

SUPERVISOR TO ATTACH  
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Write your **student number** in the boxes above.

**Letter**

# Mathematical Methods Examination 2

## Question and Answer Book

VCE Examination – Thursday 7 November 2024

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- Reading time is **15 minutes**: 11.45 am to 12 noon
- Writing time is **2 hours**: 12 noon to 2.00 pm

### Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

### Materials supplied

- Question and Answer Book of 28 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

### Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

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### Contents

	pages
<b>Section A</b> (20 questions, 20 marks)	2–10
<b>Section B</b> (5 questions, 60 marks)	11–26

## Section A – Multiple-choice questions

### Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

### Question 1

The asymptote(s) of the graph of  $y = \log_e(x + 1) - 3$  are

- A.  $x = -1$  only
- B.  $x = 1$  only
- C.  $y = -3$  only
- D.  $x = -1$  and  $y = -3$

### Question 2

A function  $g : R \rightarrow R$  has the derivative  $g'(x) = x^3 - x$ .

Given that  $g(0) = 5$ , the value of  $g(2)$  is

- A. 2
- B. 3
- C. 5
- D. 7

### Question 3

A discrete random variable  $X$  is defined using the probability distribution below, where  $k$  is a positive real number.

$x$	0	1	2	3	4
$\Pr(X=x)$	$2k$	$3k$	$5k$	$3k$	$2k$

Find  $\Pr(X < 4 \mid X > 1)$ .

- A.  $\frac{13}{15}$
- B.  $\frac{11}{13}$
- C.  $\frac{4}{5}$
- D.  $\frac{8}{15}$

**Question 4**

If  $\int_a^b f(x)dx = -5$  and  $\int_a^c f(x)dx = 3$ , where  $a < b < c$ , then  $\int_b^c 2f(x)dx$  is equal to

- A.  $-16$
- B.  $16$
- C.  $-4$
- D.  $4$

**Question 5**

Consider the functions  $f: (1, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = x^2 - 4x$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g(x) = e^{-x}$ .

The range of the composite function  $g(f(x))$  is

- A.  $(0, e^3)$
- B.  $(0, e^3]$
- C.  $(0, e^4)$
- D.  $(0, e^4]$

**Question 6**

Consider the function  $f(x) = \frac{2x+1}{3-x}$  with domain  $x \in \mathbb{R} \setminus \{3\}$ .

The inverse of  $f$  is

- A.  $f^{-1}(x) = \frac{3x-1}{x+2}$  with domain  $x \in \mathbb{R} \setminus \{3\}$
- B.  $f^{-1}(x) = 3 - \frac{7}{x+2}$  with domain  $x \in \mathbb{R} \setminus \{-2\}$
- C.  $f^{-1}(x) = 3 + \frac{5}{x+2}$  with domain  $x \in \mathbb{R} \setminus \{-2\}$
- D.  $f^{-1}(x) = \frac{1-3x}{x+2}$  with domain  $x \in \mathbb{R} \setminus \{-2\}$

**Question 7**

A fair six-sided die is repeatedly rolled. What is the minimum number of rolls required so that the probability of rolling a six at least once is greater than 0.95?

- A. 15
- B. 16
- C. 17
- D. 18

**Question 8**

Some values of the functions  $f: R \rightarrow R$  and  $g: R \rightarrow R$  are shown below.

$x$	1	2	3
$f(x)$	0	4	5
$g(x)$	3	4	-5

The graph of the function  $h(x) = f(x) - g(x)$  must have an  $x$ -intercept at

- A. (2, 0)
- B. (3, 0)
- C. (4, 0)
- D. (5, 0)

**Question 9**

At a Year 12 formal, 45% of the students travelled to the event in a hired limousine, while the remaining 55% were driven to the event by a parent.

Of the students who travelled in a hired limousine, 30% had a professional photo taken.

Of the students who were driven by a parent, 60% had a professional photo taken.

Given that a student had a professional photo taken, what is the probability that the student travelled to the event in a hired limousine?

- A.  $\frac{1}{8}$
- B.  $\frac{27}{200}$
- C.  $\frac{9}{31}$
- D.  $\frac{22}{31}$

**Question 10**

Suppose a function  $f: [0, 5] \rightarrow R$  and its derivative  $f': [0, 5] \rightarrow R$  are defined and continuous on their domains. If  $f'(2) < 0$  and  $f'(4) > 0$ , which one of these statements must be true?

- A.  $f$  is strictly decreasing on  $[0, 2]$ .
- B.  $f$  does not have an inverse function.
- C.  $f$  is positive on  $[4, 5]$ .
- D.  $f$  has a local minimum at  $x = 3$ .

**Question 11**

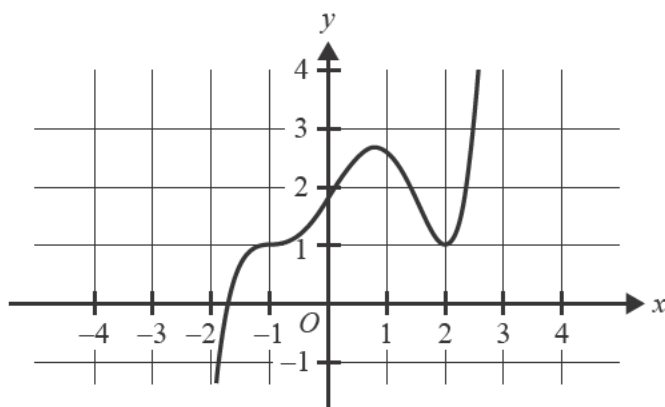
Twelve students sit in a classroom, with seven students in the first row and the other five students in the second row. Three students are chosen randomly from the class.

The probability that exactly two of the three students chosen are in the first row is

- A.  $\frac{7}{44}$
- B.  $\frac{21}{44}$
- C.  $\frac{5}{22}$
- D.  $\frac{245}{576}$

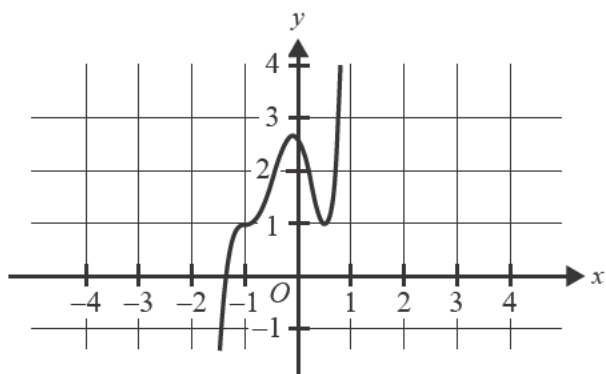
**Question 12**

The graph of  $y = f(x)$  is shown below.

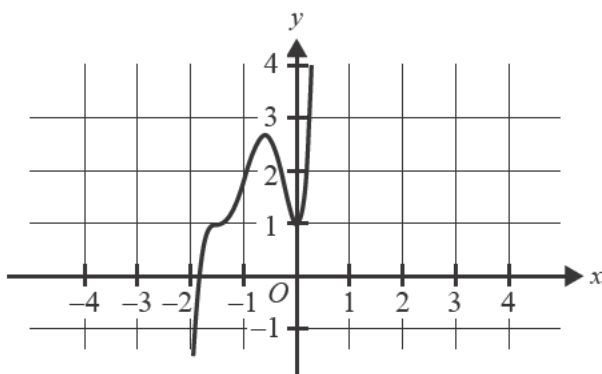


Which of the following options best represents the graph of  $y = f(2x + 1)$ ?

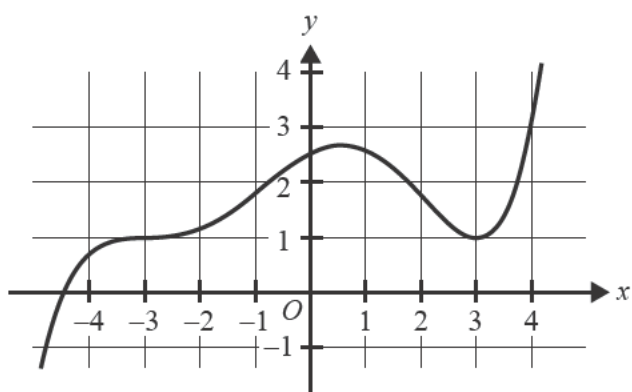
A.



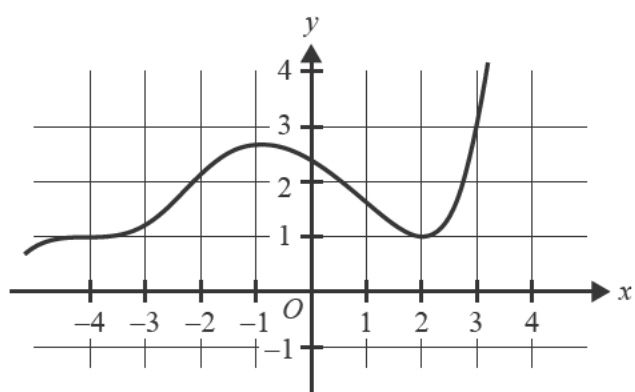
B.



C.



D.



**Question 13**

The function  $f : (0, \infty) \rightarrow \mathbb{R}$ ,  $f(x) = \frac{x}{2} + \frac{2}{x}$  is mapped to the function  $g$  with the following sequence of transformations:

1. dilation by a factor of 3 from the  $y$ -axis
2. translation by 1 unit in the negative direction of the  $y$ -axis.

The function  $g$  has a local minimum at the point with the coordinates

- A.  $(6, 1)$
- B.  $\left(\frac{2}{3}, 1\right)$
- C.  $(2, 5)$
- D.  $\left(2, -\frac{1}{3}\right)$

**Question 14**

Let  $h$  be the probability density function for a continuous random variable  $X$ , where

$$h(x) = \begin{cases} \frac{x}{6} + k & -3 \leq x < 0 \\ -\frac{x}{2} + k & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

and  $k$  is a positive real number.

The value of  $\Pr(X < 0.5)$  is

- A.  $\frac{1}{2}$
- B.  $\frac{15}{16}$
- C.  $\frac{3}{16}$
- D.  $\frac{49}{48}$

**Question 15**

The points of inflection of the graph of  $y = 2 - \tan\left(\pi\left(x - \frac{1}{4}\right)\right)$  are

- A.  $\left(k + \frac{1}{4}, 2\right), k \in \mathbb{Z}$
- B.  $\left(k - \frac{1}{4}, 2\right), k \in \mathbb{Z}$
- C.  $\left(k + \frac{1}{4}, -2\right), k \in \mathbb{Z}$
- D.  $\left(k - \frac{3}{4}, -2\right), k \in \mathbb{Z}$

**Question 16**

Suppose that a differentiable function  $f: \mathbb{R} \rightarrow \mathbb{R}$  and its derivative  $f': \mathbb{R} \rightarrow \mathbb{R}$  satisfy  $f(4) = 25$  and  $f'(4) = 15$ .

Determine the gradient of the tangent line to the graph of  $y = \sqrt{f(x)}$  at  $x = 4$ .

- A.  $\sqrt{15}$
- B.  $\frac{1}{10}$
- C.  $\frac{15}{2}$
- D.  $\frac{3}{2}$

**Question 17**

Consider the algorithm below, which prints the roots of the cubic polynomial  $f(x) = x^3 - 2x^2 - 9x + 18$ .

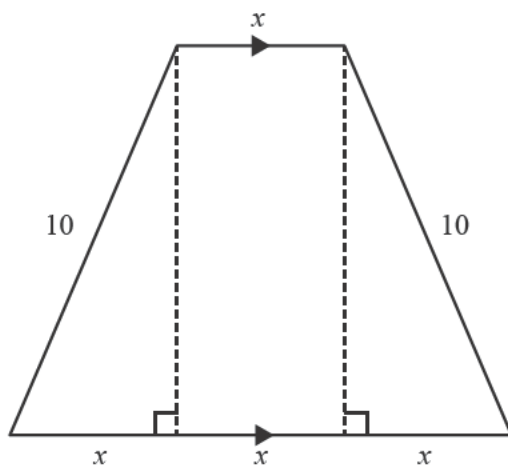
```
define f(x)
    return  $(x^3 - 2x^2 - 9x + 18)$ 
c ← f(0)
if c < 0 then
    c ← (-c)
end if
while c > 0
    if f(c) = 0 then
        print c
    end if
    if f(-c) = 0 then
        print -c
    end if
    c ← c - 1
end while
```

In order, the algorithm prints the values

- A. -3, 3, 2
- B. -3, 2, 3
- C. 3, 2, -3
- D. 3, -3, 2

**Question 18**

Find the value of  $x$  which maximises the area of the trapezium below.



- A. 10
- B.  $5\sqrt{2}$
- C. 7
- D.  $\sqrt{10}$

**Question 19**

Consider the normal random variable  $X$  that satisfies  $\Pr(X < 10) = 0.2$  and  $\Pr(X > 18) = 0.2$ .

The value of  $\Pr(X < 12)$  is closest to

- A. 0.134
- B. 0.297
- C. 0.337
- D. 0.365

**Question 20**

The function  $f: \mathbb{R} \rightarrow \mathbb{R}$  has an average value  $k$  on the interval  $[0, 2]$  and satisfies  $f(x) = f(x + 2)$  for

all  $x \in \mathbb{R}$ . The value of the definite integral  $\int_2^6 f(x)dx$  is

- A.  $2k$
- B.  $3k$
- C.  $4k$
- D.  $6k$

Do not write in this area.

## Section B

### Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

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### Question 1 (12 marks)

Consider the function  $f: R \rightarrow R$ ,  $f(x) = (x+1)(x+a)(x-2)(x-2a)$  where  $a \in R$ .

- a. State, in terms of  $a$  where required, the values of  $x$  for which  $f(x) = 0$ . 1 mark

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- b. Find the values of  $a$  for which the graph of  $y = f(x)$  has

- i. exactly three  $x$ -intercepts. 2 marks

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- ii. exactly four  $x$ -intercepts. 1 mark

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**Question 1** continues on the next page.

- c. Let  $g$  be the function  $g : \mathbb{R} \rightarrow \mathbb{R}$ ,  $g(x) = (x+1)^2(x-2)^2$ , which is the function  $f$  where  $a = 1$ .

i. Find  $g'(x)$ .

1 mark

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ii. Find the coordinates of the local maximum of  $g$ .

1 mark

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iii. Find the values of  $x$  for which  $g'(x) > 0$ .

1 mark

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iv. Consider the two tangent lines to the graph of  $y = g(x)$  at the points where

$$x = \frac{-\sqrt{3}+1}{2} \text{ and } x = \frac{\sqrt{3}+1}{2}.$$

Determine the coordinates of the point of intersection of these two tangent lines.

2 marks

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- d. Let  $g$  remain as the function  $g : R \rightarrow R$ ,  $g(x) = (x+1)^2(x-2)^2$ , which is the function  $f$  where  $a = 1$ .

Let  $h$  be the function  $h : R \rightarrow R$ ,  $h(x) = (x+1)(x-1)(x+2)(x-2)$ , which is the function  $f$  where  $a = -1$ .

- i. Using translations only, describe a sequence of transformations of  $h$ , for which its image would have a local maximum at the same coordinates as that of  $g$ .

1 mark

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- ii. Using a dilation and translations, describe a different sequence of transformations of  $h$ , for which its image would have both local minimums at the same coordinates as that of  $g$ .

2 marks

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**Question 2** (11 marks)

A model for the temperature in a room, in degrees Celsius, is given by

$$f(t) = \begin{cases} 12 + 30t & 0 \leq t \leq \frac{1}{3} \\ 22 & t > \frac{1}{3} \end{cases}$$

where  $t$  represents time in hours after a heater is switched on.

- a.** Express the derivative  $f'(t)$  as a hybrid function.

2 marks

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- b.** Find the average rate of change in temperature predicted by the model between  $t = 0$  and  $t = \frac{1}{2}$ .

Give your answer in degrees Celsius per hour.

1 mark

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- c.** Another model for the temperature in the room is given by  $g(t) = 22 - 10e^{-6t}$ ,  $t \geq 0$ .

- i.** Find the derivative  $g'(t)$ .

1 mark

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- ii.** Find the value of  $t$  for which  $g'(t) = 10$ .

Give your answer correct to three decimal places.

1 mark

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- d. Find the time  $t \in (0, 1)$  when the temperatures predicted by the models  $f$  and  $g$  are equal.

Give your answer correct to two decimal places.

1 mark

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- e. Find the time  $t \in (0, 1)$  when the difference between the temperatures predicted by the two models is the greatest.

Give your answer correct to two decimal places.

1 mark

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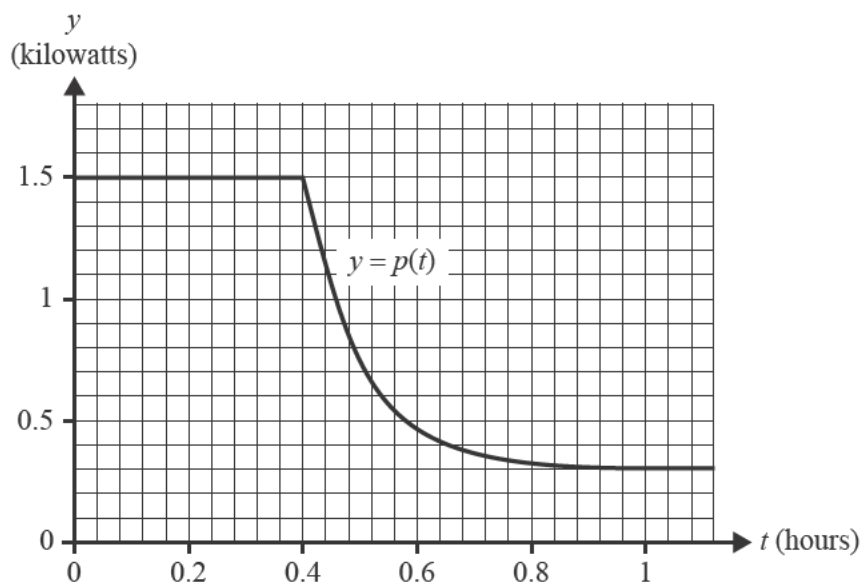
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- f. The amount of power, in kilowatts, used by the heater  $t$  hours after it is switched on, can be modelled by the continuous function  $p$ , whose graph is shown below.

$$p(t) = \begin{cases} 1.5 & 0 \leq t \leq 0.4 \\ 0.3 + Ae^{-10t} & t > 0.4 \end{cases}$$

The amount of energy used by the heater, in kilowatt hours, can be estimated by evaluating the area between the graph of  $y = p(t)$  and the  $t$ -axis.



- i. Given that  $p(t)$  is continuous for  $t \geq 0$ , show that  $A = 1.2e^4$ .

1 mark

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- ii. Find how long it takes, after the heater is switched on, until the heater has used 0.5 kilowatt hours of energy.

Give your answer in hours.

1 mark

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- iii. Find how long it takes, after the heater is switched on, until the heater has used 1 kilowatt hour of energy.

Give your answer in hours, correct to two decimal places.

2 marks

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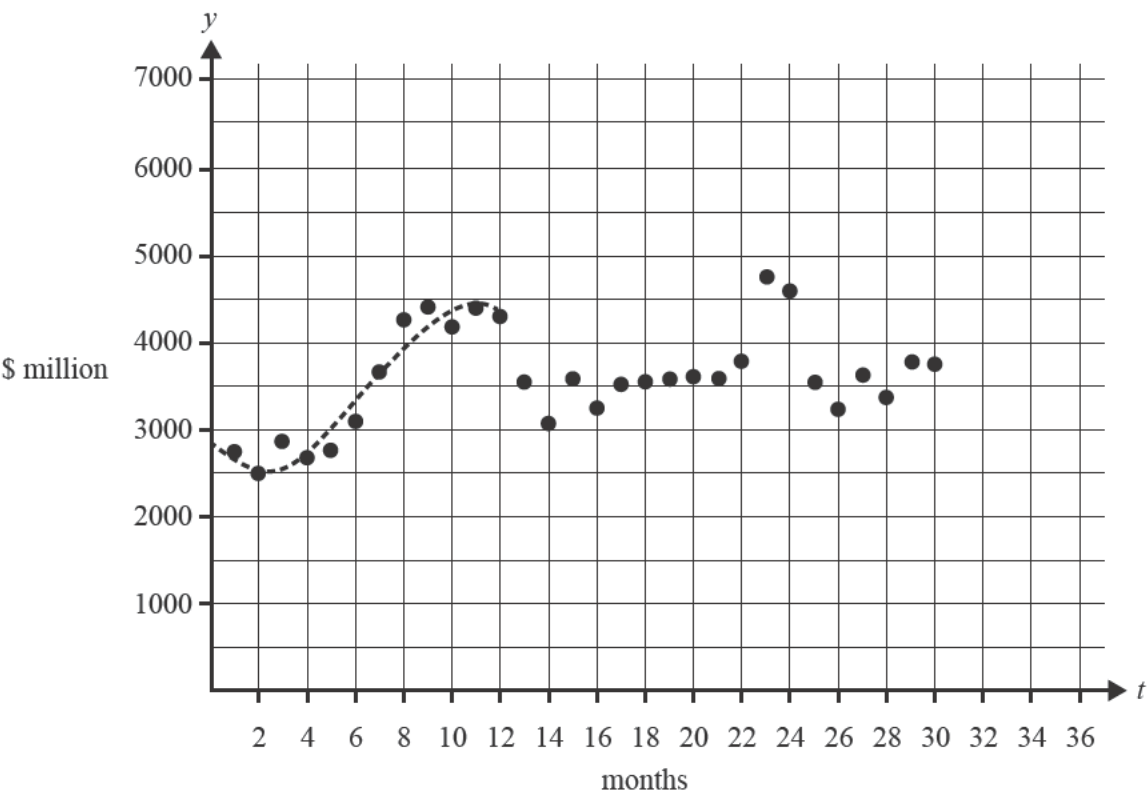
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Question 3 (11 marks)

The points shown on the chart below represent monthly online sales in Australia.

The variable  $y$  represents sales in millions of dollars.

The variable  $t$  represents the month when the sales were made, where  $t = 1$  corresponds to January 2021,  $t = 2$  corresponds to February 2021 and so on.



- a. A cubic polynomial  $p : (0, 12] \rightarrow R, p(t) = at^3 + bt^2 + ct + d$  can be used to model monthly online sales in 2021.

The graph of  $y = p(t)$  is shown as a dashed curve on the set of axes above.

It has a local minimum at  $(2, 2500)$  and a local maximum at  $(11, 4400)$ .

- i. Find, correct to two decimal places, the values of  $a, b, c$  and  $d$ .

3 marks

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- ii. Let  $q : (12, 24] \rightarrow \mathbb{R}$ ,  $q(t) = p(t - h) + k$  be a cubic function obtained by translating  $p$ , which can be used to model monthly online sales in 2022.

Find the values of  $h$  and  $k$  such that the graph of  $y = q(t)$  has a local maximum at  $(23, 4750)$ .

2 marks

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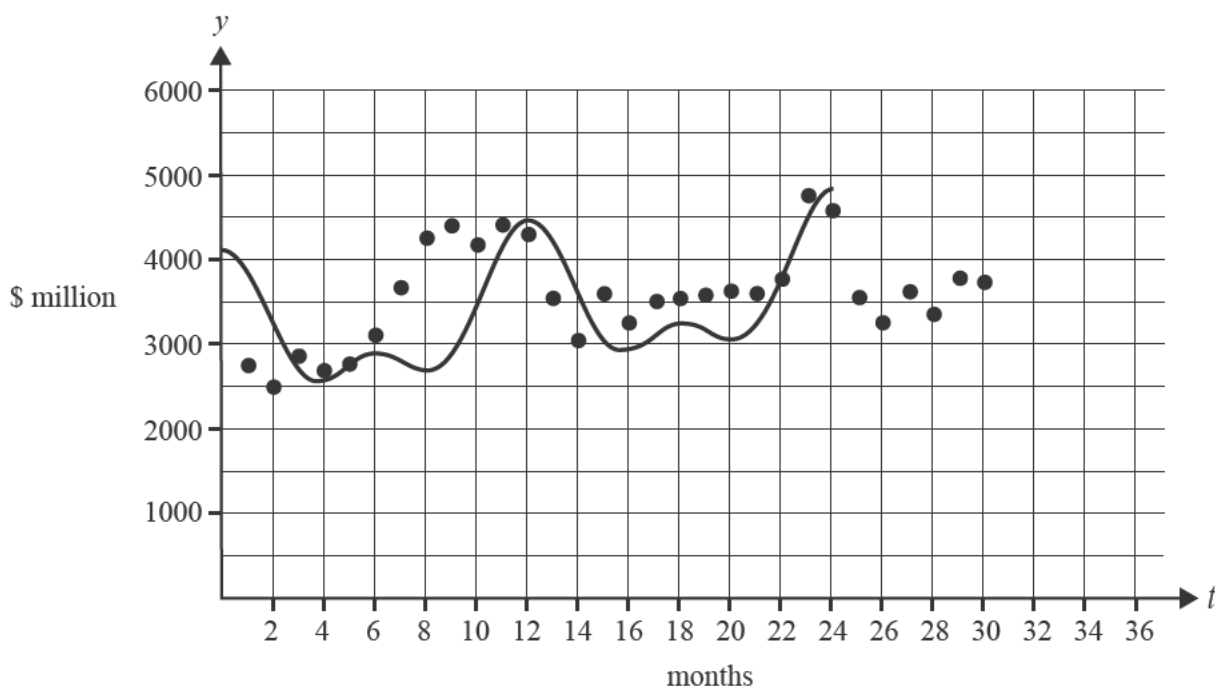


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- b. Another function  $f$  can be used to model monthly online sales, where

$$f : (0, 36] \rightarrow \mathbb{R}, f(t) = 3000 + 30t + 700 \cos\left(\frac{\pi t}{6}\right) + 400 \cos\left(\frac{\pi t}{3}\right)$$

Part of the graph of  $f$  is shown on the axes below.



- i. Complete the graph of  $f$  on the set of axes above until December 2023, that is, for  $t \in (24, 36]$ .

Label the endpoint at  $t = 36$  with its coordinates.

2 marks

- ii. The function  $f$  predicts that every 12 months, monthly online sales increase by  $n$  million dollars.

Find the value of  $n$ .

1 mark

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- iii. Find the derivative  $f'(t)$ .

1 mark

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- iv. Hence, find the maximum instantaneous rate of change for the function  $f$ , correct to the nearest million dollars per month, and the values of  $t$  in the interval  $(0, 36]$  when this maximum rate occurs, correct to one decimal place.

2 marks

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**Question 4 (15 marks)**

At an airport, luggage is weighed before it is checked in.

The mass of each piece of luggage, in kilograms, is modelled by a continuous random variable  $X$ , whose probability density function is

$$f(x) = \begin{cases} \frac{1}{67500} x^2 (30 - x) & 0 \leq x \leq 30 \\ 0 & \text{elsewhere} \end{cases}$$

A piece of luggage is labelled as heavy if its mass exceeds 23 kg.

- a.** Write a definite integral which gives the probability that a piece of luggage is labelled as heavy.

1 mark

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- b.** i. Find the mean of  $X$ .

1 mark

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- ii. Find the standard deviation of  $X$ .

2 marks

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- iii. Given that the mass of a piece of luggage is more than the mean, find the probability that it is labelled as heavy, correct to three decimal places.

2 marks

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**Question 4** continues on the next page.

Use the following information to answer parts c and d of this question.

Of the travellers flying from the airport

- 10% do not check in any luggage
- 40% check in exactly one piece of luggage
- 50% check in exactly two pieces of luggage.

Assume that the mass of each piece of luggage is independent of the number of pieces checked in by each traveller.

Use the value of 0.234 for the probability that a piece of luggage is labelled as heavy.

c. Let  $W$  be the discrete random variable that represents the number of pieces of luggage labelled as **heavy** checked in by each traveller.

i. Show that  $\Pr(W = 2) = 0.027$ , correct to three decimal places. 1 mark

ii. Complete the table below for the probability distribution  $W$ , correct to three decimal places. 2 marks

$w$	0	1	2
$\Pr(W = w)$			0.027

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- d. On a particular day, a random sample of 35 pieces of luggage was selected at the airport.

Let  $\hat{P}$  be the random variable that represents the proportion of luggage labelled as heavy in random samples of 35.

- i. Find  $\Pr(\hat{P} > 0.2)$ , correct to three decimal places.

2 marks

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- ii. Determine the probability that  $\hat{P}$  lies within one standard deviation of its mean, correct to three decimal places. Do **not** use a normal approximation.

2 marks

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- e. i. In one random sample of 50 pieces of luggage, 10 are labelled as heavy.

Use this sample to find an approximate 90% confidence interval for  $p$ , the population proportion of luggage labelled as heavy, correct to three decimal places.

1 mark

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- ii. A second random sample of 50 pieces of luggage is selected. Using this sample, the approximate 90% confidence interval for  $p$ , the population proportion of luggage labelled as heavy, is **wider** than the one obtained above in **part e.i.**

State the minimum and maximum possible number of pieces of luggage labelled as heavy in the second sample.

1 mark

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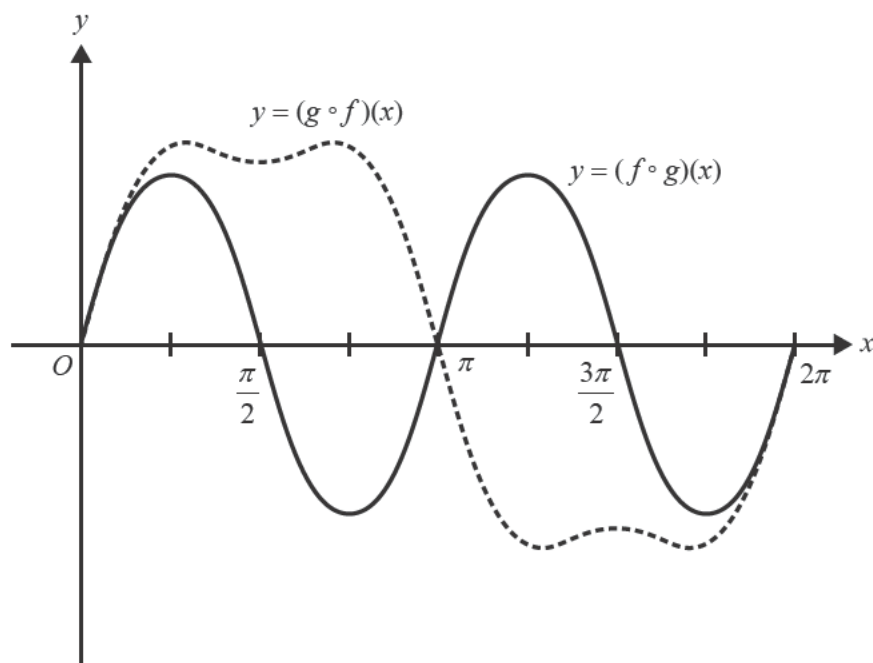
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**Question 5** (11 marks)

The graph below shows the compositions  $g \circ f$  and  $f \circ g$ , where  $f(x) = \sin(x)$  and  $g(x) = \sin(2x)$ .



- a. i. The graph of  $y = (g \circ f)(x)$  has a local maximum whose  $x$ -value lies in the interval  $\left[0, \frac{\pi}{2}\right]$ .

Find the coordinates of this local maximum, correct to one decimal place.

1 mark

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- ii. State the range of  $g \circ f$  where  $x \in [0, 2\pi]$ .

1 mark

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- b. i. Find the derivative of  $f \circ g$ .

1 mark

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- ii. Show that the equation  $\cos(\sin(2x)) = 0$  has no real solutions.

2 marks

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- iii. Find the  $x$ -values of the stationary points of  $f \circ g$  where  $x \in [0, 2\pi]$ .

1 mark

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- iv. Find the range of  $f \circ g$  where  $x \in [0, 2\pi]$ .

1 mark

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- c. i. Write a single definite integral that gives the area bounded by the graphs of  $y = (f \circ g)(x)$  and  $y = (g \circ f)(x)$  in the interval  $[0, 2\pi]$ . 1 mark

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- ii. Hence, state the area bounded by the graphs of  $y = (f \circ g)(x)$  and  $y = (g \circ f)(x)$  in the interval  $[0, 2\pi]$ , correct to two decimal places. 1 mark

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- d. Let  $f_1 : (0, 2\pi) \rightarrow \mathbb{R}$ ,  $f_1(x) = \sin(x)$ .  
Find all values of  $x$  in the interval  $(0, 2\pi)$  for which the composition  $f_1 \circ g$  is defined. 2 marks

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# Mathematical Methods Examination 2

## 2024 Formula Sheet

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You may keep this Formula Sheet.

**Mensuration**

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

**Calculus**

$\frac{d}{dx}(x^n) = nx^{n-1}$		$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$	
$\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$		$\int (ax+b)^n dx = \frac{1}{a(n+1)} (ax+b)^{n+1} + c, n \neq -1$	
$\frac{d}{dx}(e^{ax}) = ae^{ax}$		$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$	
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$		$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$	
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$		$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$	
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$		$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$	
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	Newton's method	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
trapezium rule approximation	$Area \approx \frac{x_n - x_0}{2n} \left[ f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 2f(x_{n-1}) + f(x_n) \right]$		

**Probability**

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$
binomial coefficient	$\binom{n}{x} = \frac{n!}{x!(n-x)!}$		

Probability distribution		Mean	Variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
binomial	$\Pr(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$	$\mu = np$	$\sigma^2 = np(1-p)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

**Sample proportions**

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{p}) = p$
standard deviation	$\text{sd}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left( \hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$

