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Write your **student number** in the boxes above. **Letter**

Specialist Mathematics Examination 2

Question and Answer Book

VCE Examination – Wednesday 13 November 2024

- Reading time is **15 minutes**: 11.45 am to 12 noon
- Writing time is **2 hours**: 12 noon to 2.00 pm

Approved materials

- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 24 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions

- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

VICTORIA State Government

Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1

Consider the statement

'for any integers m and n, if $m + n \geq 9$ then $m \geq 5$ or $n \geq 5$'.

The contrapositive of this statement is

A. if $m < 5$ or $n < 5$, then $m + n < 9$

B. if $m \geq 5$ or $n \geq 5$, then $m + n \geq 9$

C. if $m < 5$ and $n < 5$, then $m + n < 9$

D. if $m \leq 5$ and $n \leq 5$, then $m + n \leq 9$

Question 2

Consider the function f with rule $f(x) = \begin{cases} \dfrac{x^2 + 3x - 10}{x - 2}, & x \in R \setminus \{2\} \\ 7, & x = 2 \end{cases}$

Which of the following statements is correct?

A. The function f is continuous.

B. The graph of $y = f(x)$ has a vertical asymptote.

C. The graph of $y = f(x)$ has a horizontal asymptote.

D. The graph of $y = f(x)$ has a point of discontinuity.

Question 3

The graph of $f(x) = \dfrac{x - h}{(x + 1)(x - 4)}$, where $h \in R$, will have no turning points when

A. $h < -1$ and $h > 4$

B. $-4 < h < 1$

C. $-1 \leq h \leq 4$

D. $-4 \leq h \leq 1$

Do not write in this area.

Question 4

Given that $\sin(x) = a$, where $x \in \left(\frac{3\pi}{2}, 2\pi\right)$, then $\cos\left(\frac{x}{2}\right)$ is equal to

A. $-\frac{\sqrt{1+\sqrt{1-a^2}}}{\sqrt{2}}$

B. $\frac{\sqrt{1-\sqrt{a^2-1}}}{\sqrt{2}}$

C. $\frac{\sqrt{1+\sqrt{1-a^2}}}{\sqrt{2}}$

D. $-\frac{\sqrt{\sqrt{1-a^2}-1}}{\sqrt{2}}$

Question 5

If the point $z = 1 + \sqrt{3}i$ is represented on an Argand diagram, the point representing $-\overline{z}$ can be located by

A. reflecting the point representing z in the real axis.

B. rotating the point representing z anticlockwise about the origin by 90°.

C. reflecting the point representing z in the imaginary axis.

D. rotating the point representing z clockwise about the origin by 90°.

Question 6

Let $z = 3 + ki$ where $k \in R$.

A value of k that makes $z^2 + 4iz + 3$ purely imaginary is

A. -2

B. -1

C. 1

D. 2

Question 7

A solution to the differential equation

$\frac{dy}{dx} = e^{x-y}\left(\cos(x-y) - \cos(x+y)\right)$ can be found using

A. $\int e^{y} \cos(y)\, dy = 2\int e^{x} \cos(x)\, dx$

B. $\int \frac{e^{y}}{\sin(y)}\, dy = 2\int e^{-x} \sin(x)\, dx$

C. $\int \frac{e^{y}}{\sin(y)}\, dy = 2\int e^{x} \sin(x)\, dx$

D. $\int e^{-y} \sin(y)\, dy = 2\int \frac{e^{x}}{\cos(x)}\, dx$

Question 8

Consider the differential equation $\frac{dy}{dx} = xy^2$ where $y_0 = y(0) = 1$.

When Euler's method is applied using a step size of h, where $h > 0$, $y_3 = 1.126528$

The value of h is

A. 0.01

B. 0.02

C. 0.20

D. 0.36

Question 9

The length of the curve specified by $x = 1 - \cos(t)$ and $y = t - \sin(t)$, where $t \in [0, 2\pi]$, is given by

A. $\int_0^{2\pi} 2\sin\left(\frac{t}{2}\right) dt$

B. $\int_0^{2\pi} \sqrt{\left(1-\cos(t)\right)^2 + \left(t-\sin(t)\right)^2}\, dt$

C. $2\int_0^{2\pi} \left(1-\cos(t)\right) dt$

D. $\int_0^{2\pi} 2\cos\left(\frac{t}{2}\right) dt$

Question 10

The curve defined by the parametric equations

$x = 5t,\ y = 12t$, for $0 \le t \le k$

is rotated about the y-axis to form a surface of revolution.

The area of this surface is

A. $65\,k^2\pi$

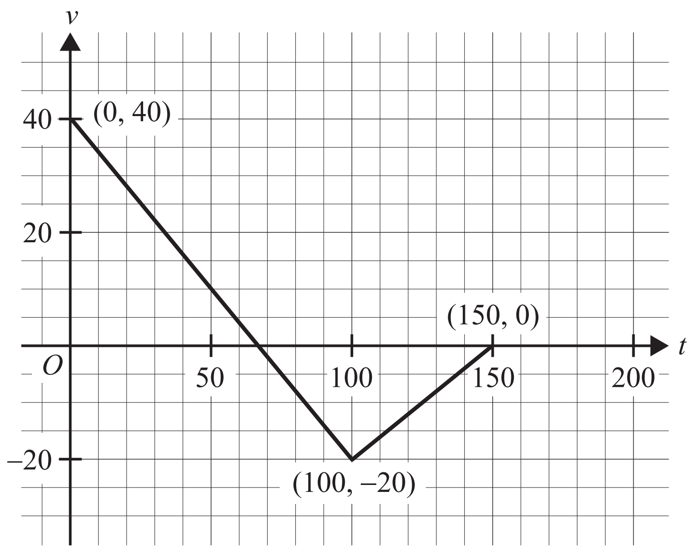
B. $130\,k^2\pi$

C. $156\,k^2\pi$

D. $825\,k^2\pi$

Question 11

The velocity–time graph of a particle moving along an east–west line with velocity v m s^{-1} at time t seconds, starting from a fixed origin O, is shown below. The graph comprises two straight line segments.



The initial velocity of the particle is 40 m s^{-1} to the east.

How far, in metres, is the particle to the east of O, 150 seconds later?

A. 450

B. 500

C. 1000

D. $\dfrac{6500}{3}$

Question 12

The position, x metres, of a particle moving in a straight line from a fixed origin O at time, t seconds, is given by $x = e^{(k-1)t}$, where $k > 1$.

The acceleration of the particle, in m s^{-2}, when $x = k + 1$ is

A. $k^2 - 1$

B. $(k^2 - 1)(k + 1)$

C. $(k^2 - 1)(k - 1)$

D. $(k - 1)^2$

Question 13

If the angle between the vectors $2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $2\underset{\sim}{i} + m\underset{\sim}{j} + 6\underset{\sim}{k}$ is $\cos^{-1}\left(\frac{13}{21}\right)$, then the value of m, where $m \in R^+$, is

A. 2

B. 3

C. 4

D. 5

Question 14

Consider the vectors $\underset{\sim}{r}$ and $\underset{\sim}{s}$ where $|\underset{\sim}{r}| = 9$ and $\underset{\sim}{s} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$.

If the vector resolute of $\underset{\sim}{r}$ in the direction of $\underset{\sim}{s}$ is equal to $-4\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k}$, then the scalar resolute of $\underset{\sim}{s}$ in the direction of $\underset{\sim}{r}$ is equal to

A. -18

B. -2

C. 2

D. 3

Question 15

The position of a moving body is given by $\underset{\sim}{r}(t) = \sin(t)\underset{\sim}{i} + \cos(2t)\underset{\sim}{j}$, where t is measured in seconds, for $t \geq 0$.

The motion of the body can be described as moving along a parabolic path given by

A. $y = 1 - 2x^2$, starting at (0, 1), reversing direction at (1, –1) and then again at (–1, –1), then returning to (0, 1) after 2π seconds.

B. $y = 1 - x^2$, starting at (1, 0), reversing direction at (–1, 0), then returning to (1, 0) after 2π seconds.

C. $y = 1 - 2x^2$, starting at (0, 1), reversing direction at (1, –1) and then again at (–1, –1), then returning to (0, 1) after π seconds.

D. $y = 1 - x^2$, starting at (1, 0), reversing direction at (–1, 0), then returning to (1, 0) after π seconds.

Question 16

Particle 1 has position vector $\underset{\sim}{r}_1(t) = \cos(t)\underset{\sim}{i} + \sin(t)\underset{\sim}{j} + \sqrt{\sin(2t)}\,\underset{\sim}{k}$ and Particle 2 has position vector $\underset{\sim}{r}_2(t) = \sin(t)\underset{\sim}{i} + \cos(t)\underset{\sim}{j} + \sqrt{\sin(2t)}\,\underset{\sim}{k}$, where t is measured in seconds and $t \in \left(0, \frac{\pi}{2}\right)$.

The number of times the **velocity** of Particle 1 is perpendicular to the **position** vector $\underset{\sim}{r}_2(t)$ during the first $\frac{\pi}{2}$ seconds is

A. 1

B. 2

C. 3

D. 4

Question 17

Consider the following parallel lines.

$L_1 : \underset{\sim}{r}_1 = \underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k} + s\left(\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}\right)$ and $L_2 : \underset{\sim}{r}_2 = -2\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k} + t\left(\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}\right)$ where $s, t \in R$.

The shortest distance between L_1 and L_2 is

A. 3

B. $\sqrt{14}$

C. $\sqrt{17}$

D. 14

Question 18

The point of intersection of the line $\underset{\sim}{r} = \underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k} + t\left(-2\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}\right)$, where $t \in R$ and the plane $3x - 2y + 4z = 5$ is

A. $(-5, -1, 2)$

B. $(-1, 2, 1)$

C. $(3, 4, 1)$

D. $(-5, 4, 7)$

Question 19

When conducting a hypothesis test, a type II error occurs when

A. a null hypothesis is not rejected when the alternative hypothesis is true.

B. a null hypothesis is rejected when it is true.

C. a null hypothesis is rejected when the alternative hypothesis is true.

D. a null hypothesis is not rejected when it is doubtful.

Question 20

The masses of avocados in a crop may be assumed to be normally distributed, with a mean of 200 grams and a standard deviation of 7.5 grams.

After an avocado of mass M grams is peeled and the stone is removed, the mass of edible flesh F grams is given by $F = 0.70M$. Four avocados are randomly selected from the crop.

What is the probability, correct to four decimal places, that a total of more than 570 grams of edible flesh is obtained?

A. 0.0868

B. 0.1705

C. 0.2128

D. 0.3170

Do not write in this area.

End of Section A

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Section B

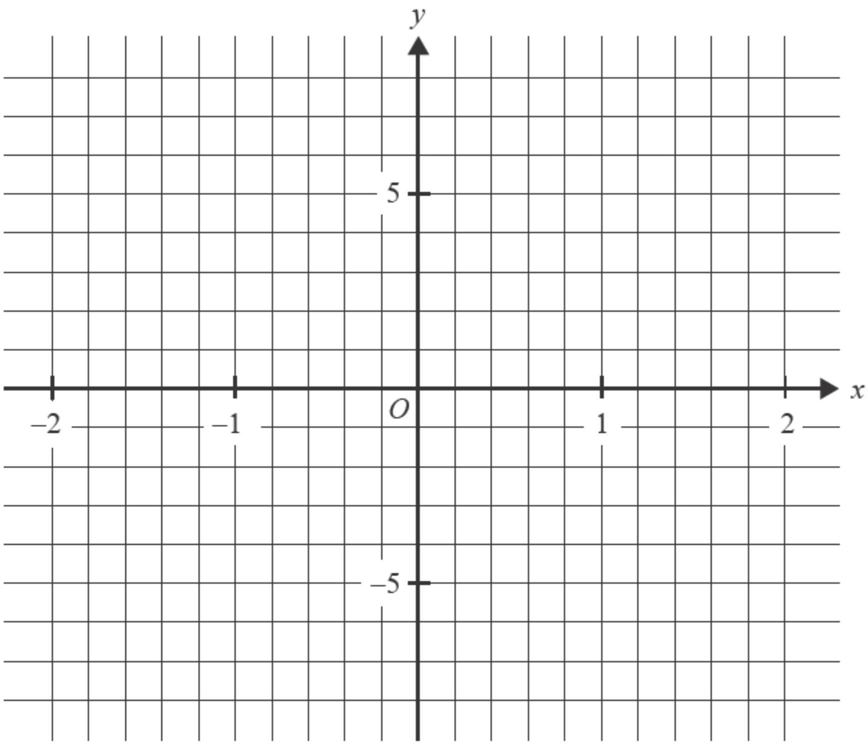
Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- Unless otherwise specified, an **exact** answer is required to a question.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- Take the **acceleration due to gravity** to have magnitude g m s^{-2}, where $g = 9.8$

Question 1 (10 marks)

Consider the function f with rule $f(x) = \dfrac{x^4 - x^2 + 1}{1 - x^2}$.

a. Sketch the graph of $y = f(x)$ on the set of axes below. Label the vertical asymptotes with their equations and label the stationary points with their coordinates. 3 marks



b. The region bounded by the graph of $y = f(x)$ and the lines $y = 1$ and $y = 6$ is rotated about the y-axis to form a solid of revolution.

i. Write down a definite integral involving only the variable y, that when evaluated, will give the volume of the solid. 2 marks

ii. Find the volume of the solid, correct to one decimal place. 1 mark

c. Now consider the function g with rule $g(x) = \dfrac{x^4 + b}{1 - x^2}$, where $b \in R$.

For what value of b will the graph of g have no asymptotes? 1 mark

d. The gradient function of g is given by $g'(x) = \dfrac{-2x\left((x^2 - 1)^2 - (b + 1)\right)}{(1 - x^2)^2}$.

For what values of b will the graph of g have exactly

i. one stationary point? 1 mark

ii. three stationary points? 1 mark

iii. five stationary points? 1 mark

Question 2 (10 marks)

a. Express the relation $|z - z_1| = |z - z_2|$ in the form $y = mx + c$, where $x, y, m, c \in R$, $z = x + iy$, $z_1 = 1 + 2i$ and $z_2 = 4$. 2 marks

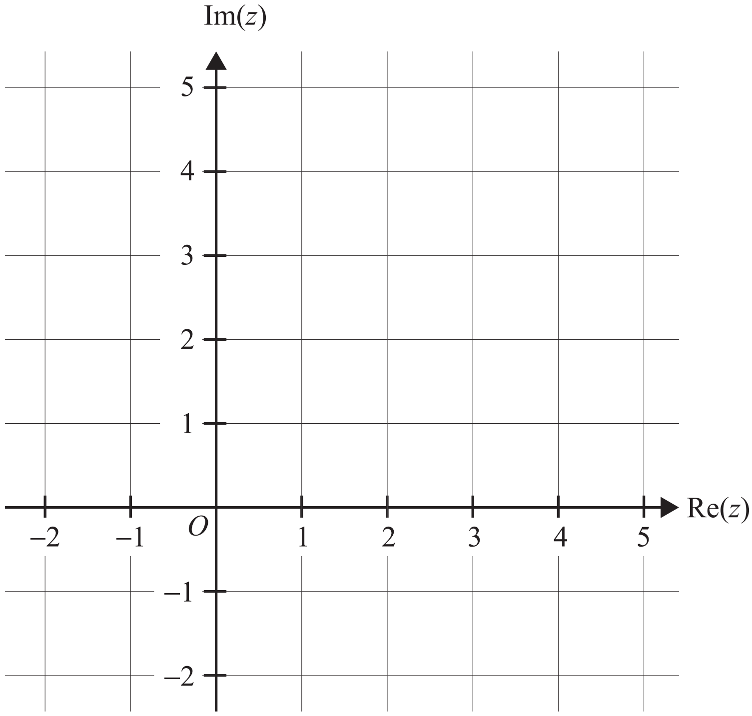
b. The line segment from $z_1 = 1 + 2i$ to $z_2 = 4$ is the diameter of a circle.

Find the equation of this circle in the form $|z - z_c| = r$, where z_c is the centre of the circle and r is the radius. 2 marks

Do not write in this area.

c. A second circle is given by $\left|z-(1+2i)\right|=2$.

Sketch this circle on the Argand diagram below, labelling the imaginary axis intercepts with their values. 2 marks



A ray originating at the point $z = 2 - i$ passes through the point $z = -2 + 3i$, cutting the second circle into two segments.

d. **i.** Sketch the ray on the Argand diagram provided in **part c**. 1 mark

ii. Find the equation of the ray in the form $\text{Arg}(z - z_0) = \theta$ where $z_0 \in C$ and θ is measured in radians in terms of π. 1 mark

e. Find the area of the minor segment formed by the intersection of the ray and the circle. 2 marks

Question 3 (10 marks)

A pollutant, at time $t = 0$ days, begins to enter a pond of still, unpolluted water at a rate of

$\frac{dV}{dt} = \frac{8t}{240 + 5t^4}$, where V is the volume of pollutant, in cubic metres, in the pond after t days.

The pollutant does not dissolve or mix, and spreads across the pond, maintaining the shape of a thin circular disc of radius $r(t)$ metres and constant depth of 1 millimetre.

a. What is the maximum rate, in cubic metres per day, at which the pollutant will enter the pond, and for what value of t will this rate occur? 1 mark

b. At what rate is the radius of the disc increasing after $t = 4$ days, where it may be assumed that the radius of the disc is 6.54 m?

Give your answer in metres per day correct to two decimal places. 3 marks

c. **i.** Use the substitution $u = \sqrt{5}\,t^2$ to express $\int \frac{8t}{240 + 5t^4}\,dt$ as an integral involving only the variable u. 1 mark

ii. Hence, or otherwise, find, in terms of t, the total volume V m^3 of pollutant that has entered the pond after t days.

Give your answer in the form $\frac{1}{a\sqrt{b}}\arctan\left(\frac{t^c}{d\sqrt{b}}\right)$, where $a, b, c, d \in Z^+$. 1 mark

d. What surface area of the pond would the coverage of the pollutant approach?

Give your answer in square metres correct to two decimal places. 2 marks

Question 3 continues on the next page.

e. The clean-up of the pond begins after five days, where the pollutant is removed at a constant rate of 0.05 cubic metres per day until the pond is free of pollutant. However, efforts to stem the flow are unsuccessful and the pollutant continues to enter the pond at a rate of $\dfrac{8t}{240 + 5t^4}$ cubic metres per day.

After how many days, from the start of the clean-up, will the pond be free of pollutant? Give your answer in days correct to one decimal place. 2 marks

Question 4 (11 marks)

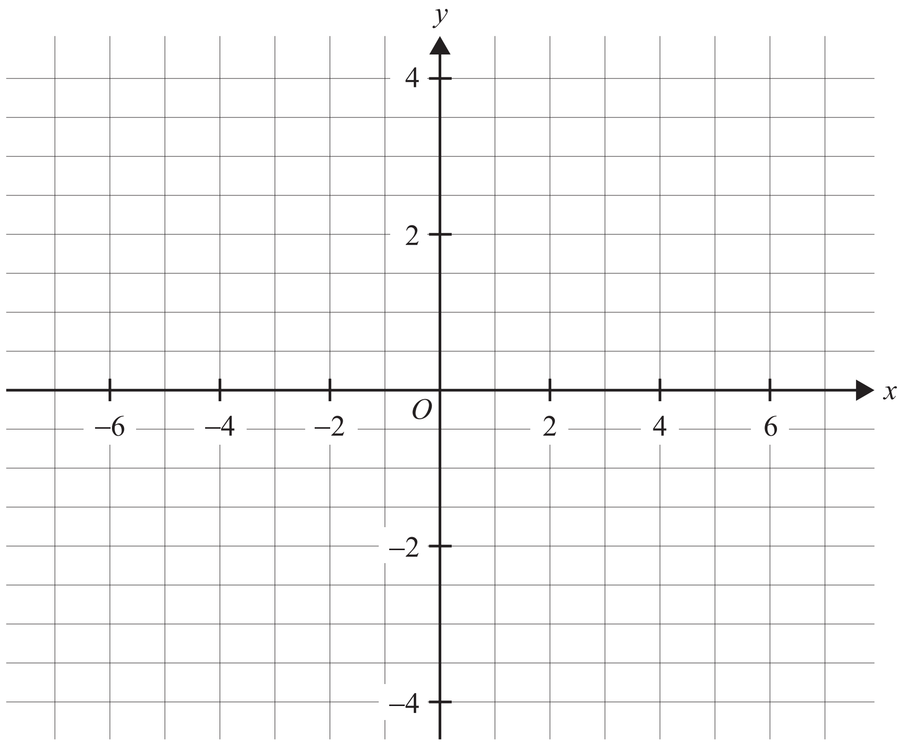
A model yacht is sailing on a lake between two buoys. Its path from one buoy to the other, relative to an origin O, is given by

$$\underset{\sim}{r}_Y(t) = 3\sec(t)\underset{\sim}{i} + 2\tan(t)\underset{\sim}{j}, \text{ where } \frac{2\pi}{3} \le t \le \frac{4\pi}{3}.$$

Displacement components are measured in metres, and time t is measured in minutes.

a. Use a trigonometric identity to show that the Cartesian equation of the path is given by $\frac{x^2}{9} - \frac{y^2}{4} = 1$. 1 mark

b. Sketch the path of the yacht on the axes below. Label the endpoints with their coordinates and show the direction of motion. 2 marks



Question 4 continues on the next page.

c. **i.** Write down an expression, in terms of $\sec(t)$, for the square of the speed of the yacht at any time, t. 1 mark

ii. Find the time, in minutes, when the minimum speed occurs.

You do not need to justify that this speed is a minimum. 1 mark

iii. State the minimum speed of the yacht in metres per minute. 1 mark

iv. State the coordinates of the yacht when the minimum speed occurs. 1 mark

d. **i.** Write down a definite integral, in terms of t, that gives the distance travelled by the yacht along the path given by $\underset{\sim}{r}_Y(t)$ over the time interval $\frac{2\pi}{3} \le t \le \frac{4\pi}{3}$. 1 mark

ii. Find the distance travelled by the yacht over this time interval.

Give your answer in metres correct to one decimal place. 1 mark

e. The position vector of a drone videoing the yacht, relative to the same origin as the yacht, O, is given by $\underset{\sim}{r}_D(t) = (2-3t)\underset{\sim}{i} + (4t-1)\underset{\sim}{j} + (6-t)\underset{\sim}{k}$, where $0 \le t \le 5$.

Displacement components are measured in metres, and time t is measured in minutes.

What is the shortest distance from the drone to the yacht, as the yacht sails along the path given by $\underset{\sim}{r}_Y(t) = 3\sec(t)\underset{\sim}{i} + 2\tan(t)\underset{\sim}{j}$, where $\frac{2\pi}{3} \le t \le \frac{4\pi}{3}$?

Give your answer in metres, correct to one decimal place. 2 marks

Question 5 (10 marks)

Consider the points $A(1, -2, 3)$ and $B(2, -5, -1)$.

a. Find a vector equation, in terms of the components $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$, for the line passing through these points. 1 mark

b. Consider the different line L_1: $\underset{\sim}{r}_1(t) = 2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k} + t\left(-\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}\right), t \in R$.

Find the shortest distance from L_1 to point A.

Give your answer in the form $\dfrac{a\sqrt{b}}{c}$ where a, b and c are positive integers. 3 marks

Do not write in this area.

c. Let C be the point $(0, 2, -5)$.

Find the Cartesian equation of the plane that contains the points A, B and C. 3 marks

d. Another plane has the Cartesian equation $2x - 3y + 4z = 12$.

This plane intersects the coordinate axes at three points, which form the vertices of a triangle.

i. Find the coordinates of these three points. 1 mark

ii. Find the area of the triangle.

Give your answer in the form $m\sqrt{n}$ where m and n are integers. 2 marks

Question 6 (9 marks)

A machine fills bottles with olive oil. The volume of olive oil dispensed into each bottle may be assumed to be normally distributed with mean μ millilitres (mL) and standard deviation $\sigma = 4.2$ mL. When the machine is working properly $\mu = 1000$.

The volume dispensed is monitored regularly by taking a random sample of nine bottles and finding the mean volume dispensed.

The machine will be paused and adjusted if the mean volume of olive oil in the nine bottles is significantly less than 1000 mL at the 5% level of significance.

When checked, a random sample of nine bottles gave a mean volume of 997.5 mL.

A one-sided statistical test is to be performed.

a. Write down suitable null and alternative hypotheses H_0 and H_1 for the test. 1 mark

b. **i.** Find the p value for this test correct to three decimal places. 1 mark

ii. Using the p value found in **part b.i**, state with a reason whether the machine should be paused. 1 mark

c. Assuming that the mean volume dispensed by the machine each time is in fact 997 mL and not 1000 mL, find the probability of a type II error for the test using nine bottles at the 5% level of significance. Assume that the population standard deviation is 4.2 mL, and give your answer correct to two decimal places. 2 marks

d. Let $\overline{X}$ denote the sample mean of a random sample of nine bottles. As a quality-control measure, the machine will be paused if $\overline{X} < a$ or if $\overline{X} > b$, where $\Pr\left(\overline{X} < a\right) = 0.01$ and $\Pr\left(\overline{X} > b\right) = 0.01$.

Assume $\mu = 1000$ mL and $\sigma = 4.2$ mL.

Find the values of a and b correct to one decimal place. 1 mark

A new machine is purchased, and it is observed that the volume dispensed by the new machine in 50 randomly chosen bottles provided a sample mean of 1005 mL and a sample standard deviation of 4 mL.

e. Find a 95% confidence interval for the population mean volume dispensed by the new machine, giving values correct to one decimal place. You may assume a population standard deviation of 4 mL. 1 mark

f. Forty samples, each consisting of 50 randomly chosen bottles, are taken, and a 95% confidence interval is calculated for each sample.

In how many of these confidence intervals would the population mean volume dispensed by the machine be expected to lie? 1 mark

g. What minimum size sample should be used so that, with 95% confidence, the sample mean is within 1 mL of the population mean volume dispensed by the new machine? Assume a population standard deviation of 4 mL. 1 mark

End of Question and Answer Book

Specialist Mathematics Examination 2

2024 Formula Sheet

You may keep this Formula Sheet.

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Mensuration

area of a circle segment	$\frac{r^2}{2}(\theta - \sin(\theta))$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$	sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
volume of a pyramid	$\frac{1}{3}Ah$	cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Algebra, number and structure (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$	$\lvert z \rvert = \sqrt{x^2 + y^2} = r$	
$-\pi < \operatorname{Arg}(z) \le \pi$	$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$	
$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$	de Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$

Data analysis, probability and statistics

for independent random variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(aX_1 + b) = a\,\mathrm{E}(X_1) + b$ $\mathrm{E}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1\mathrm{E}(X_1) + a_2\mathrm{E}(X_2) + \ldots + a_n\mathrm{E}(X_n)$	
	$\mathrm{Var}(aX_1 + b) = a^2\mathrm{Var}(X_1)$ $\mathrm{Var}(a_1X_1 + a_2X_2 + \ldots + a_nX_n)$ $= a_1^2\mathrm{Var}(X_1) + a_2^2\mathrm{Var}(X_2) + \ldots + a_n^2\mathrm{Var}(X_n)$	
for independent identically distributed variables $X_1, X_2, \ldots, X_n$	$\mathrm{E}(X_1 + X_2 + \ldots + X_n) = n\mu$	
	$\mathrm{Var}(X_1 + X_2 + \ldots + X_n) = n\sigma^2$	
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	
distribution of sample mean $\bar{X}$	mean	$E(\bar{X}) = \mu$
	variance	$\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = n x^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \ n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = a e^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\cot(ax)\right) = -a\,\mathrm{cosec}^2(ax)$	$\int \mathrm{cosec}^2(ax) dx = -\frac{1}{a}\cot(ax) + c$
$\frac{d}{dx}\left(\sec(ax)\right) = a\sec(ax)\tan(ax)$	$\int \sec(ax)\tan(ax)\, dx = \frac{1}{a}\sec(ax) + c$
$\frac{d}{dx}\left(\mathrm{cosec}(ax)\right) = -a\,\mathrm{cosec}(ax)\cot(ax)$	$\int \mathrm{cosec}(ax)\cot(ax)\, dx = -\frac{1}{a}\mathrm{cosec}(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(ax)\right) = \frac{a}{\sqrt{1-(ax)^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\cos^{-1}(ax)\right) = \frac{-a}{\sqrt{1-(ax)^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \ a > 0$
$\frac{d}{dx}\left(\tan^{-1}(ax)\right) = \frac{a}{1+(ax)^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n\, dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, \quad n \neq -1$
	$\int \frac{1}{ax+b} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$

Calculus – continued

product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
integration by parts	$\int u\frac{dv}{dx}\,dx = u\,v - \int v\frac{du}{dx}\,dx$
Euler's method	If $\frac{dy}{dx} = f(x,\ y)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + h \times f\left(x_n, y_n\right)$
arc length parametric	$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area Cartesian about x-axis	$\int_{x_1}^{x_2} 2\pi y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$
surface area Cartesian about y-axis	$\int_{y_1}^{y_2} 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$
surface area parametric about x-axis	$\int_{t_1}^{t_2} 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
surface area parametric about y-axis	$\int_{t_1}^{t_2} 2\pi x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

Kinematics

acceleration	$a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$	
constant acceleration formulas	$v = u + at$	$s = ut + \dfrac{1}{2}at^2$
	$v^2 = u^2 + 2as$	$s = \dfrac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r}(t) = x(t)\underset{\sim}{i} + y(t)\underset{\sim}{j} + z(t)\underset{\sim}{k}$	$\lvert\underset{\sim}{r}(t)\rvert = \sqrt{x(t)^2 + y(t)^2 + z(t)^2}$
	$\dot{\underset{\sim}{r}}(t) = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$
for $\underset{\sim}{r}_1 = x_1\underset{\sim}{i} + y_1\underset{\sim}{j} + z_1\underset{\sim}{k}$ and $\underset{\sim}{r}_2 = x_2\underset{\sim}{i} + y_2\underset{\sim}{j} + z_2\underset{\sim}{k}$	vector scalar product $\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = \lvert\underset{\sim}{r}_1\rvert\lvert\underset{\sim}{r}_2\rvert\cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$
	vector cross product $\underset{\sim}{r}_1 \times \underset{\sim}{r}_2 = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = (y_1z_2 - y_2z_1)\underset{\sim}{i} + (x_2z_1 - x_1z_2)\underset{\sim}{j} + (x_1y_2 - x_2y_1)\underset{\sim}{k}$
vector equation of a line	$\underset{\sim}{r}(t) = \underset{\sim}{r}_1 + t\underset{\sim}{r}_2 = (x_1 + x_2t)\underset{\sim}{i} + (y_1 + y_2t)\underset{\sim}{j} + (z_1 + z_2t)\underset{\sim}{k}$
parametric equation of a line	$x(t) = x_1 + x_2t \quad y(t) = y_1 + y_2t \quad z(t) = z_1 + z_2t$
vector equation of a plane	$\underset{\sim}{r}(s, t) = \underset{\sim}{r}_0 + s\underset{\sim}{r}_1 + t\underset{\sim}{r}_2$ $= (x_0 + x_1s + x_2t)\underset{\sim}{i} + (y_0 + y_1s + y_2t)\underset{\sim}{j} + (z_0 + z_1s + z_2t)\underset{\sim}{k}$
parametric equation of a plane	$x(s, t) = x_0 + x_1s + x_2t, \; y(s, t) = y_0 + y_1s + y_2t, \; z(s, t) = z_0 + z_1s + z_2t$
Cartesian equation of a plane	$ax + by + cz = d$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \dfrac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\sin(2x) = 2\sin(x)\cos(x)$	
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	$\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$
$\sin^2(ax) = \dfrac{1}{2}\left(1-\cos(2ax)\right)$	$\cos^2(ax) = \dfrac{1}{2}\left(1+\cos(2ax)\right)$

End of Formula Sheet