VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2006

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

									Letter
Figures									
Words									

FURTHER MATHEMATICS

Written examination 2

Wednesday 1 November 2006

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 32 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core

Table 1 shows the heights (in cm) of three groups of randomly chosen boys aged 18 months, 27 months and 36 months respectively.

Table 1.

	height (cm)	
18 months	27 months	36 months
76.0	82.0	88.0
78.5	83.1	88.8
78.6	84.0	90.0
80.0	86.8	92.3
80.5	87.2	93.0
81.2	87.6	94.1
82.8	88.3	94.2
83.2	90.7	95.8
83.4	91.0	96.9
83.7	92.3	97.1
85.8	92.5	97.8
86.6	93.1	99.2
87.3	94.8	100.6
89.8	97.2	103.8

Question 1

a. Complete Table 2 by calculating the standard deviation of the heights of the **18-month-old** boys. Write your answer correct to one decimal place.

Table 2.

age	18 months	27 months	36 months
mean	82.7	89.3	95.1
standard deviation		4.5	4.5

1 mark

A **27-month-old** boy has a height of 83.1 cm.

b. Calculate his standardised height (*z* score) relative to this sample of 27-month-old boys.

Write your answer correct to one decimal place.

1 mark

Core – Question 1 – continued

The heights of the **36-month-old** boys are normally distributed.

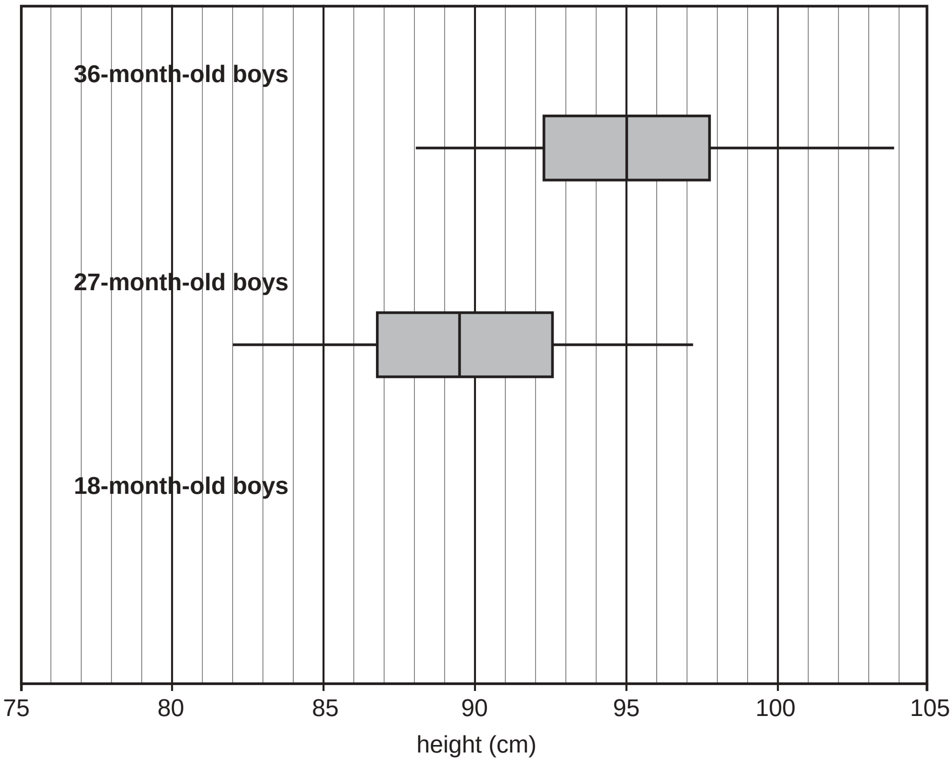
A 36-month-old boy has a standardised height of 2.

c. Approximately what percentage of 36-month-old boys will be shorter than this child?

percentage =

1 mark

Using the data from Table 1, boxplots have been constructed to display the distributions of heights of 36-month-old and 27-month-old boys as shown below.



d. Complete the display by constructing and drawing a boxplot that shows the distribution of heights for the **18-month-old** boys.

2 marks

e. Use the appropriate boxplot to determine the median height (in centimetres) of the **27-month-old** boys.

1 mark

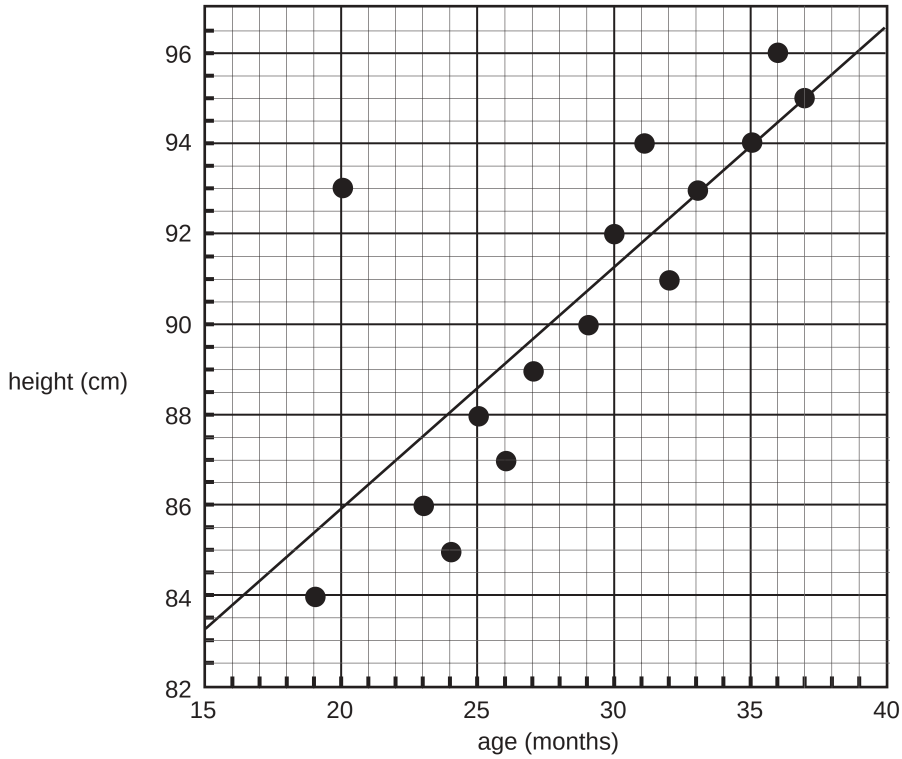
The three parallel boxplots suggest that *height* and *age* (18 months, 27 months, 36 months) are **positively** related.

f. Explain why, giving reference to an appropriate statistic.

1 mark

Question 2

The heights (in cm) and ages (in months) of a different random sample of 15 boys have been plotted in the scatterplot below. The least squares regression line has been fitted to the data.



The equation of the least squares regression line is ***height* = 75.4 + 0.53 × *age***

The correlation coefficient is ***r* = 0.7541**

a. Complete the following sentence.

On average, the height of a boy increases by ☐ cm for each one-month increase in age.

1 mark

b. **i.** Evaluate the coefficient of determination.

Write your answer, as a percentage, correct to one decimal place.

____

ii. Interpret the coefficient of determination in terms of the variables *height* and *age*.

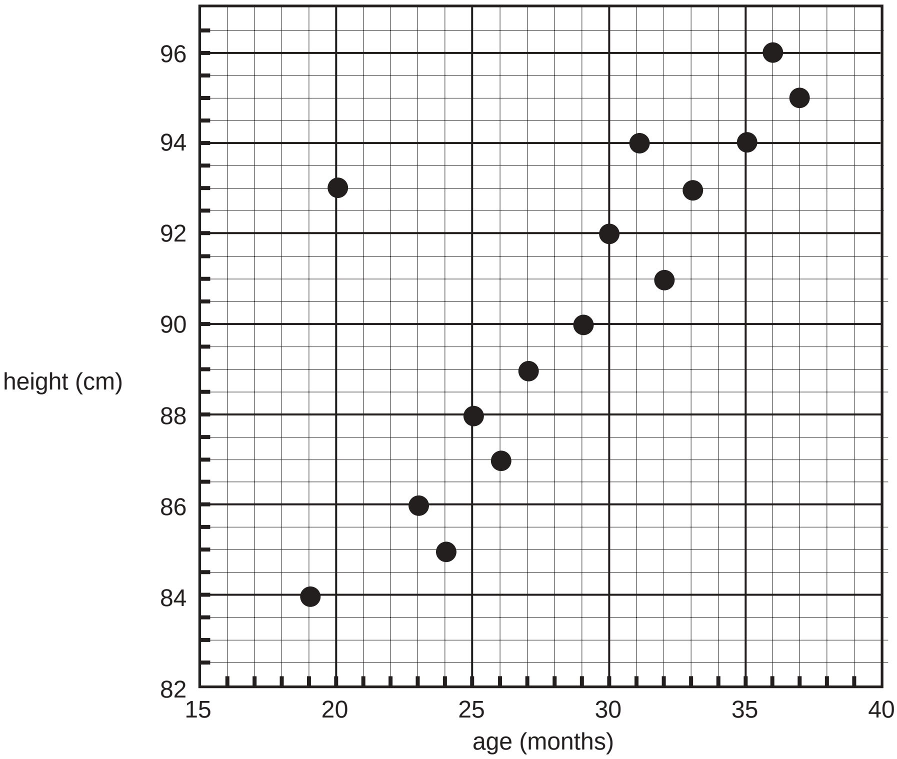
____

____

1 + 1 = 2 marks

Question 3

The heights (in cm) and ages (in months) of the 15 boys are shown in the scatterplot below.



a. Fit a **three median** line to the scatterplot. **Circle** the three points you used to determine this three median line.

2 marks

b. Determine the equation of the three median line. Write the equation in terms of the variables *height* and *age* and give the slope and intercept correct to one decimal place.

2 marks

c. Explain why the three median line might model the relationship between *height* and *age* better than the least squares regression line.

1 mark

Total 15 marks

END OF CORE
TURN OVER

Module 1: Number patterns

Question 1

It is estimated that the trees in an orchard contain 48 000 kg of fruit.

Each day 3 000 kg of fruit is picked from the trees.

a. How many kilograms of fruit remain on the trees at the end of the second day?

1 mark

b. The number of kilograms of fruit, F_n, remaining on the trees at the end of the nth day can be written as

$$F_n = 48\,000 + d \times n$$

The value of d is

$d = \boxed{}$

1 mark

c. How many days, in total, will it take to pick all the fruit from the trees?

1 mark

Question 2

A new type of fruit tree is planted in the orchard.
In the first month after planting, the gardeners worked 625 hours.
In the second and third months after planting, the gardeners worked 500 hours and 400 hours respectively.
Suppose this decreasing pattern of work continues.

The number of hours the gardeners work each month follows a geometric sequence.

a. Show that the common ratio of this sequence is $r = 0.8$

1 mark

b. Determine the number of hours the gardeners will work in the fifth month.

1 mark

c. Write an expression that gives the number of hours, H_n, the gardeners will work in the nth month after planting.

1 mark

d. How many more hours will the gardeners work in the sixth month than in the seventh month?

Write your answer correct to the nearest hour.

1 mark

e. In which month will the gardeners first work less than 100 hours?

1 mark

In the first three months after planting, the gardeners worked a total of 1525 hours.

f. How many hours, in total, will the gardeners work in the next nine months?

Write your answer correct to the nearest hour.

2 marks

Question 3

The water used in the orchard is stored in a tank.
Each afternoon, 10% of the volume of water in the tank is used.
Each evening, 2 000 litres of water is added to the tank.
This pattern continues each day.

The volume of water, V_n, in the tank on the morning of the nth day is modelled by the difference equation

$$V_{n+1} = rV_n + d \quad \text{where } V_1 = 45\,000 \text{ litres.}$$

a. Find r and d.

r =

d =

2 marks

b. Determine how many litres of water will be in the tank on the morning of the fourth day.

1 mark

c. On the morning of which day will the volume of water in the tank first be below 30 000 litres?

1 mark

d. In the long term, how many litres of water will be in the tank each morning?
Write your answer correct to the nearest litre.

1 mark

Total 15 marks

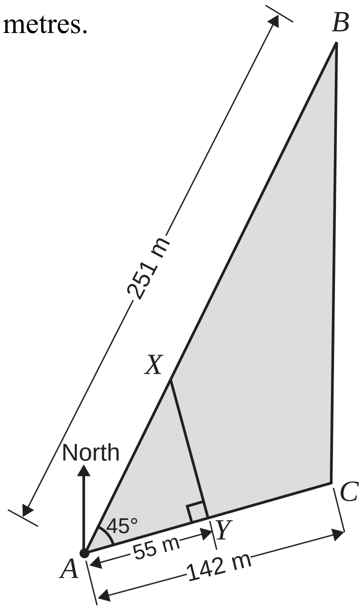
END OF MODULE 1

Working space

Module 2: Geometry and trigonometry

Question 1

A farmer owns a flat allotment of land in the shape of triangle *ABC* shown below.
Boundary *AB* is 251 metres.
Boundary *AC* is 142 metres.
Angle *BAC* is 45°.



A straight track, *XY*, runs perpendicular to the boundary *AC*.
Point *Y* is 55 m from *A* along the boundary *AC*.

a. Determine the size of angle *AXY*.

1 mark

b. Determine the length of *AX*.
Write your answer, in metres, correct to one decimal place.

1 mark

c. The bearing of *C* from *A* is 078°.
Determine the bearing of *B* from *A*.

1 mark

d. Determine the shortest distance from *X* to *C*.
Write your answer, in metres, correct to one decimal place.

2 marks

Module 2: Geometry and trigonometry – Question 1 – continued

e. Determine the area of triangle *ABC* correct to the nearest square metre.

1 mark

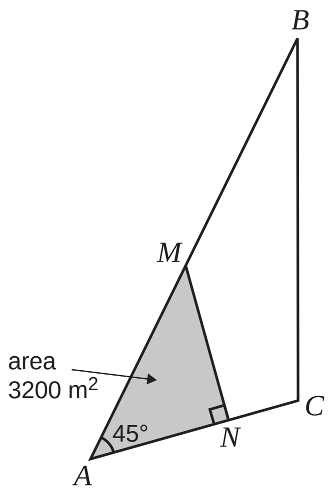
The length of the boundary *BC* is 181 metres (correct to the nearest metre).

f. **i.** Use the cosine rule to show how this length can be found.

ii. Determine the size of angle *ABC*.
Write your answer, in degrees, correct to one decimal place.

1 + 1 = 2 marks

A farmer plans to build a fence, *MN*, perpendicular to the boundary *AC*.
The land enclosed by triangle *AMN* will have an area of 3200 m^2.



g. Determine the length of the fence *MN*.

2 marks

Question 2

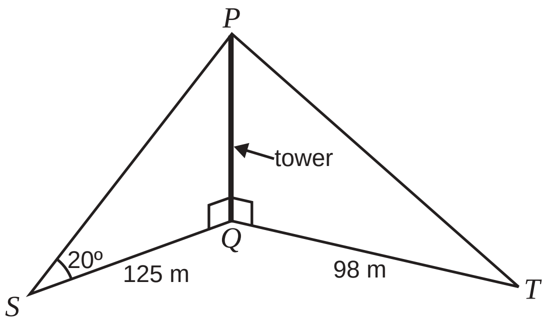
The allotment of land contains a communications tower, *PQ*.

Points *S*, *Q* and *T* are situated on level ground.

From *S* the angle of elevation of *P* is 20°.

Distance *SQ* is 125 metres.

Distance *TQ* is 98 metres.



a. Determine the height, *PQ*, of the communications tower.

Write your answer, in metres, correct to one decimal place.

1 mark

b. Determine the angle of depression of *T* from *P*.

Write your answer, in degrees, correct to one decimal place.

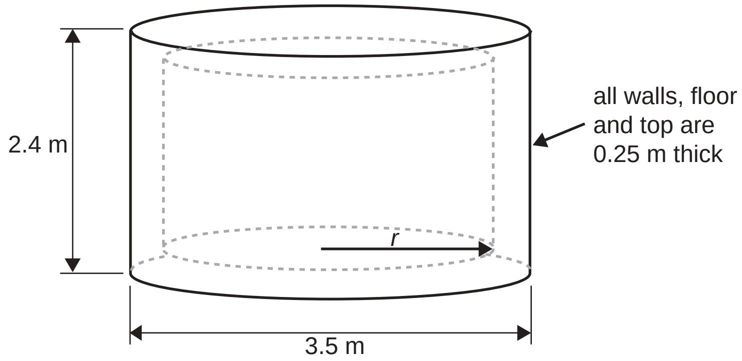
1 mark

Question 3

A **closed** cylindrical water tank has external diameter 3.5 metres.

The external height of the tank is 2.4 metres.

The walls, floor and top of the tank are made of concrete 0.25 m thick.



a. What is the internal radius, r, of the tank?

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1 mark

b. Determine the maximum amount of water this tank can hold.

Write your answer correct to the nearest cubic metre.

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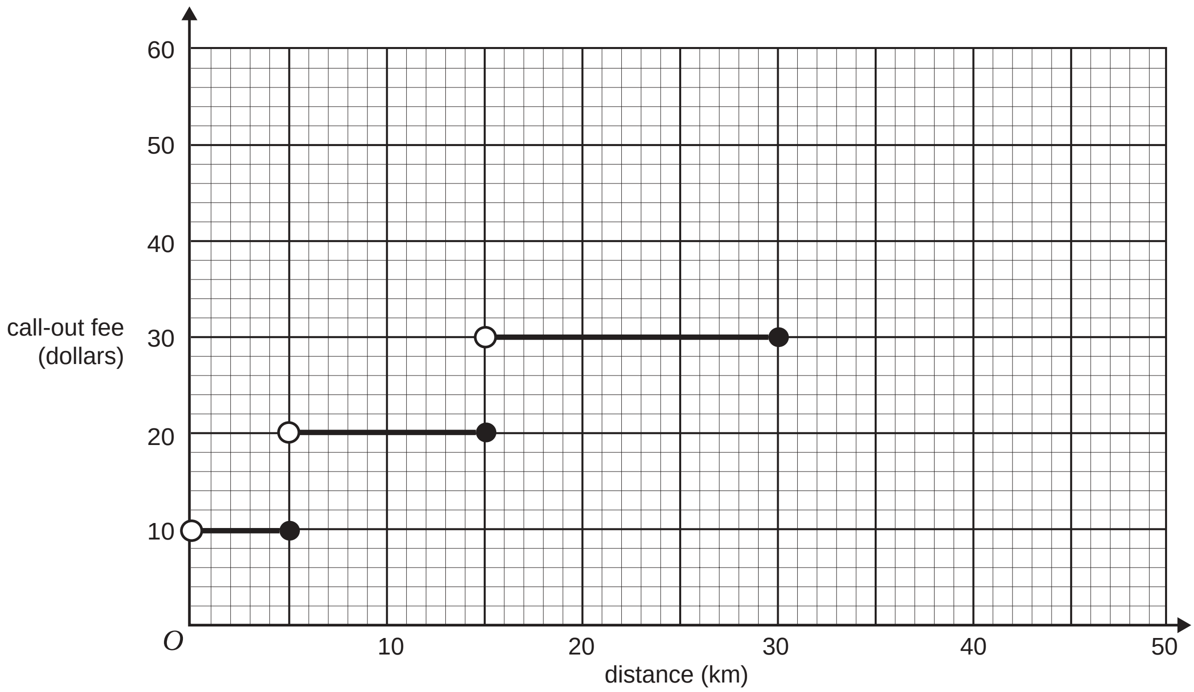
2 marks

Total 15 marks

Module 3: Graphs and relations

Question 1

Harry operates a mobile pet care service. The call-out fee charged depends on the distance he has to travel to tend to a pet. The call-out fees for distances up to 30 km are shown on the graph below.



a. According to this graph

i. what is the call-out fee to travel a distance of 20 km?

ii. what is the maximum distance travelled for a call-out fee of $10?

1 + 1 = 2 marks

A call-out fee of $50 is charged to travel distances of more than 30 km but less than or equal to 40 km.

b. **Draw** this information on the graph above.

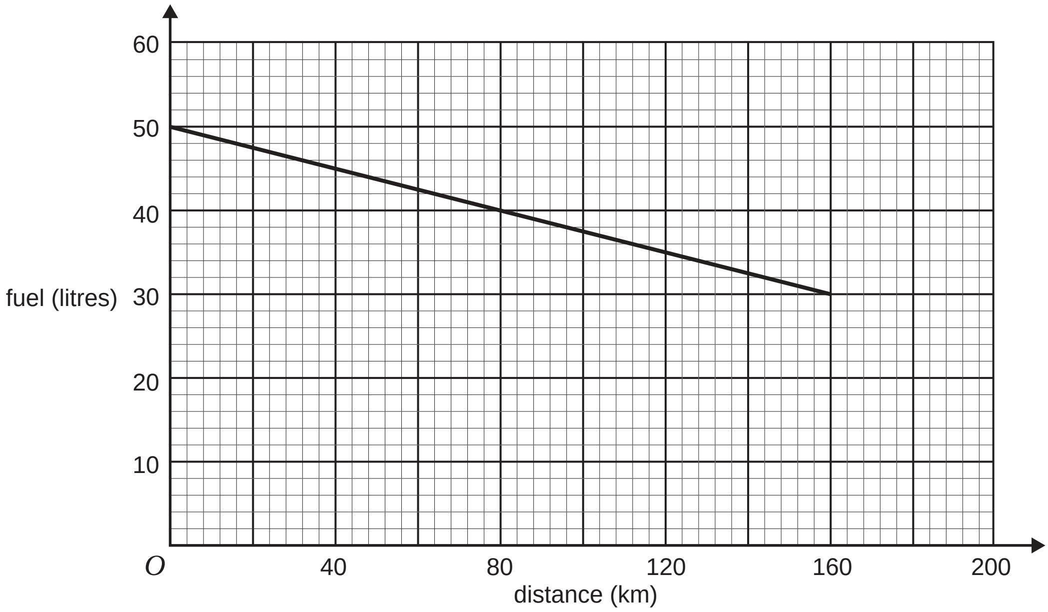
1 mark

Question 2

In one particular week, Harry began with 50 litres of fuel in the tank of his van.

After he had travelled 160 km there were 30 litres of fuel left in the tank of his van.

The amount of fuel remaining in the tank of Harry's van followed a linear trend as shown in the graph below.



a. Determine the equation of the line shown in the graph above.

2 marks

Assume this linear trend continues and that Harry does not add fuel to the tank of his van.

b. How much **further** will he be able to travel before the tank is empty?

1 mark

Harry stopped to refuel his van when there were 12 litres of fuel left in the tank.

He completely filled the tank in $3\frac{1}{2}$ minutes when fuel was flowing from the pump at a rate of 18 litres per minute.

c. How much fuel does the tank hold when it is completely full?

Write your answer in litres.

1 mark

Question 3

Harry offers dog washing and dog clipping services.

Let x be the number of dogs washed in one day
y be the number of dogs clipped in one day.

It takes 20 minutes to wash a dog and 25 minutes to clip a dog.
There are 200 minutes available each day to wash and clip dogs.

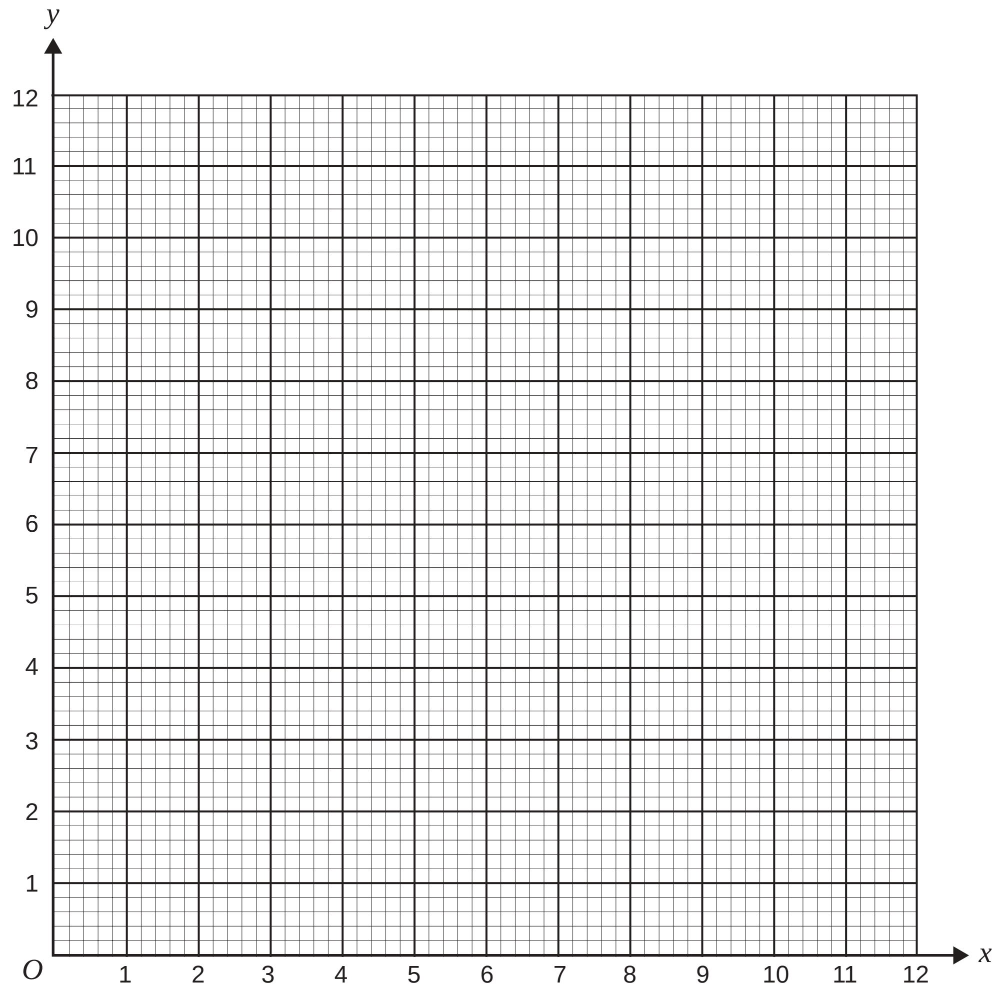
This information can be written as Inequalities 1 to 3.

Inequality 1: $x \geq 0$

Inequality 2: $y \geq 0$

Inequality 3: $20x + 25y \leq 200$

a. Draw the line that represents $20x + 25y = 200$ on the graph below.



1 mark

In any one day the number of dogs clipped is **at least** twice the number of dogs washed.

b. Write an inequality to describe this information in terms of x and y.

Inequality 4:

1 mark

c. **i.** On the graph on page 18 **draw** and clearly indicate the **boundaries** of the region represented by Inequalities 1 to 4.

ii. On a day when exactly five dogs are clipped, what is the maximum number of dogs that could be washed?

2 + 1 = 3 marks

The profit from washing one dog is \$40 and the profit from clipping one dog is \$30.

Let P be the total profit obtained in one day from washing and clipping dogs.

d. Write an equation for the total profit, P, in terms of x and y.

1 mark

e. **i.** Determine the number of dogs that should be washed and the number of dogs that should be clipped in one day in order to maximise the total profit.

ii. What is the maximum total profit that can be obtained from washing and clipping dogs in one day?

1 + 1 = 2 marks

Total 15 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1

A company purchased a machine for \$60 000.

For taxation purposes the machine is depreciated over time.

Two methods of depreciation are considered.

a. Flat rate depreciation

The machine is depreciated at a flat rate of 10% of the purchase price each year.

i. By how many dollars will the machine depreciate annually?

ii. Calculate the value of the machine after three years.

iii. After how many years will the machine be \$12 000 in value?

1 + 1 + 1 = 3 marks

b. Reducing balance depreciation

The value, V, of the machine after n years is given by the formula $V = 60\,000 \times (0.85)^n$

i. By what percentage will the machine depreciate annually?

ii. Calculate the value of the machine after three years.

iii. At the end of which year will the machine's value first fall below \$12 000?

1 + 1 + 1 = 3 marks

c. At the end of which year will the value of the machine **first** be less using flat rate depreciation than it will be using reducing balance depreciation?

2 marks

Module 4: Business-related mathematics – continued

Question 2

It is estimated that inflation will average 2% per annum over the next eight years.

If a new machine costs $60 000 now, calculate the cost of a similar new machine in eight years time, adjusted for inflation. Assume no other cost change.

Write your answer correct to the nearest dollar.

1 mark

Question 3

The company prepares for this expenditure by establishing three different investments.

a. $7 000 is invested at a simple interest rate of 6.25% per annum for eight years.

Determine the total value of this investment at the end of eight years.

2 marks

b. $10 000 is invested at an interest rate of 6% per annum compounding quarterly for eight years.

Determine the total value of this investment at the end of eight years.

Write your answer correct to the nearest dollar.

1 mark

c. $500 is deposited into an account with an interest rate of 6.5% per annum compounding monthly.

Deposits of $200 are made to this account on the last day of each month after interest has been paid.

Determine the total value of this investment at the end of eight years.

Write your answer correct to the nearest dollar.

1 mark

Question 4

The company anticipates that it will need to borrow \$20 000 to pay for the new machine.

It expects to take out a reducing balance loan with interest calculated monthly at a rate of 10% per annum.

The loan will be fully repaid with 24 equal monthly instalments.

Determine the total amount of interest that will be paid on this loan.

Write your answer to the nearest dollar.

2 marks

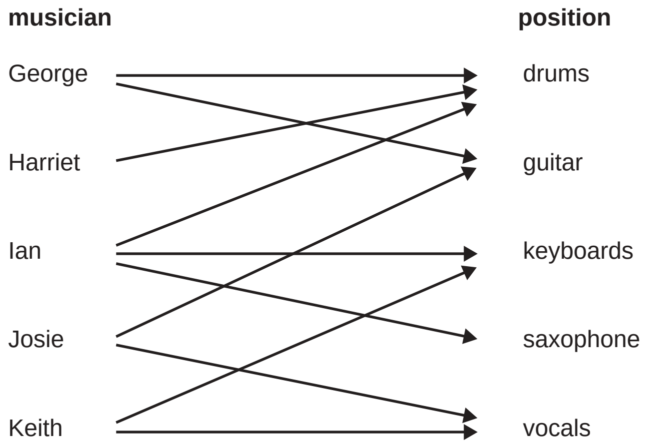
Total 15 marks

END OF MODULE 4

Module 5: Networks and decision mathematics

Question 1

George, Harriet, Ian, Josie and Keith are a group of five musicians.
They are forming a band where each musician will fill one position only.
The following bipartite graph illustrates the positions that each is able to fill.



a. Which musician **must** play the guitar?

1 mark

b. Complete the table showing the positions that the following musicians **must** fill in the band.

Person	Position
Harriet	
Ian	
Keith	

2 marks

Question 2

The five musicians compete in a music trivia game.

Each musician competes once against every other musician.

In each game there is a winner and a loser.

The results are represented in the dominance matrix, Matrix 1, and also in the **incomplete** directed graph below.

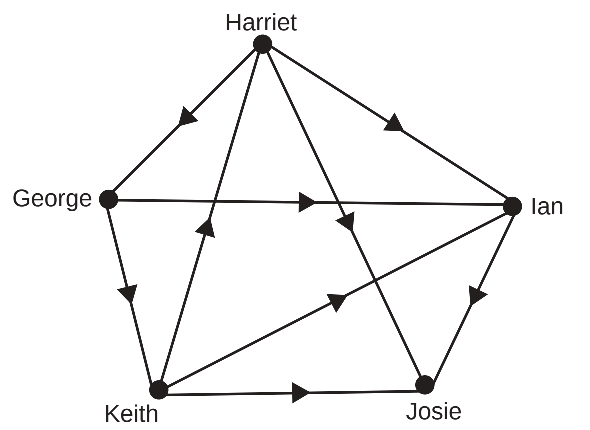
On the directed graph an arrow from Harriet to George shows that Harriet won against George.

Matrix 1

loser

$$\textbf{winner}\quad \begin{array}{c} \\ G \\ H \\ I \\ J \\ K \end{array} \begin{array}{c} \begin{array}{ccccc} G & H & I & J & K \end{array} \\ \begin{bmatrix} \mathbf{0} & 0 & 1 & 0 & 1 \\ 1 & \mathbf{0} & 1 & 1 & 0 \\ 0 & 0 & \mathbf{0} & 1 & 0 \\ 1 & 0 & 0 & \mathbf{0} & 0 \\ 0 & 1 & 1 & 1 & \mathbf{0} \end{bmatrix} \end{array}$$

Directed graph



a. Explain why the figures in bold in Matrix 1 are all zero.

1 mark

One of the edges on the directed graph is missing.

b. Using the information in Matrix 1, **draw** in the missing edge on the directed graph above and clearly show its **direction**.

1 mark

The results of each trivia contest (one-step dominances) are summarised as follows.

Musician	**Dominance value (wins)**
George	2
Harriet	3
Ian	1
Josie	1
Keith	3

In order to rank the musicians from first to last in the trivia contest, two-step (two-edge) dominances will be considered.

The following **incomplete** matrix, Matrix 2, shows two-step dominances.

Matrix 2

$$\begin{array}{c c} & \begin{array}{ccccc} G & H & I & J & K \end{array} \\ \begin{array}{c} G \\ H \\ I \\ J \\ K \end{array} & \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & x & 0 \end{bmatrix} \end{array}$$

c. Explain the two-step dominance that George has over Ian.

1 mark

d. Determine the value of the entry x in Matrix 2.

1 mark

e. Taking into consideration both the one-step and two-step dominances, determine which musician was ranked first and which was ranked last in the trivia contest.

First

Last

2 marks

Question 3

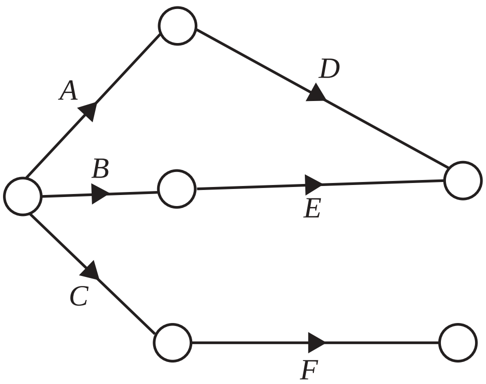
The five musicians are to record an album. This will involve nine activities.

The activities and their immediate predecessors are shown in the following table.

The duration of each activity is not yet known.

Activity	Immediate predecessors
A	–
B	–
C	–
D	*A*
E	*B*
F	*C*
G	*D, E*
H	*F*
I	*G, H*

a. Use the information in the table above to complete the network below by including activities *G*, *H* and *I*.



2 marks

There is only one critical path for this project.

b. How many **non-critical** activities are there?

1 mark

The following table gives the earliest start times (EST) and latest start times (LST) for **three of the activities only**. All times are in hours.

Activity	**EST**	**LST**
A	0	2
C	0	1
I	12	12

c. Write down the critical path for this project.

1 mark

The minimum time required for this project to be completed is 19 hours.

d. What is the duration of activity *I*?

1 mark

The duration of activity *C* is 3 hours.

e. Determine the maximum combined duration of activities *F* and *H*.

1 mark

Total 15 marks

Working space

Module 6: Matrices

Question 1

A manufacturer sells three products, A, B and C, through outlets at two shopping centres, Eastown (E) and Noxland (N).

The number of units of each product sold per month through each shop is given by the matrix Q, where

$$Q = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 2500 & 3400 & 1890 \\ 1765 & 4588 & 2456 \end{bmatrix} \end{array} \begin{array}{c} \\ E \\ N \end{array}$$

a. Write down the order of matrix Q.

1 mark

The matrix P, shown below, gives the selling price, in dollars, of products A, B, C.

$$P = \begin{bmatrix} 14.50 \\ 21.60 \\ 19.20 \end{bmatrix} \begin{array}{c} A \\ B \\ C \end{array}$$

b. **i.** Evaluate the matrix M, where $M = QP$.

ii. What information does the elements of matrix M provide?

1 + 1 = 2 marks

c. Explain why the matrix PQ is **not** defined.

1 mark

Question 2

A new shopping centre called Shopper Heaven (S) is about to open. It will compete for customers with Eastown (E) and Noxland (N).

Market research suggests that each shopping centre will have a regular customer base but attract and lose customers on a weekly basis as follows.

80% of Shopper Heaven customers will return to Shopper Heaven next week

12% of Shopper Heaven customers will shop at Eastown next week

8% of Shopper Heaven customers will shop at Noxland next week

76% of Eastown customers will return to Eastown next week

9% of Eastown customers will shop at Shopper Heaven next week

15% of Eastown customers will shop at Noxland next week

85% of Noxland customers will return to Noxland next week

10% of Noxland customers will shop at Shopper Heaven next week

5% of Noxland customers will shop at Eastown next week

a. Enter this information into transition matrix T as indicated below (express percentages as proportions, for example write 76% as 0.76).

this week

$$T = \begin{array}{c} \quad S \qquad E \qquad N \\ \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \end{array} \begin{array}{l} S \\ E \\ N \end{array} \quad \textbf{next week}$$

2 marks

During the week that Shopper Heaven opened, it had 300 000 customers.

In the same week, Eastown had 120 000 customers and Noxland had 180 000 customers.

b. Write this information in the form of a column matrix, K_0, as indicated below.

$$K_0 = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \begin{array}{l} S \\ E \\ N \end{array}$$

1 mark

c. Use T and K_0 to **write** and **evaluate** a matrix product that determines the number of customers expected at each of the shopping centres during the following week.

2 marks

d. Show by calculating at least two appropriate state matrices that, in the long term, the number of customers expected at each centre each week is given by the matrix

$$K = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$

2 marks

Question 3

Market researchers claim that the ideal number of bookshops (x), sports shoe shops (y) and music stores (z) for a shopping centre can be determined by solving the equations

$$2x + y + z = 12$$
$$x - y + z = 1$$
$$2y - z = 6$$

a. Write the equations in matrix form using the following template.

$$\begin{bmatrix} \quad & \quad & \quad \\ \\ \\ \end{bmatrix} \begin{bmatrix} \quad \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \quad \\ \\ \\ \end{bmatrix}$$

1 mark

b. Do the equations have a unique solution? Provide an explanation to justify your response.

____

____

____

1 mark

c. Write down an inverse matrix that can be used to solve these equations.

1 mark

d. Solve the equations and hence write down the estimated ideal number of bookshops, sports shoe shops and music stores for a shopping centre.

1 mark

Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:	$z = \dfrac{x - \bar{x}}{s_x}$
least squares line:	$y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value:	residual value = actual value – predicted value
seasonal index:	seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series:	$a + (a + d) + \ldots + (a + (n-1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$
geometric series:	$a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1-r^n)}{1-r}, r \neq 1$
infinite geometric series:	$a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1-r}, \lvert r \rvert < 1$

Module 2: Geometry and trigonometry

area of a triangle:	$\dfrac{1}{2}bc\sin A$
Heron's formula:	$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a+b+c)$
circumference of a circle:	$2\pi r$
area of a circle:	πr^2
volume of a sphere:	$\dfrac{4}{3}\pi r^3$
surface area of a sphere:	$4\pi r^2$
volume of a cone:	$\dfrac{1}{3}\pi r^2 h$
volume of a cylinder:	$\pi r^2 h$
volume of a prism:	area of base × height
volume of a pyramid:	$\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET