1) = 0.7365, \text{ correct to four decimal places}$$

**Question 3h.**

$$X_2 \sim \text{Bi}(100, 0.05), \Pr(\hat{P}_A > 0.04 | \hat{P}_A < 0.08) = \frac{\Pr(5 \leq X_1 \leq 7)}{\Pr(X_1 \leq 7)} = \frac{0.4361}{0.8720} = 0.5000,$$

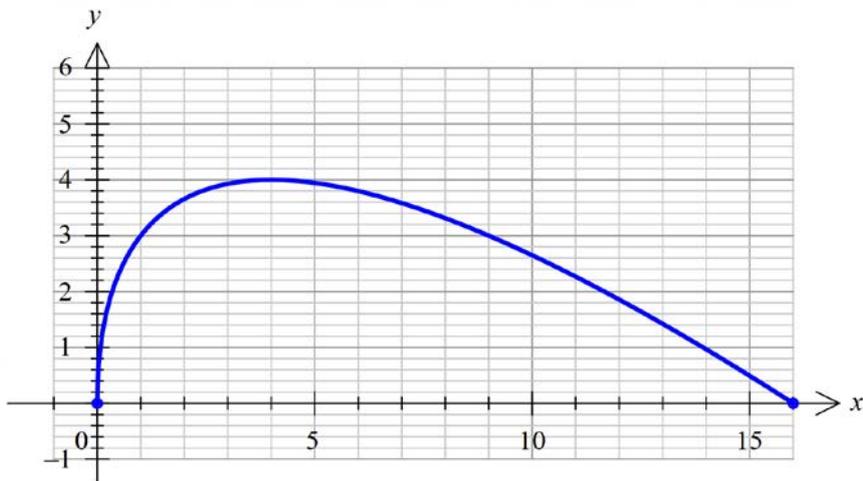
correct to four decimal places

**Question 3i.**

A 90% confidence interval for the population proportion from this sample is (0.02, 0.10), correct to two decimal places

**Question 4a.**

$$f: [0, 16] \rightarrow R, f(x) = 4\sqrt{x} - x, \text{ maximum occurs when } x = 4 \text{ and is } f(4) = 4$$

**Question 4b.****Question 4ci.**

$$A = \int_0^{16} f(x)dx = \frac{128}{3} \text{ square units}$$

**Question 4cii.**

$$A_{\Delta OCX} = \frac{16 \times f(c)}{2}, \text{ maximum occurs when } c = 4 \text{ and the maximum area is 32 square units}$$

**Question 4d.**

$$4\sqrt{b} - b = 4\sqrt{a} - a, \quad 4\sqrt{b} - 4\sqrt{a} = b - a, \quad 4(\sqrt{b} - \sqrt{a}) = (\sqrt{b} - \sqrt{a})(\sqrt{b} + \sqrt{a}), \quad \sqrt{b} = 4 - \sqrt{a},$$

$$b = (\sqrt{a} - 4)^2$$

**Question 4ei.**

$$\text{Area of rectangle } A_R = (b - a)f(a) = \left( (4 - \sqrt{a})^2 - a \right) (4\sqrt{a} - a) = 8(2 - \sqrt{a})(4\sqrt{a} - a) \text{ or}$$

$$8a^{\frac{3}{2}} - 48a + 64\sqrt{a} \text{ square units}$$

**Question 4eii.**

$$A'_R(a) = 0 \text{ or find maximum of } A_R, \quad a = \frac{8}{3}(2 - \sqrt{3}), \quad b = \frac{8}{3}(2 + \sqrt{3})$$

**Question 4eiii.**

$$A_R\left(\frac{8}{3}(2 - \sqrt{3})\right) = \frac{128\sqrt{3}}{9} \text{ square units}$$

**Question 4fi.**

$$A_T = \frac{1}{2}(16 + (b - a)) \times f(a), \quad A'_T(a) = 0, \quad a = \frac{16}{9}, \quad A_T = \frac{1024}{27} \text{ square units}$$

**Question 4fii.**

$$\frac{A_T}{A} = \frac{\frac{1024}{27}}{\frac{128}{3}} = \frac{8}{9}$$