VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2008

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

FURTHER MATHEMATICS

Written examination 2

Wednesday 5 November 2008

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
5	5	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 35 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Diagrams are not to scale unless specified otherwise.

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Core 4

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Core

Question 1

In a small survey, twenty-five Year 8 girls were asked what they did (walked, sat, stood, ran) for most of the time during a typical school lunch time.

Their responses are recorded below.

sat	stood	sat	ran	sat
walked	walked	sat	walked	ran
sat	walked	walked	walked	ran
walked	ran	walked	ran	walked
ran	sat	ran	ran	walked

Use the data to

a. complete the following frequency table

Activity	**Frequency**
walked	
sat or stood	
ran	
Total	25

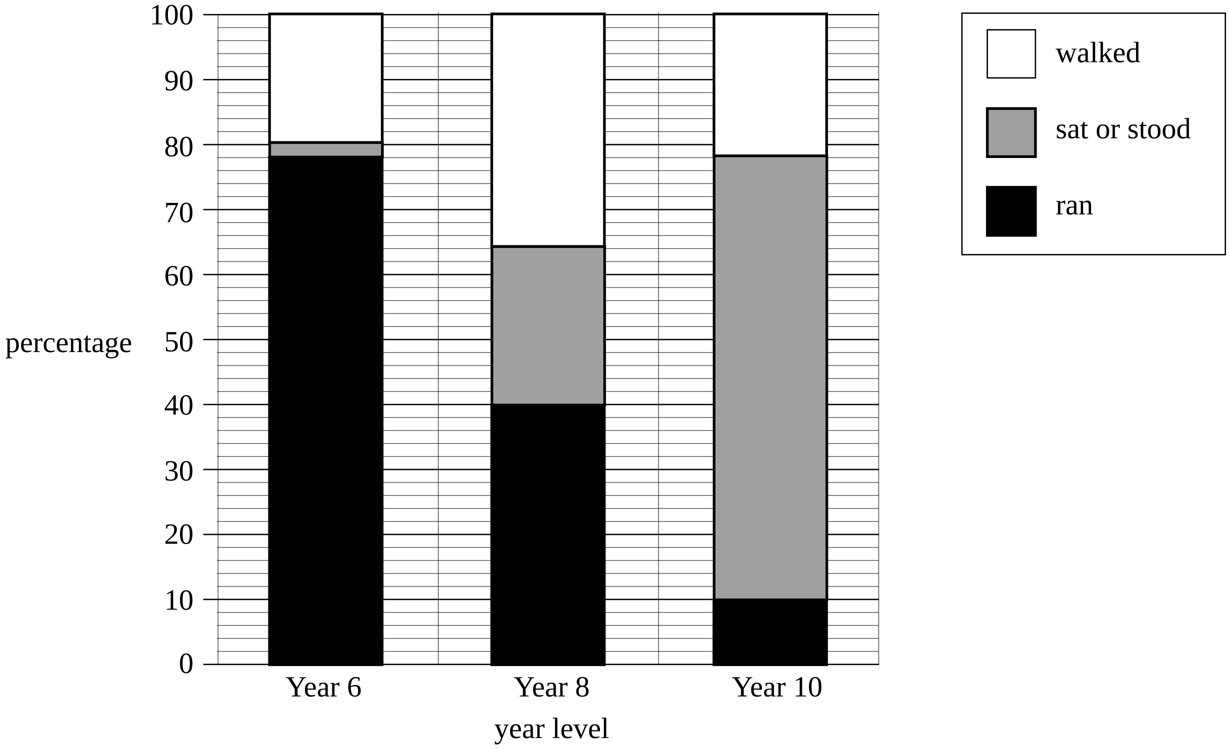
1 mark

b. determine the percentage of Year 8 girls who ran for most of the time during a typical school lunch time.

1 mark

Question 2

In a larger survey, Years 6, 8 and 10 girls were asked what they did (walked, sat, stood, ran) for most of the time during a typical school lunch time. The results are displayed in the percentage segmented bar chart below.

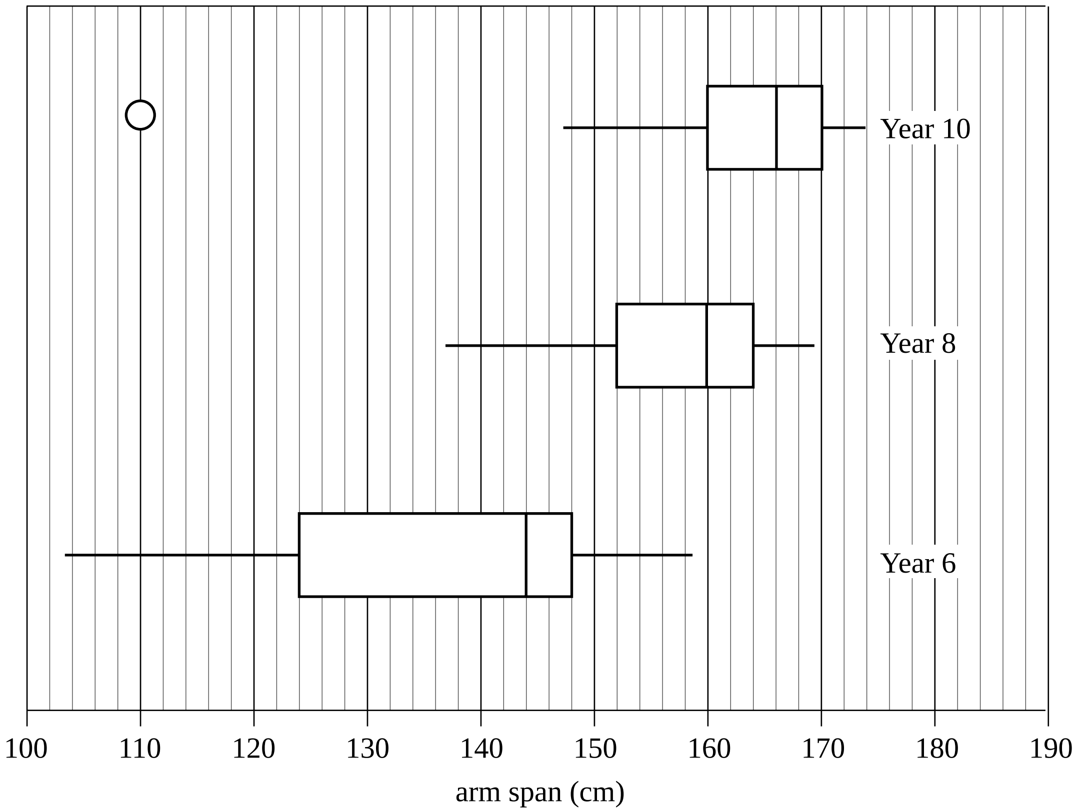


Does the percentage segmented bar chart support the opinion that, for these girls, the lunch time activity (walked, sat or stood, ran) undertaken is associated with year level? Justify your answer by quoting appropriate percentages.

2 marks

Question 3

The arm spans (in cm) were also recorded for each of the Years 6, 8 and 10 girls in the larger survey. The results are summarised in the three parallel box plots displayed below.



a. Complete the following sentence.

The middle 50% of Year 6 students have an arm span between ☐ and ☐ cm.

1 mark

b. The three parallel box plots suggest that arm span and year level are associated.
Explain why.

---

1 mark

c. The arm span of 110 cm of a Year 10 girl is shown as an outlier on the box plot. This value is an error. Her real arm span is 140 cm. If the error is corrected, would this girl's arm span still show as an outlier on the box plot? Give reasons for your answer showing an appropriate calculation.

---

2 marks

Question 4

The arm spans (in cm) and heights (in cm) for a group of 13 boys have been measured. The results are displayed in the table below.

Arm span (cm)	**Height (cm)**
152	152
153	155
174	168
141	149
170	172
165	168
163	163
155	157
165	165
152	150
143	146
156	153
174	174

The aim is to find a linear equation that allows arm span to be predicted from height.

a. What will be the independent variable in the equation?

1 mark

b. Assuming a linear association, determine the equation of the least squares regression line that enables *arm span* to be predicted from *height*. Write this equation in terms of the variables *arm span* and *height*. Give the coefficients correct to two decimal places.

2 marks

c. Using the equation that you have determined in **part b.**, interpret the slope of the least squares regression line in terms of the variables *height* and *arm span*.

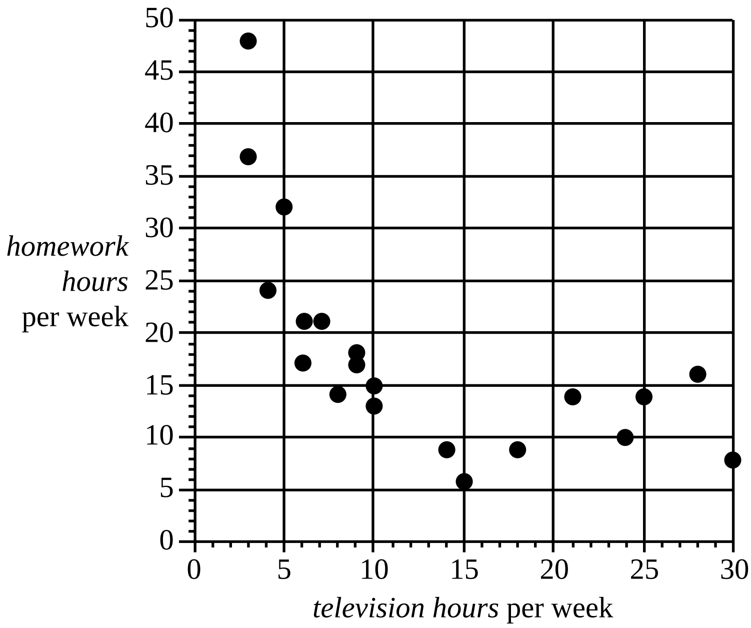
1 mark

Question 5

The number of hours spent doing homework each week (*homework hours*) and the number of hours spent watching television each week (*television hours*) were recorded for a group of 20 Year 12 students.

The results are displayed in the table below and a scatterplot constructed as shown.

television hours	***homework hours***
6	17
28	16
14	9
6	21
9	17
30	8
10	15
3	48
3	37
18	9
7	21
9	18
4	24
25	14
8	14
24	10
21	14
5	32
15	6
10	13



The relationship between *homework hours* per week and *television hours* per week is clearly nonlinear.

A **reciprocal** transformation applied to the variable, *television hours*, can be used to linearise the scatterplot.

a. Apply this reciprocal transformation to the data and determine the equation of the least squares regression line that allows *homework hours* to be predicted from the reciprocal of *television hours*.

Write the coefficients correct to two decimal places.

---

2 marks

b. If a student spends 12 hours per week watching television, use the least squares regression line to predict the number of hours that the student spends doing homework. Give your answer correct to one decimal place.

---

1 mark

Total 15 marks

END OF CORE

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Module 1: Number patterns

Question 1

David operates a large farm. Initially he has 1250 cows.

He plans to sell 150 cows each year.

Assume that no new cows arrive on the farm and that none die.

a. How many cows will David have on the farm after two years?

1 mark

b. Complete the missing number in the box below to provide an equation which, when evaluated, will give the number of cows, C_5 , on the farm after five years.

$$C_5 = 1250 + 5 \times \boxed{}$$

1 mark

c. For how many full years can David sell 150 cows each year?

1 mark

Question 2

The difference equation below provides a more realistic model for the number of cows, C_n, on David's farm after n years. It takes into account births, deaths and the sale of 150 cows each year.

$$C_n = 1.08C_{n-1} - 150 \qquad C_0 = 1250$$

a. Use this difference equation to predict the number of cows on the farm after

i. one year ______________________________

ii. six years. ______________________________

1 + 1 = 2 marks

b. According to this difference equation, the number of cows increases by a certain percentage before the 150 cows are sold each year.

i. What is this percentage?

____

ii. For how full many years can David sell 150 cows each year?

____

1 + 1 = 2 marks

Suppose David wanted to keep 1250 cows on the farm each year. The difference equation used to model the number of cows, C_n, on David's farm after n years is given below.

$$C_n = 1.08C_{n-1} - k \qquad C_0 = 1250$$

where k represents the number of cows that David will sell each year.

c. Determine the value of k.

____

1 mark

Question 3

David also keeps deer on the farm. He plans to increase the number of deer each year.

The number of deer on the farm each year will form the terms of a geometric sequence with a common ratio of 1.5.

David began with 32 deer on the farm.

a. How many deer will be on the farm after three years?

1 mark

b. After how many years will the number of deer on the farm first exceed 820?

1 mark

c. How many more deer will there be on the farm after five years than there were after one year?

1 mark

d. Write an expression in terms of n that can be used to predict the number of deer on the farm after n years.

1 mark

e. Let D_n be the number of deer on the farm after n years.

Write a difference equation for D_n in terms of D_{n-1} that can be used to predict the number of deer on the farm after n years.

1 mark

Question 4

A more realistic model for the number of deer, D_n, on the farm after n years is given by the difference equation below. This difference equation takes into account births, deaths and sale of deer each year.

$$D_n = 1.4D_{n-1} - 10 \qquad D_0 = 32$$

To predict the number of cows, C_n, on the farm after n years, David uses the difference equation

$$C_n = 1.08C_{n-1} - 150 \qquad C_0 = 1250$$

Determine how many years it will be before the number of deer on the farm first exceeds the number of cows on the farm.

2 marks

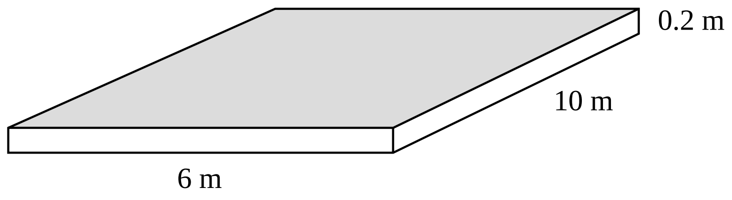
Total 15 marks

END OF MODULE 1
TURN OVER

Module 2: Geometry and trigonometry

Question 1

A shed is built on a concrete slab. The concrete slab is a rectangular prism 6 m wide, 10 m long and 0.2 m deep.



a. Determine the volume of the concrete slab in m^3.

1 mark

b. On a plan of the concrete slab, a 3 cm line is used to represent a length of 6 m.

i. What scale factor is used to draw this plan?

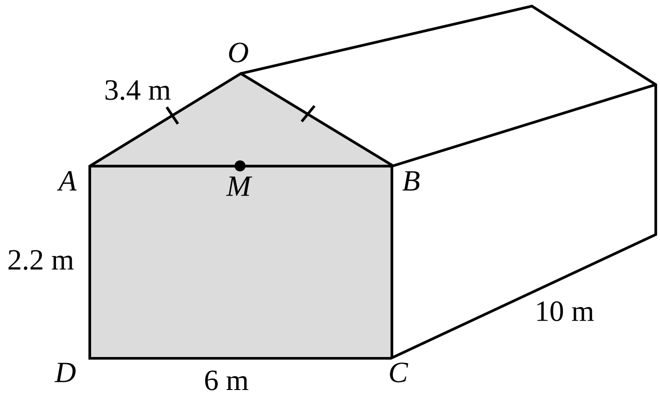
The top surface of the concrete slab shaded in the diagram above has an area of 60 m^2.

ii. What is the area of this surface on the plan?

1 + 1 = 2 marks

Question 2

The shed has the shape of a prism. Its front face, *AOBCD*, is shaded in the diagram below. *ABCD* is a rectangle and *M* is the mid point of *AB*.



a. Show that the length of *OM* is 1.6 m.

1 mark

b. Show that the area of the front face of the shed, *AOBCD*, is 18 m^2.

1 mark

c. Find the volume of the shed in m^3.

1 mark

d. All inside surfaces of the shed, including the floor, will be painted.

i. Find the total area that will be painted in m^2.

One litre of paint will cover an area of 16 m^2.

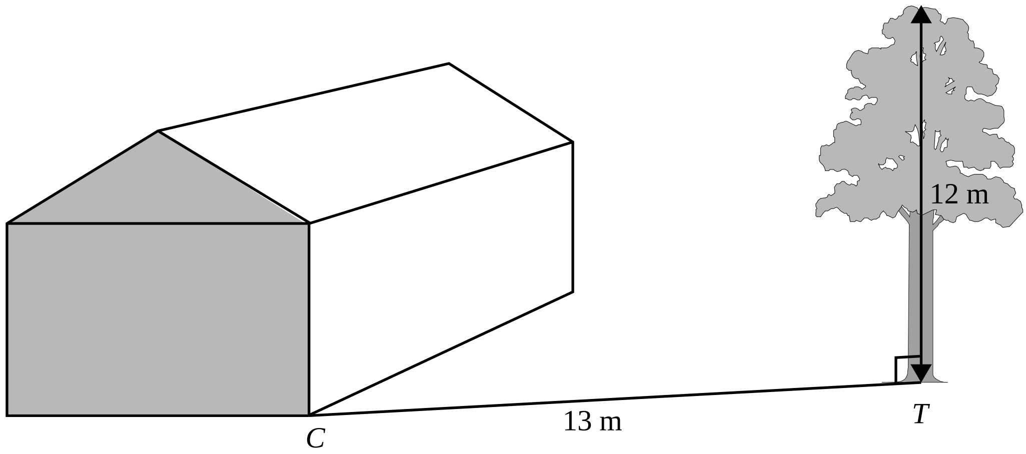
ii. Determine the number of litres of paint that is required.

2 + 1 = 3 marks

Question 3

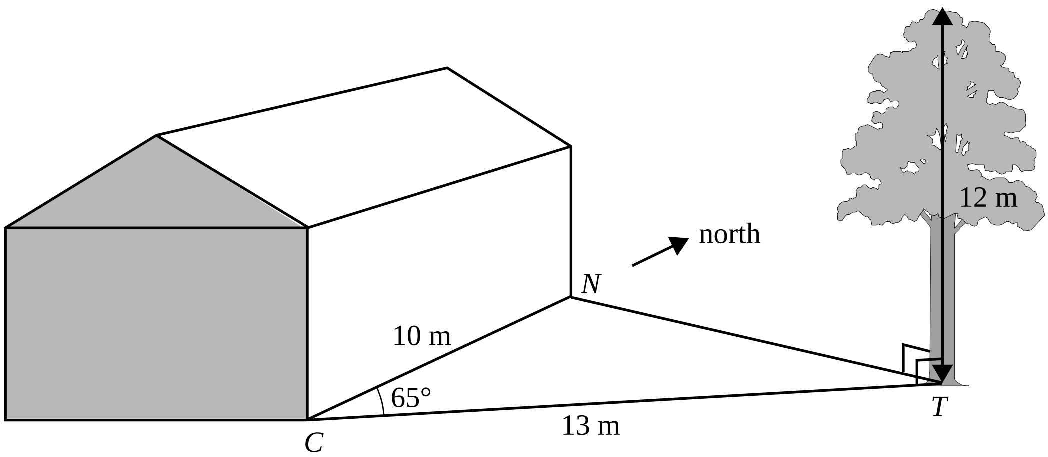
A tree, 12 m tall, is growing at point *T* near the shed.

The distance, *CT*, from corner *C* of the shed to the centre base of the tree is 13 m.



a. Calculate the angle of elevation of the top of the tree from point *C*.
Write your answer, in degrees, correct to one decimal place.

1 mark



N and *C* are two corners at the base of the shed. *N* is due north of *C*.

The angle, *TCN*, is 65°.

b. Show that, correct to one decimal place, the distance, *NT*, is 12.6 m.

1 mark

c. Calculate the angle, *CNT*, correct to the nearest degree.

1 mark

d. Determine the bearing of *T* from *N*. Write your answer correct to the nearest degree.

1 mark

e. Is it possible for the tree to hit the shed if it falls?
Explain your answer showing appropriate calculations.

2 marks

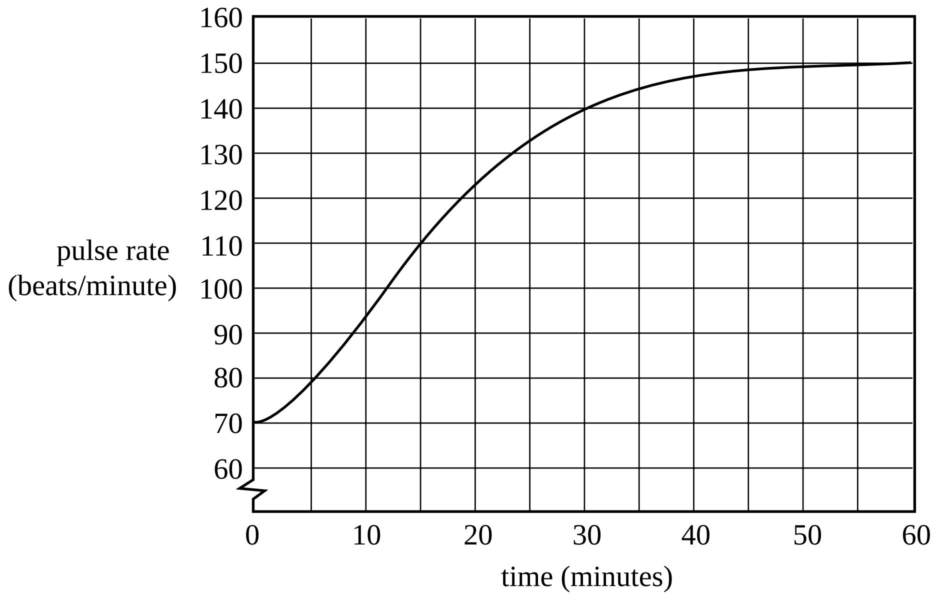
Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

Question 1

Tiffany's pulse rate (in beats/minute) during the first 60 minutes of a long-distance run is shown in the graph below.



a. What was Tiffany's pulse rate (in beats/minute) 15 minutes after she started her run?

1 mark

b. By how much did Tiffany's pulse rate increase over the first 60 minutes of her run?

Write your answer in beats/minute.

1 mark

c. The recommended maximum pulse rate for adults during exercise is determinded by subtracting the person's age in years from 220.

i. Write an equation in terms of the variables *maximum pulse rate* and *age* that can be used to determine a person's recommended maximum pulse rate from his or her age.

The target zone for aerobic exercise is between 60% and 75% of a person's maximum pulse rate.

Tiffany is 20 years of age.

ii. Determine the values between which Tiffany's pulse rate should remain so that she exercises within her target zone.

Write your answers correct to the nearest whole number.

1 + 1 = 2 marks

Question 2

Tiffany decides to enter a charity event involving running and cycling.

There is a \$35 fee to enter.

a. Write an equation that gives the total amount, R dollars, collected from entry fees when there are x competitors in the event.

1 mark

The event costs the organisers \$50 625 plus \$12.50 per competitor.

b. Write an equation that gives the total cost, C, in dollars, of the event when there are x competitors.

1 mark

c. **i.** Determine the number of competitors required for the organisers to break even.

The number of competitors who entered the event was 8670.

ii. Determine the profit made by the organisers.

1 + 1 = 2 marks

Question 3

The event involves running for 10 km and cycling for 30 km.

Let x be the time taken (in minutes) to run 10 km
y be the time taken (in minutes) to cycle 30 km

Event organisers set constraints on the time taken, in minutes, to run and cycle during the event.
Inequalities 1 to 6 below represent all time constraints on the event.

Inequality 1: $x \geq 0$

Inequality 2: $y \geq 0$

Inequality 3: $x \leq 120$

Inequality 4: $y \leq 150$

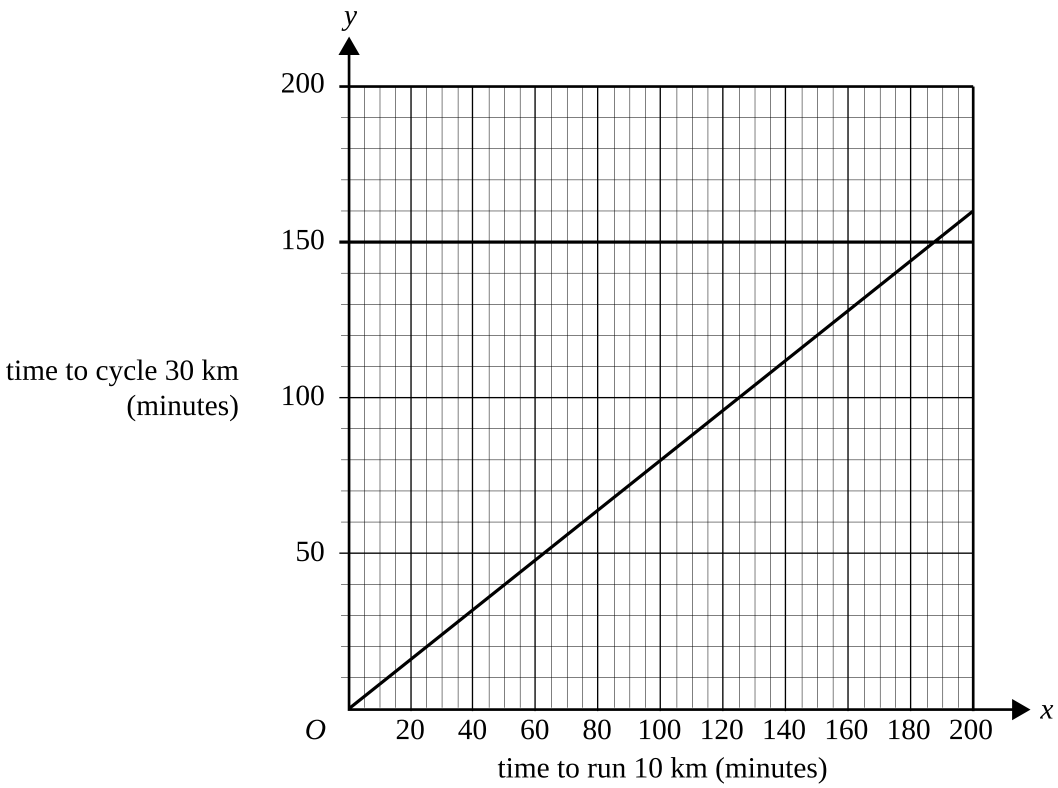
Inequality 5: $y \leq 1.5x$

Inequality 6: $y \geq 0.8x$

a. Explain the meaning of Inequality 3 in terms of the context of this problem.

1 mark

The lines $y = 150$ and $y = 0.8x$ are drawn on the graph below.



b. On the graph above

i. **draw** and **label** the lines $x = 120$ and $y = 1.5x$

ii. clearly **shade** the feasible region represented by Inequalities 1 to 6.

2 + 1 = 3 marks

One competitor, Jenny, took 100 minutes to complete the run.

c. Between what times, in minutes, can she complete the cycling and remain within the constraints set for the event?

1 mark

d. Competitors who complete the event in 90 minutes or less qualify for a prize.

Tiffany qualified for a prize.

i. Determine the maximum number of minutes for which Tiffany could have cycled.

ii. Determine the maximum number of minutes for which Tiffany could have run.

1 + 1 = 2 marks

Total 15 marks

END OF MODULE 3

TURN OVER

Module 4: Business-related mathematics

Question 1

The bank statement below shows the transactions on Michelle's account for the month of July.

Date	Description of Transaction	Withdrawals	Deposits	Balance
01 July	**Opening Balance**			**6250.67**
11 July	Deposit – cash			6870.67
14 July	Withdrawal – cheque	749.81		6120.86
19 July	Deposit – Internet transfer		838.23	6959.09
31 July	**Closing Balance**			**6959.09**

a. What amount, in dollars, was deposited in cash on 11 July?

1 mark

Interest for this account is calculated on the minimum monthly balance at a rate of 3% per annum.

b. Calculate the interest for July, correct to the nearest cent.

2 marks

Question 2

Michelle decided to invest some of her money at a higher interest rate. She deposited \$3000 in an account paying 8.2% per annum, compounding half yearly.

a. Write down an expression involving the compound interest formula that can be used to find the value of Michelle's \$3000 investment at the end of two years. Find this value correct to the nearest cent.

2 marks

b. How much interest will the \$3000 investment earn over a four-year period?

Write your answer correct to the nearest cent.

1 mark

Question 3

Michelle purchased a \$17 000 car. The car's value depreciates at the rate of 10% per annum using the reducing balance method.

a. By what amount, in dollars, does the car's value depreciate during Michelle's third year of ownership?

2 marks

b. After how many years of ownership will the car's value first be below \$7000?

1 mark

Question 4

Michelle intends to keep the \$17 000 car for 15 years. At the end of this time its value will be \$3500.

a. By what amount, in dollars, would the car's value depreciate annually if Michelle used the flat rate method of depreciation?

1 mark

b. Determine the annual flat rate of depreciation correct to one decimal place.

1 mark

Question 5

Michelle took a reducing balance loan for \$15 000 to purchase her car. Interest is calculated monthly at a rate of 9.4% per annum.

In order to repay the loan Michelle will make a number of equal monthly payments of \$350.

The final repayment will be less than \$350.

a. How many equal monthly payments of \$350 will Michelle need to make?

1 mark

b. How much of the principal does Michelle have left to pay immediately after she makes her final \$350 payment? Find this amount correct to the nearest dollar.

1 mark

Exactly one year after Michelle established her loan the interest rate increased to 9.7% per annum. Michelle decided to increase her monthly payment so that the loan would be fully paid in three years (exactly four years from the date the loan was established).

c. What is the new monthly payment Michelle will make? Write your answer correct to the nearest cent.

2 marks

Total 15 marks

END OF MODULE 4

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Module 5: Networks and decision mathematics

Question 1

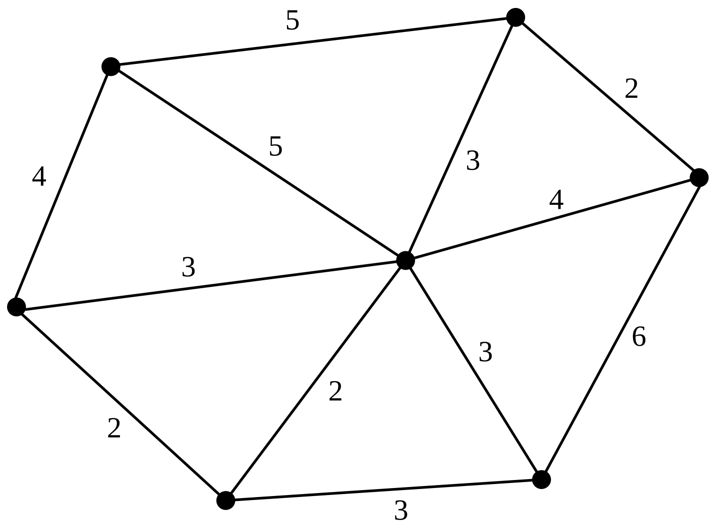
James, Dante, Tahlia and Chanel are four children playing a game.

In this children's game, seven posts are placed in the ground.

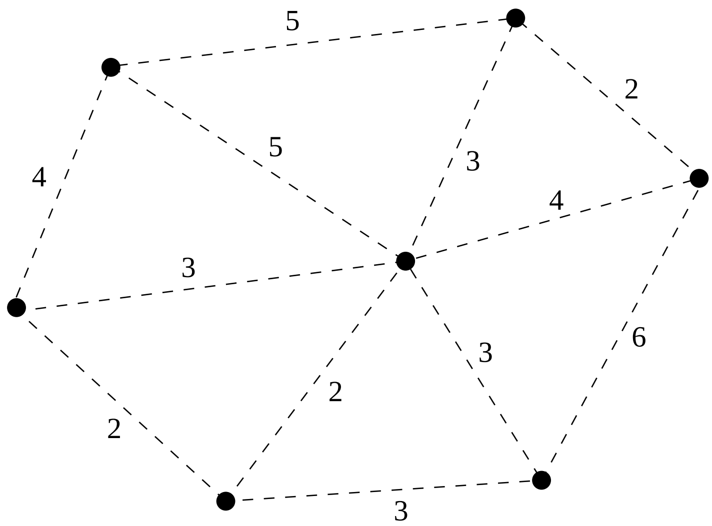
The network below shows distances, in metres, between the seven posts.

The aim of the game is to connect the posts with ribbon using the shortest length of ribbon.

This will be a minimal spanning tree.



a. Draw in a minimal spanning tree for this network on the diagram below.



1 mark

b. Determine the length, in metres, of this minimal spanning tree.

---

1 mark

c. How many different minimal spanning trees can be drawn for this network?

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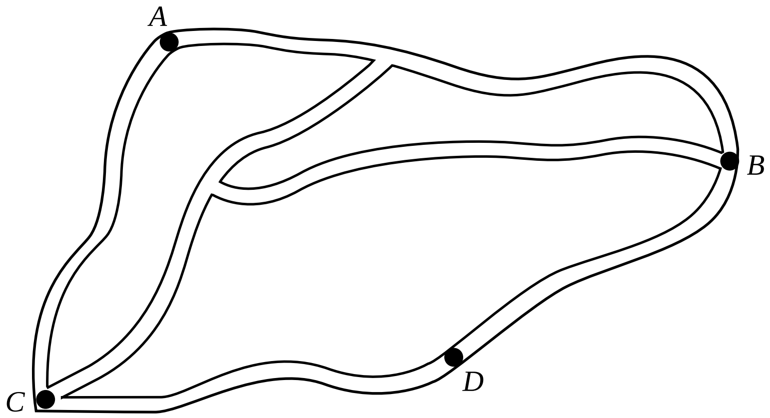
1 mark

Question 2

The four children each live in a different town.

The following is a map of the roads that link the four towns, A, B, C and D.



a. How many different ways may a vehicle travel from town A to town D without travelling along any road more than once?

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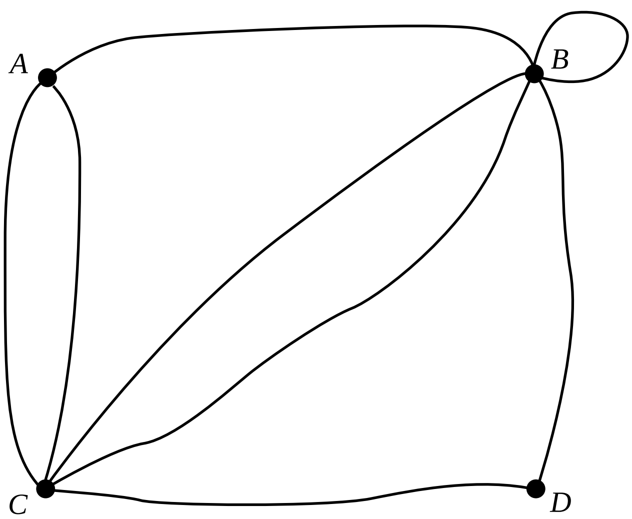
1 mark

James' father has begun to draw a network diagram that represents all the routes between the four towns on the map. This is shown below.



In this network, vertices represent towns and edges represent routes between towns that do not pass through any other town.

b. **i.** One more edge needs to be added to complete this network. **Draw** in this edge clearly on the diagram above.

ii. With reference to the network diagram, explain why a motorist at A could not drive each of these routes once only and arrive back at A.

---

1 + 1 = 2 marks

Question 3

Each child is to be driven by his or her parents to one of four different concerts. The following table shows the distance that each car would have to travel, in kilometres, to each of the four concerts.

	Concert 1	**Concert 2**	**Concert 3**	**Concert 4**
James	10	16	18	20
Dante	9	14	19	15
Tahlia	15	13	20	18
Chanel	10	15	21	16

The concerts will be allocated so as to minimise the total distance that must be travelled to take the children to the concerts. The hungarian algorithm is to be used to find this minimum value.

a. Step 1 of the hungarian algorithm is to subtract the minimum entry in each row from each element in the row. **Complete** step 1 for Tahlia by writing the missing values in the table below.

	Concert 1	**Concert 2**	**Concert 3**	**Concert 4**
James	0	6	8	10
Dante	0	5	10	6
Tahlia				
Chanel	0	5	11	6

1 mark

After further steps of the hungarian algorithm have been applied, the table is as follows.

	Concert 1	**Concert 2**	**Concert 3**	**Concert 4**
James	0	5	0	4
Dante	0	4	2	0
Tahlia	3	0	0	0
Chanel	0	4	3	0

It is now possible to allocate each child to a concert.

b. Explain why this table shows that Tahlia should attend Concert 2.

1 mark

c. Determine the concerts that could be attended by James, Dante and Chanel to minimise the total distance travelled. Write your answers in the table below.

	Concert
James	
Dante	
Tahlia	2
Chanel	

1 mark

d. Determine the minimum total distance, in kilometres, travelled by the four cars.

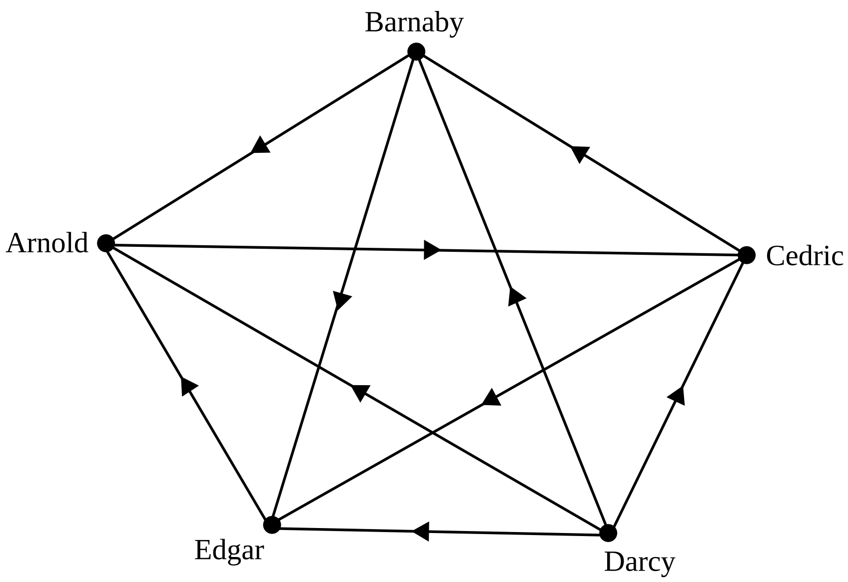
1 mark

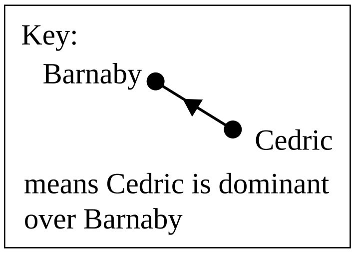
Question 4

The children are taken to the zoo where they observe the behaviour of five young male lion cubs. The lion cubs are named Arnold, Barnaby, Cedric, Darcy and Edgar.

A dominance hierarchy has emerged within this group of lion cubs.

In the directed graph below, the directions of the arrows show which lions are dominant over others.





a. Name the two pairs of lion cubs who have equal totals of one-step dominances.

2 marks

b. Over which lion does Cedric have both a one-step dominance and a two-step dominance?

1 mark

In determining the final order of dominance, the number of one-step dominances and two-step dominances are added together.

c. Complete the table below for the final order of dominance.

Final order of dominance	**Lion**
1st	Darcy
2nd	
3rd	
4th	
5th	

1 mark

Over time, the pattern of dominance changes until each lion cub has a one-step dominance over two other lion cubs.

d. Determine the total number of two-step dominances for this group of five lion cubs.

1 mark

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

Two subjects, Biology and Chemistry, are offered in the first year of a university science course.

The matrix N lists the number of students enrolled in each subject.

$$N = \begin{array}{c} \begin{bmatrix} 460 \\ 360 \end{bmatrix} \end{array} \begin{array}{l} \text{Biology} \\ \text{Chemistry} \end{array}$$

The matrix P lists the proportion of these students expected to be awarded an A, B, C, D or E grade in each subject.

$$P = \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ [0.05 \quad 0.125 \quad 0.175 \quad 0.45 \quad 0.20] \end{array}$$

a. Write down the order of matrix P.

1 mark

b. Let the matrix $R = NP$.

i. Evaluate the matrix R.

ii. Explain what the matrix **element** R_{24} represents.

1 + 1 = 2 marks

c. Students enrolled in Biology have to pay a laboratory fee of \$110, while students enrolled in Chemistry pay a laboratory fee of \$95.

i. Write down a clearly labelled row matrix, called *F*, that lists these fees.

ii. Show a matrix calculation that will give the total laboratory fees, *L*, paid in dollars by the students enrolled in Biology and Chemistry. Find this amount.

1 + 1 = 2 marks

Question 2

The following transition matrix, T, is used to help predict class attendance of History students at the university on a lecture-by-lecture basis.

$$\begin{array}{cccc} & \multicolumn{2}{c}{\text{this lecture}} & \\ & \textit{attend} & \textit{not attend} & \\ T = & \left[\begin{array}{cc} 0.90 & 0.20 \\ 0.10 & 0.80 \end{array}\right] & & \begin{array}{l} \textit{attend} \\ \textit{not attend} \end{array} \quad \text{next lecture} \end{array}$$

S_1 is the attendance matrix for the first History lecture.

$$S_1 = \begin{bmatrix} 540 \\ 36 \end{bmatrix} \begin{array}{l} \textit{attend} \\ \textit{not attend} \end{array}$$

S_1 indicates that 540 History students attended the first lecture and 36 History students did not attend the first lecture.

a. Use T and S_1 to

i. determine S_2 the attendance matrix for the second lecture

ii. predict the number of History students attending the fifth lecture.

1 + 1 = 2 marks

b. Write down a matrix equation for S_n in terms of T, n and S_1.

1 mark

The History lecture can be transferred to a smaller lecture theatre when the number of students predicted to attend falls below 400.

c. For which lecture can this first be done?

1 mark

d. In the long term, how many History students are predicted to attend lectures?

1 mark

Question 3

The bookshop manager at the university has developed a matrix formula for determining the number of Mathematics and Physics textbooks he should order each year.

For 2009, the starting point for the formula is the column matrix S_{2008}. This lists the number of Mathematics and Physics textbooks sold in 2008.

$$S_{2008} = \begin{bmatrix} 456 \\ 350 \end{bmatrix} \begin{array}{l} \textit{Mathematics} \\ \textit{Physics} \end{array}$$

O_{2009} is a column matrix listing the number of Mathematics and Physics textbooks to be ordered for 2009.

O_{2009} is given by the matrix formula

$$O_{2009} = A\ S_{2008} + B \qquad \text{where } A = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.68 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 18 \\ 12 \end{bmatrix}$$

a. Determine O_{2009}

1 mark

The matrix formula above only allows the manager to predict the number of books he should order one year ahead. A new matrix formula enables him to determine the number of books to be ordered two or more years ahead.

The new matrix formula is

$$O_{n+1} = C\,O_n - D$$

where O_n is a column matrix listing the number of Mathematics and Physics textbooks to be ordered for year n.

Here, $C = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}$ and $D = \begin{bmatrix} 40 \\ 38 \end{bmatrix}$

The number of books ordered in 2008 was given by

$$O_{2008} = \begin{bmatrix} 500 \\ 360 \end{bmatrix} \begin{array}{l} \textit{Mathematics} \\ \textit{Physics} \end{array}$$

b. Use the new matrix formula to determine the number of Mathematics textbooks the bookshop manager should order in 2010.

2 marks

Question 4

By the end of each academic year, students at the university will have either passed, failed or deferred the year.

Experience has shown that

- 88% of students who pass this year will also pass next year
- 10% of students who pass this year will fail next year
- 2% of students who pass this year will defer next year

- 52% of students who fail this year will pass next year
- 44% of students who fail this year will fail next year
- 4% of students who fail this year will defer next year

- 65% of students who defer this year will pass next year
- 10 % of students who defer this year will fail next year
- 25% of students who defer this year will defer next year.

Twelve hundred and thirty students began a business degree in 2007.

By the end of the 2007 academic year, 880 students had passed, 230 had failed, while 120 had deferred the year.

No students have dropped out of the business degree permanently.

Use this information to predict the number of business students who will **defer** the 2009 academic year.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET