VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2009

FURTHER MATHEMATICS
Written examination 1

Monday 2 November 2009

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 40 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Core: Data analysis

The following information relates to Questions 1 to 3.

The back-to-back ordered stem plot below shows the female and male smoking rates, expressed as a percentage, in 18 countries.

smoking rates (%)

female												male								
				9	9	9	7	7	6	5	1	7	9							
8	6	5	5	5	5	5	3	2	1	0	2	2	4	4	4	5	6	7	7	7
											3	0	0	1	1	6	9			
											4	7								

Question 1

For these 18 countries, the lowest female smoking rate is

A. 5%
B. 7%
C. 9%
D. 15%
E. 19%

Question 2

For these 18 countries, the interquartile range (IQR) of the female smoking rates is

A. 4
B. 6
C. 19
D. 22
E. 23

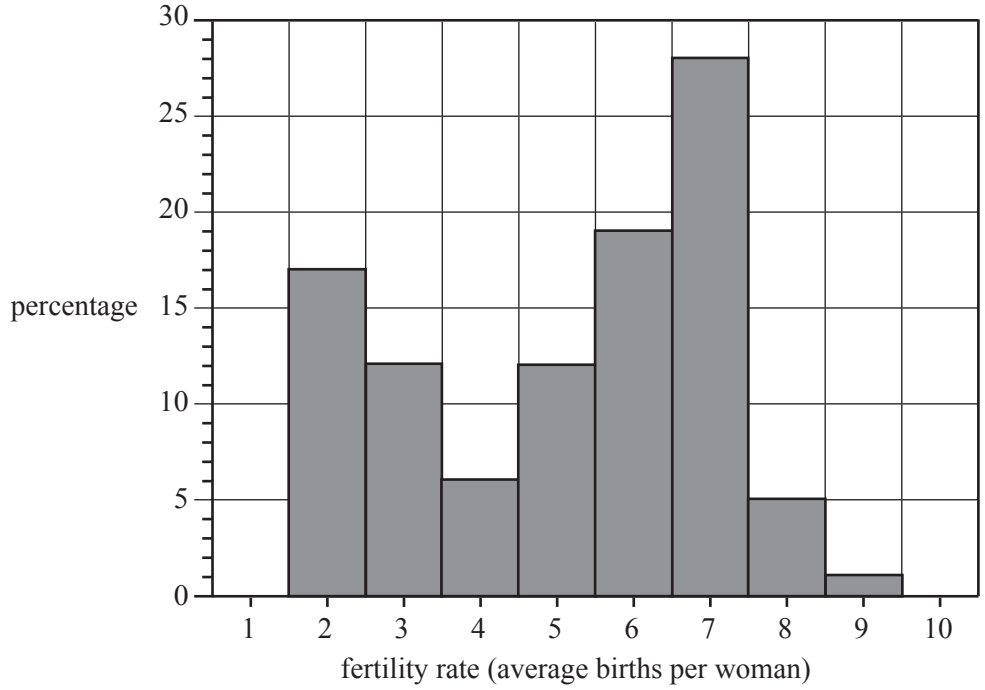
Question 3

For these 18 countries, the smoking rates for females are generally

A. lower and less variable than the smoking rates for males.
B. lower and more variable than the smoking rates for males.
C. higher and less variable than the smoking rates for males.
D. higher and more variable than the smoking rates for males.
E. about the same as the smoking rates for males.

The following information relates to Questions 4 to 6.

The percentage histogram below shows the distribution of the fertility rates (in average births per woman) for 173 countries in 1975.



Question 4

In 1975, the percentage of these 173 countries with fertility rates of 4.5 or greater was closest to

A. 12%

B. 35%

C. 47%

D. 53%

E. 65%

Question 5

In 1975, for these 173 countries, fertility rates were most frequently

A. less than 2.5

B. between 1.5 and 2.5

C. between 2.5 and 4.5

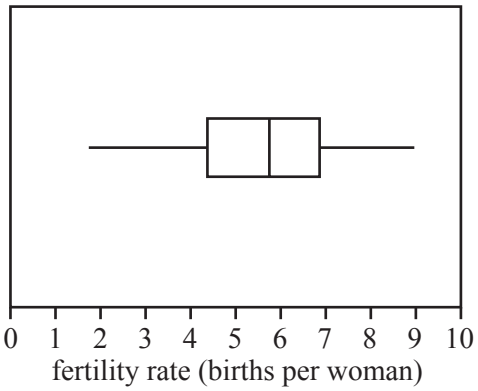
D. between 6.5 and 7.5

E. greater than 7.5

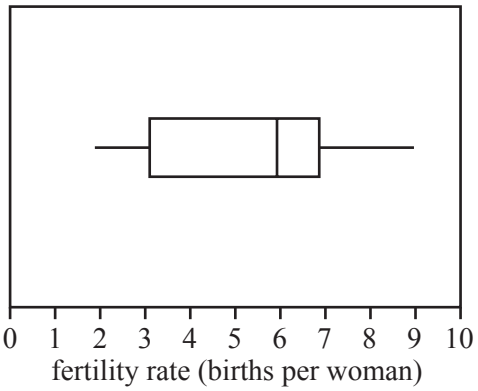
Question 6

Which one of the boxplots below could best be used to represent the same fertility rate data as displayed in the percentage histogram?

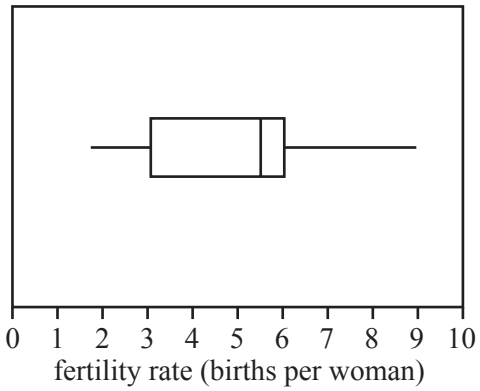
A.



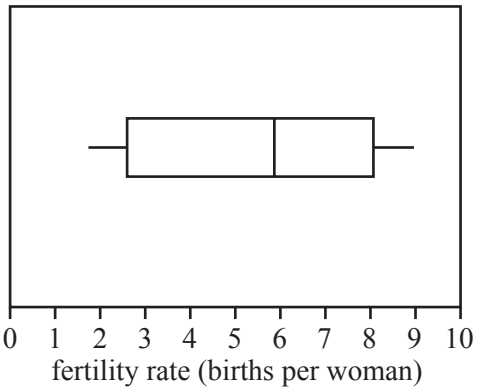
B.



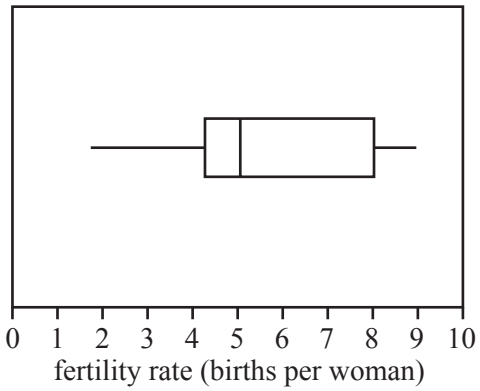
C.



D.



E.



Question 7

The level of oil use in certain countries is approximately normally distributed with a mean of 42.2 units and a standard deviation of 10.2 units.

The percentage of these countries in which the level of oil use is greater than 32 units is closest to

A. 5%

B. 16%

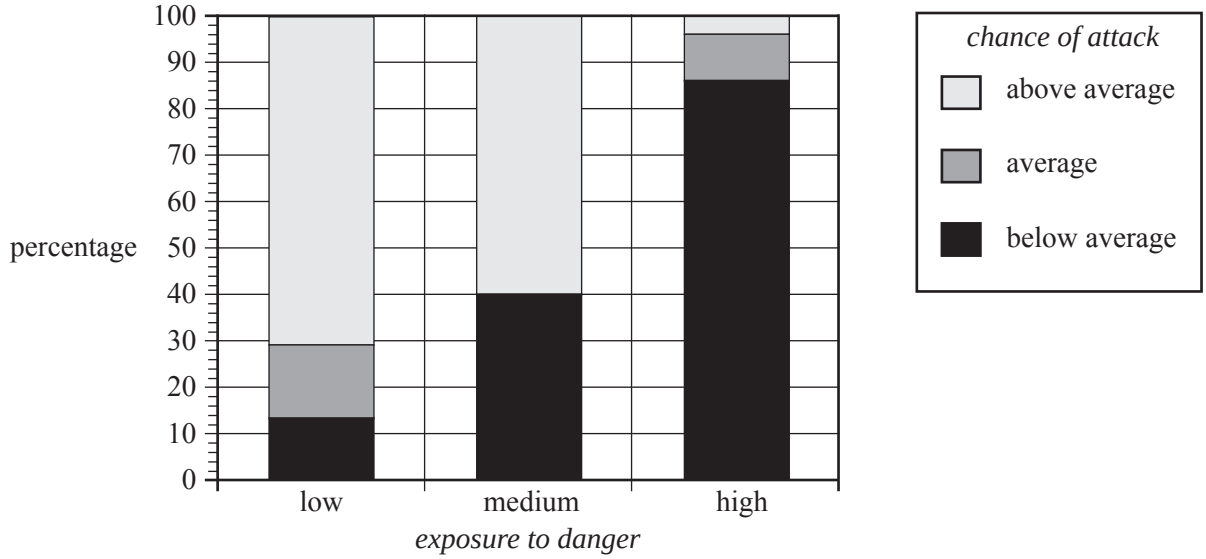
C. 34%

D. 84%

E. 97.5%

Question 8

An animal study was conducted to investigate the relationship between *exposure to danger* during sleep (high, medium, low) and *chance of attack* (above average, average, below average). The results are summarised in the percentage segmented bar chart below.



The percentage of animals whose *exposure to danger* during sleep is high, and whose *chance of attack* is below average, is closest to

A. 4%

B. 12%

C. 28%

D. 72%

E. 86%

The following information relates to Questions 9 and 10.

The table below lists the average *life span* (in years) and average *sleeping time* (in hours/day) of 12 animal species.

species	**life span (years)**	**sleeping time (hours/day)**
baboon	27	10
cow	30	4
goat	20	4
guinea pig	8	8
horse	46	3
mouse	3	13
pig	27	8
rabbit	18	8
rat	5	13
red fox	10	10
rhesus monkey	29	10
sheep	20	4

Question 9

Using *sleeping time* as the independent variable, a least squares regression line is fitted to the data.

The equation of the least squares regression line is closest to

A. *life span* = 38.9 – 2.36 × *sleeping time*.

B. *life span* = 11.7 – 0.185 × *sleeping time*.

C. *life span* = –0.185 – 11.7 × *sleeping time*.

D. *sleeping time* = 11.7 – 0.185 × *life span*.

E. *sleeping time* = 38.9 – 2.36 × *life span*.

Question 10

The value of Pearson's product-moment correlation coefficient for *life span* and *sleeping time* is closest to

A. –0.6603

B. –0.4360

C. –0.1901

D. 0.4360

E. 0.6603

Question 11

The table below lists the average *body weight* (in kg) and average *brain weight* (in g) of nine animal species.

species	body weight (kg)	brain weight (g)
baboon	10.55	179.5
cat	3.30	25.6
goat	27.70	115.0
guinea pig	1.04	5.5
rabbit	2.50	12.1
rat	0.28	1.9
red fox	4.24	50.4
rhesus monkey	6.80	179.0
sheep	55.50	175.0

A least squares regression line is fitted to the data using *body weight* as the independent variable.

The equation of the least squares regression line is

$$\textit{brain weight} = 49.4 + 2.68 \times \textit{body weight}$$

This equation is then used to predict the *brain weight* (in g) of the baboon.

The residual value (in g) for this prediction will be closest to

A. –351

B. –102

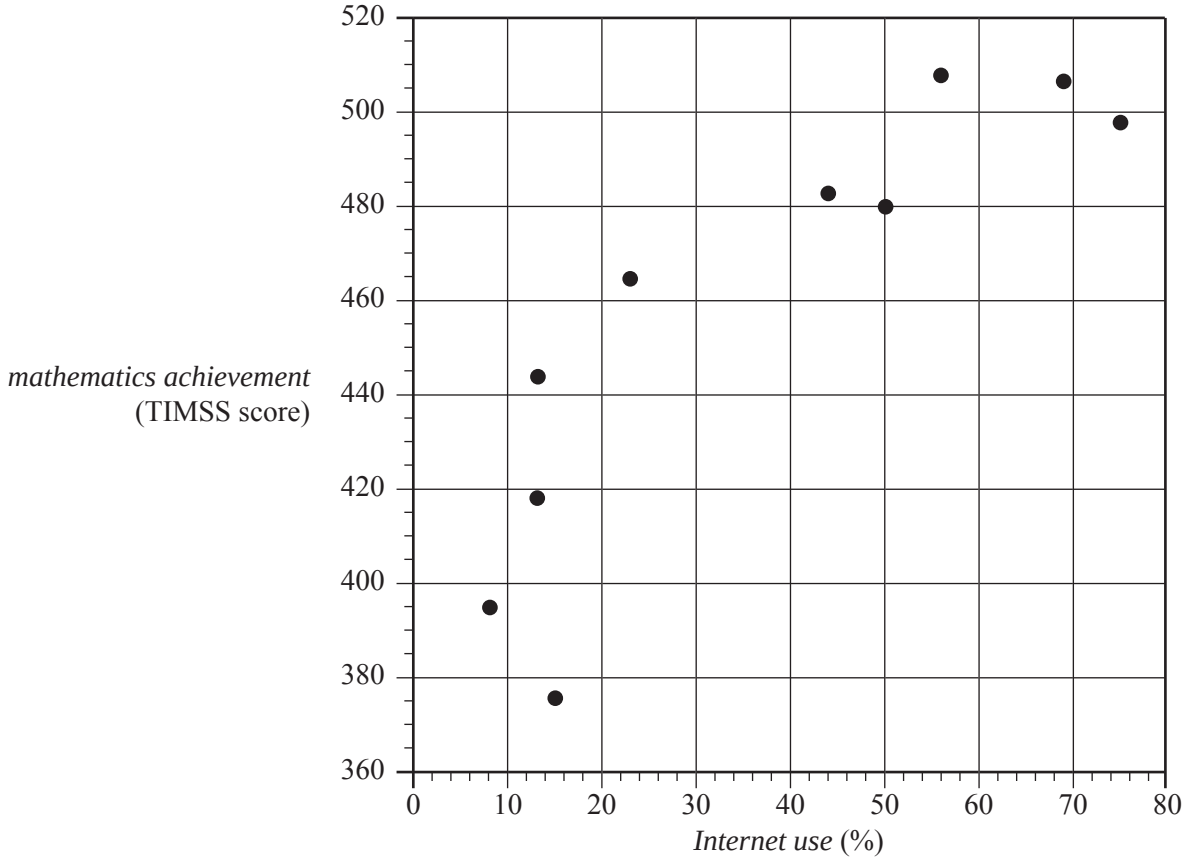
C. –78

D. 78

E. 102

Question 12

The *mathematics achievement* level (TIMSS score) for grade 8 students and the general rate of *Internet use* (%) for 10 countries are displayed in the scatterplot below.



To linearise the data, it would be best to plot

A. *mathematics achievement* against *Internet use*.

B. log (*mathematics achievement*) against *Internet use*.

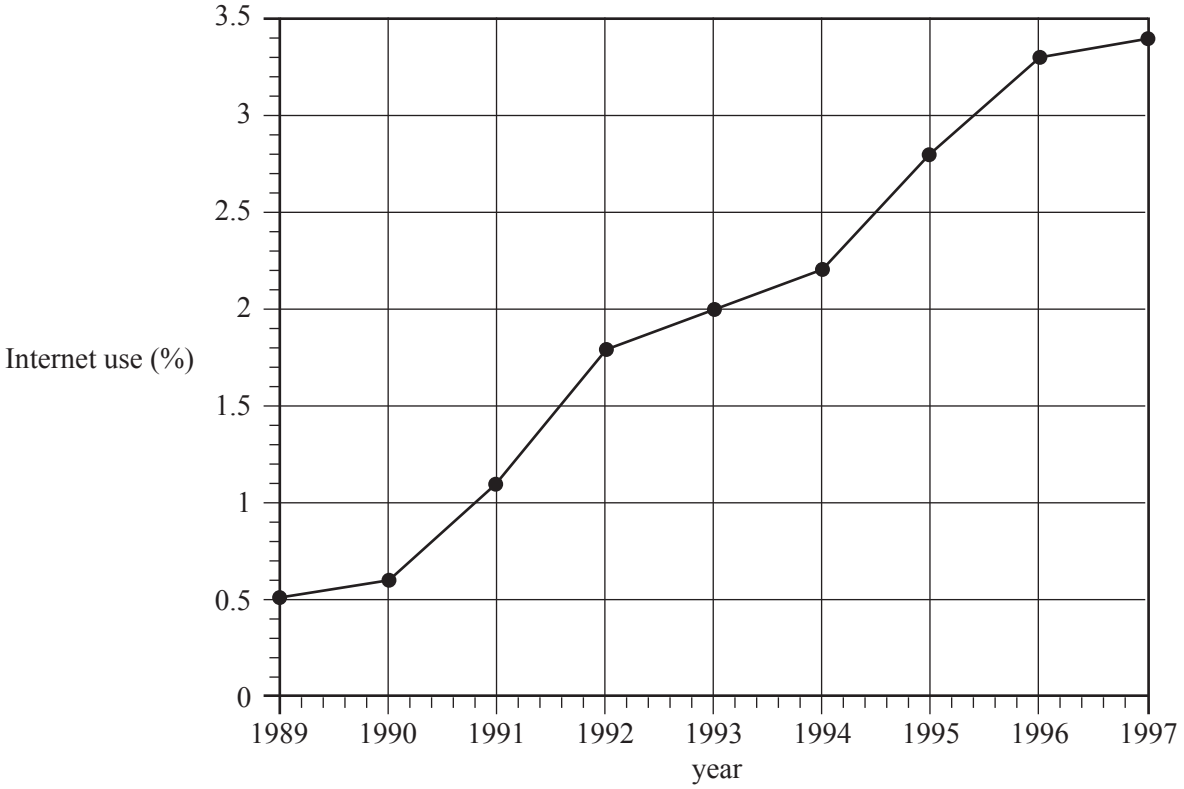
C. *mathematics achievement* against log (*Internet use*).

D. *mathematics achievement* against $(\textit{Internet use})^2$.

E. $\dfrac{1}{\textit{mathematics achievement}}$ against *Internet use*.

Question 13

The time series plot below shows the growth in Internet use (%) in a country from 1989 to 1997 inclusive.



If a three-median line is fitted to the data it would show that, on average, the increase in Internet use per year was closest to

A. 0.33%

B. 0.36%

C. 0.41%

D. 0.45%

E. 0.49%

END OF SECTION A

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The first six terms of a Fibonacci related sequence are shown below.

4, 7, 11, 18, 29, 47 . . .

The next term in the sequence is

A. 58
B. 65
C. 76
D. 94
E. 123

Question 2

Each week a young boy saves an amount of his pocket money. The amount saved forms part of an arithmetic sequence. The table shows the amounts he saves in weeks 1 to 3.

week number	amount saved ($)
1	0.40
2	0.55
3	0.70

If he continues to save in this way, the amount he will save in week eight is

A. $1.45
B. $1.60
C. $1.65
D. $7.40
E. $8.00

Question 3

The first four terms of a geometric sequence are 6400, t_2, 8100, –9112.5

The value of t_2 is

A. –7250
B. –7200
C. –1700
D. 7200
E. 7250

Question 4

The sum of the infinite geometric sequence 96, – 48, 24, –12, 6 . . . is equal to

A. 64

B. 66

C. 68

D. 144

E. 192

Question 5

On Monday morning, Jim told six friends a secret. On Tuesday morning, those six friends each told the secret to six other friends who did not know it. The secret continued to spread in this way on Wednesday, Thursday and Friday mornings.

The total number of people (not counting Jim) who will know the secret on Friday afternoon is

A. 259

B. 1296

C. 1555

D. 7776

E. 9330

Question 6

The nth term of a sequence is given by $t_n = 100 - 20n$, where $n = 1, 2, 3, 4 \ldots$

A difference equation that generates the same sequence is

A. $t_{n+1} = 100 - 20t_n \quad t_1 = 80$

B. $t_{n+1} = 100t_n - 20 \quad t_1 = 1$

C. $t_{n+1} = 80t_n \quad t_1 = 80$

D. $t_{n+1} = 100 - t_n \quad t_1 = 20$

E. $t_{n+1} = t_n - 20 \quad t_1 = 80$

Question 7

The difference equation $u_{n+1} = 4u_n - 2$ generates a sequence.

If $u_2 = 2$, then u_4 will be equal to

A. 4

B. 8

C. 22

D. 40

E. 42

Question 8

A patient takes 15 milligrams of a prescribed drug at the start of each day.

Over the next 24 hours, 85% of the drug in his body is used. The remaining 15% stays in his body.

Let D_n be the number of milligrams of the drug in the patient's body immediately after taking the drug at the start of the nth day.

A difference equation for determining D_{n+1}, the number of milligrams in the patient's body immediately after taking the drug at the start of the n+1th day, is given by

A. $D_{n+1} = 85D_n + 15 \qquad D_1 = 15$

B. $D_{n+1} = 0.85D_n + 15 \qquad D_1 = 15$

C. $D_{n+1} = 0.15D_n + 15 \qquad D_1 = 15$

D. $D_{n+1} = 0.15D_n + 0.85 \qquad D_1 = 15$

E. $D_{n+1} = 15D_n + 85 \qquad D_1 = 15$

Question 9

There are 10 checkpoints in a 4500 metre orienteering course.

Checkpoint 1 is the start and checkpoint 10 is the finish.

The distance between successive checkpoints increases by 50 metres as each checkpoint is passed.

The distance, in metres, between checkpoint 2 and checkpoint 3 is

A. 225

B. 275

C. 300

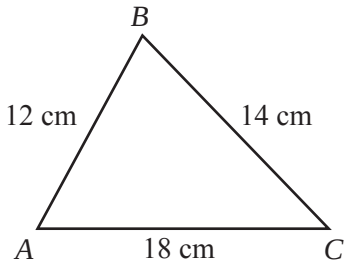
D. 350

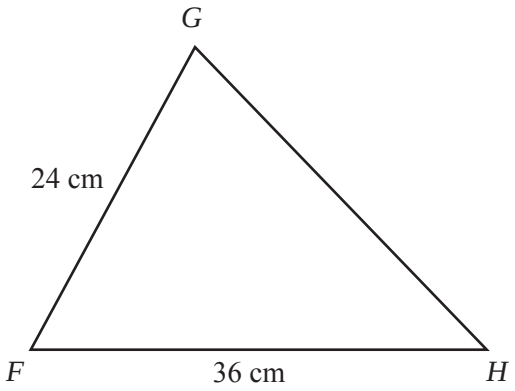
E. 400

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1





The two triangles, *ABC* and *FGH*, are similar.

The length *GH* is

A. 14 cm

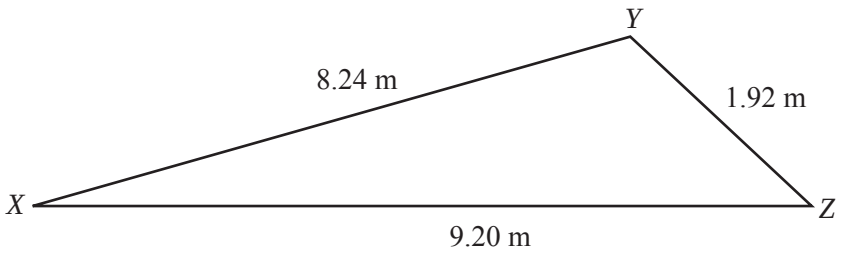
B. 24 cm

C. 26 cm

D. 28 cm

E. 32 cm

Question 2



The area (in m^2) of triangle *XYZ* can be found using Heron's formula $A = \sqrt{s(s-a)(s-b)(s-c)}$, with $a = 1.92$, $b = 8.24$, $c = 9.20$ and $s =$

A. 4.40

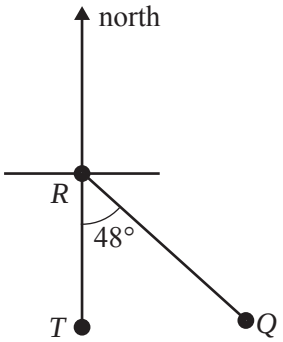
B. 6.45

C. 9.20

D. 9.68

E. 19.36

Question 3

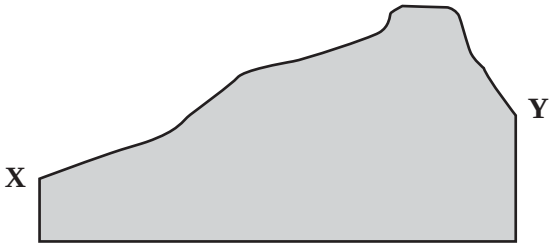


The locations of three towns, *Q*, *R* and *T*, are shown in the diagram above.
Town *T* is due south of town *R*.
The angle *TRQ* is 48°.
The bearing of town *R* from town *Q* is

A. 048°
B. 132°
C. 138°
D. 228°
E. 312°

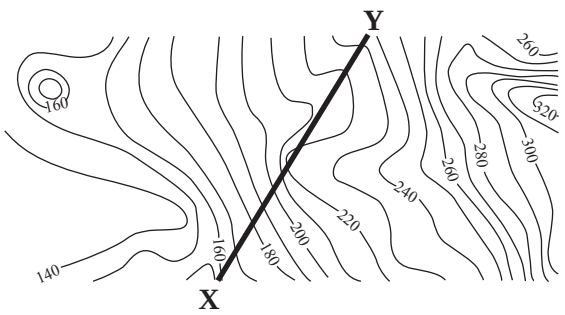
Question 4

The diagram below shows the cross-sectional profile along a line *XY* drawn on a contour map.

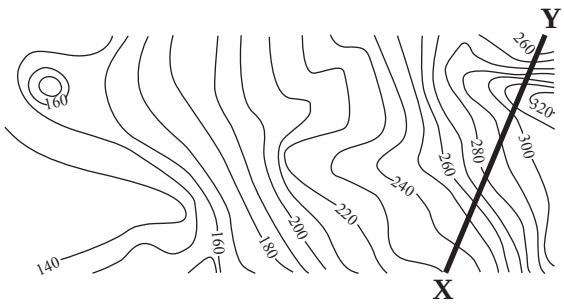


From which one of the following could this profile have been drawn?

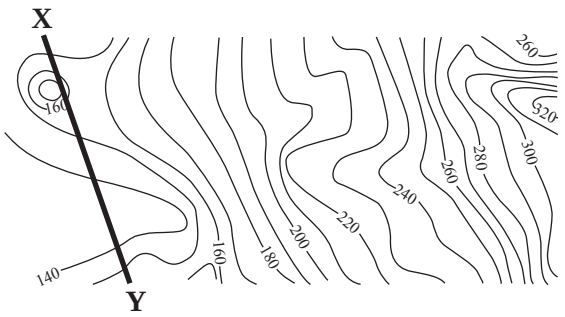
A.



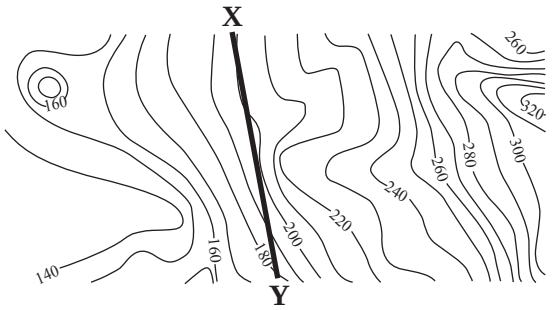
B.



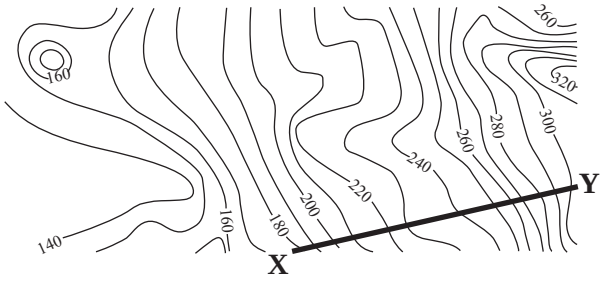
C.



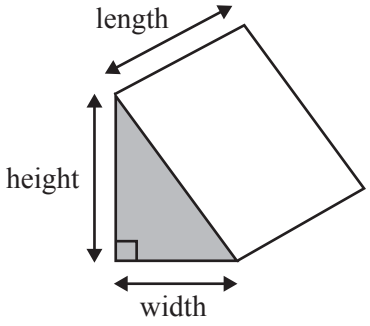
D.



E.



Question 5



A right triangular prism has a volume of 160 cm^3.

A second right triangular prism is made with the same width, twice the height and three times the length of the prism shown.

The volume of the second prism (in cm^3) is

A. 320

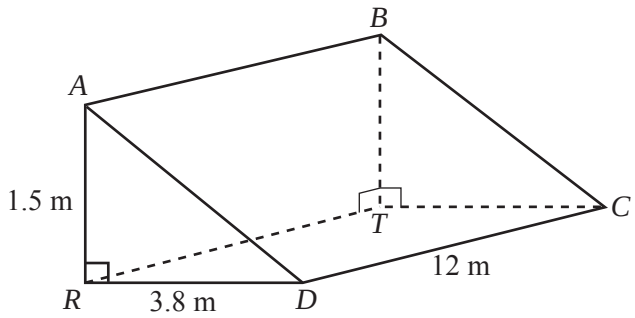
B. 640

C. 960

D. 1280

E. 1920

The following information relates to Questions 6 and 7.



ABCD is a sloping rectangular roof above a horizontal rectangular ceiling, *TCDR*.

$AB = DC = 12$ metres

$RD = TC = 3.8$ metres

$AR = BT = 1.5$ metres

Question 6

The angle of depression of *D* from *A* is closest to

A. 21.5°

B. 23.3°

C. 66.7°

D. 68.5°

E. 111.5°

Question 7

The angle *ACR* is closest to

A. 6.80°

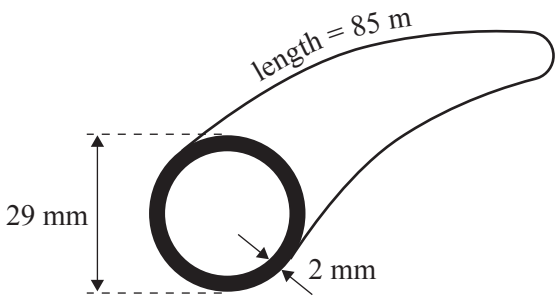
B. 6.84°

C. 7.13°

D. 18.80°

E. 21.54°

Question 8



A hose with a circular cross-section is 85 metres long.

The outside diameter of the hose is 29 millimetres. Its walls are 2 millimetres thick.

One litre of water occupies a volume of 1000 cm^3.

When the hose is full with water, the volume it holds (in litres) is closest to

A. 4

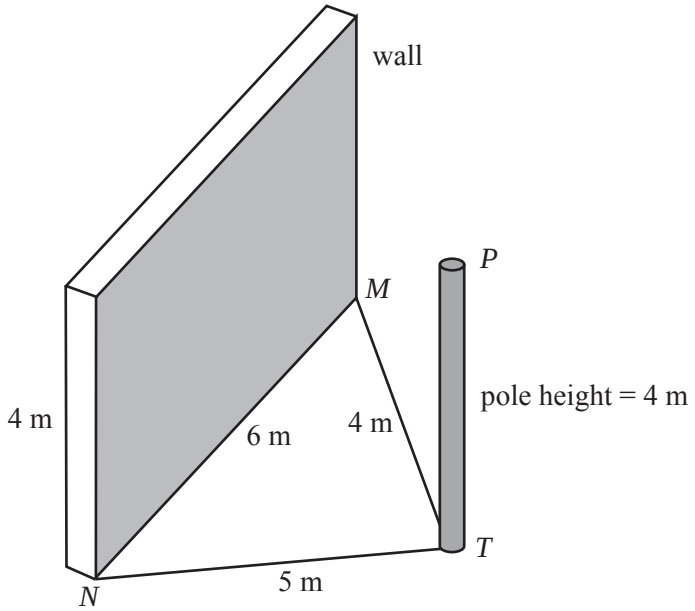
B. 42

C. 49

D. 56

E. 167

Question 9



A vertical pole, *TP*, is 4 metres tall and stands on level ground near a vertical wall.

The wall is 6 metres long and 4 metres high.

The base of the pole, *T*, is 5 metres from one end of the wall at *N* and 4 metres from the other end of the wall at *M*.

The pole falls and hits the wall.

The maximum height above ground level at which the pole could hit the wall is closest to

A. 0 m

B. 1.5 m

C. 2.3 m

D. 2.7 m

E. 3.3 m

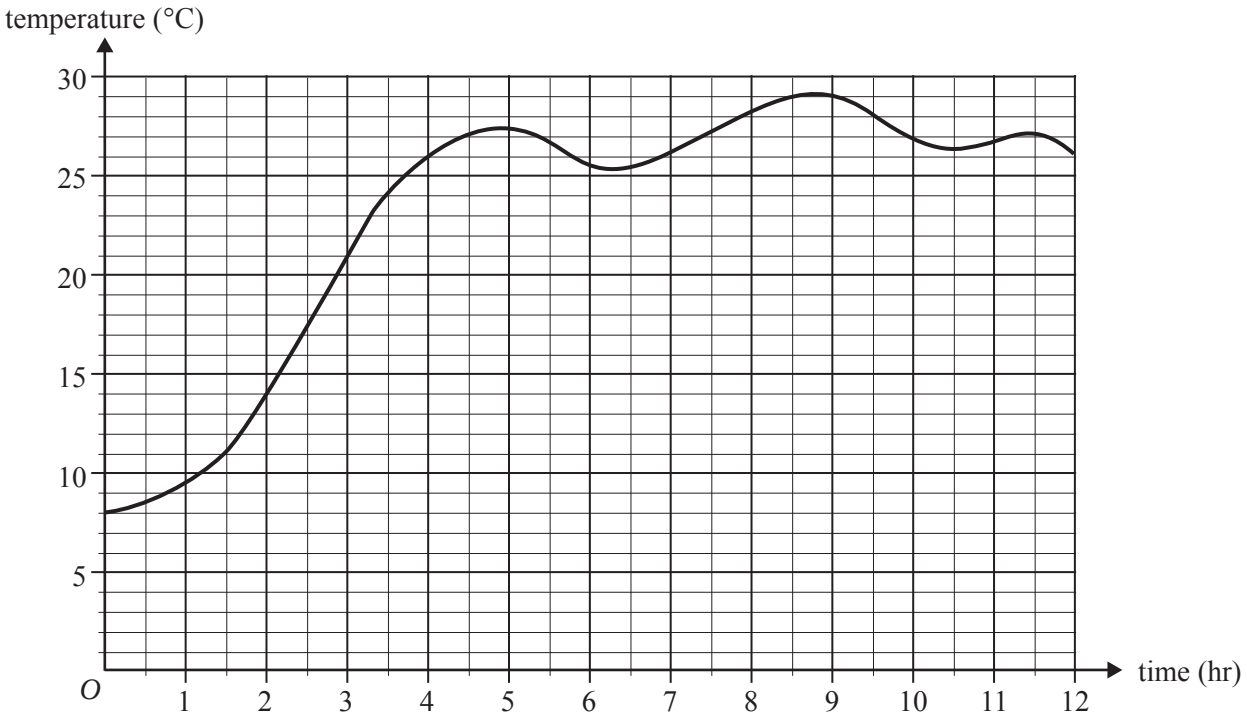
Working space

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

The following information refers to Questions 1 to 3.

The graph below shows the water temperature in a fish tank over a 12-hour period.



Question 1

Over the 12-hour period, the temperature of the tank is increasing most rapidly

A. during the first 2 hours.

B. from 2 to 4 hours.

C. from 4 to 6 hours.

D. from 6 to 8 hours.

E. from 8 to 10 hours.

Question 2

The fish tank is considered to be a safe environment for a type of fish if the water temperature is maintained between 24°C and 28°C.

Over the 12-hour period, the length of time (in hours) that the environment was safe for this type of fish was closest to

A. 1.5

B. 5.0

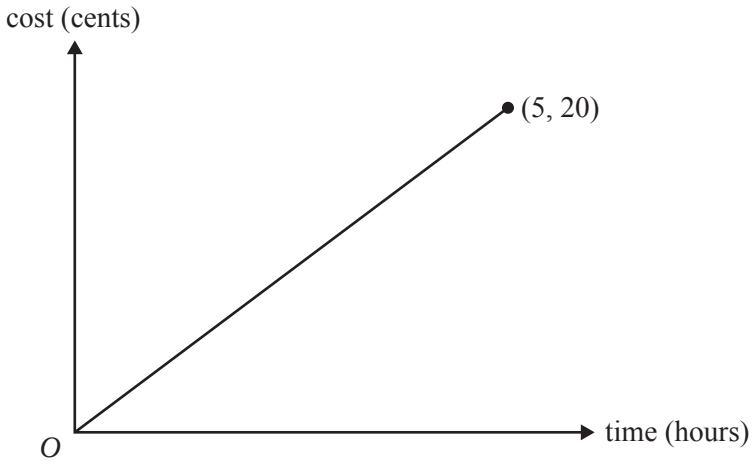
C. 7.0

D. 8.5

E. 10.5

Question 3

The graph below can be used to determine the cost (in cents) of heating the fish tank during the first five hours of heating.



The cost of heating the tank for one hour is

A. 4 cents.

B. 5 cents.

C. 15 cents.

D. 20 cents.

E. 100 cents.

Question 4

The total playing time of three CDs and four DVDs is 690 minutes.

The total playing time of five CDs and seven DVDs is 1192 minutes.

All of the CDs have the same playing time as each other and all of the DVDs have the same playing time as each other.

Let x be the playing time of a CD.

Let y be the playing time of a DVD.

The set of simultaneous linear equations that can be solved to find the playing time of a CD and the playing time of a DVD is

A. $4x + 3y = 690$
$7x + 5y = 1192$

B. $3x + 4y = 690$
$5x + 7y = 1192$

C. $3x + 5y = 690$
$4x + 7y = 1192$

D. $3x + 4y = 1192$
$5x + 7y = 690$

E. $4x + 3y = 1192$
$7x + 5y = 690$

The following information refers to Questions 5 and 6.

Kathy is a tutor who offers tutorial sessions for English and History students.

Question 5

An English tutorial session takes 1.5 hours.
A History tutorial session take 30 minutes.
Kathy has no more than 15 hours available in a week for tutorial sessions.

Let x represent the number of English tutorial sessions Kathy has each week.
Let y represent the number of History tutorial sessions Kathy has each week.

An inequality representing the constraint on Kathy's tutorial time each week (in hours) is

A. $1.5x + 30y = 15$

B. $1.5x + 30y \geq 15$

C. $1.5x + 30y \leq 15$

D. $1.5x + 0.5y \geq 15$

E. $1.5x + 0.5y \leq 15$

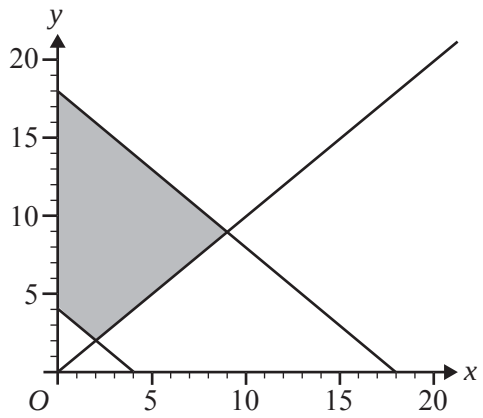
Question 6

Kathy prefers to have no more than 18 tutorial sessions in total each week.
She prefers to have at least 4 English tutorial sessions.
She also prefers to have at least as many History tutorial sessions as English tutorial sessions.

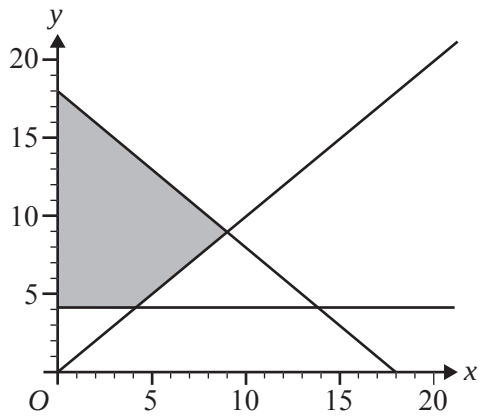
Let x represent the number of English tutorial sessions Kathy has each week.
Let y represent the number of History tutorial sessions Kathy has each week.

The shaded region that satisfies all of these constraints is

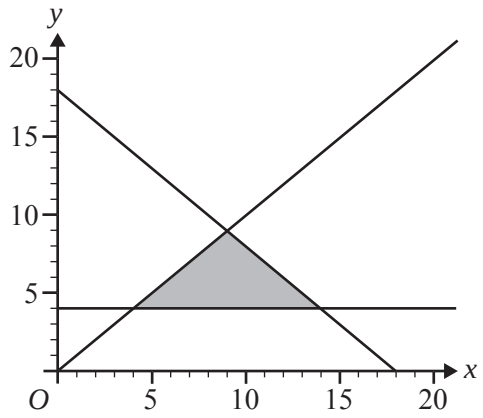
A.



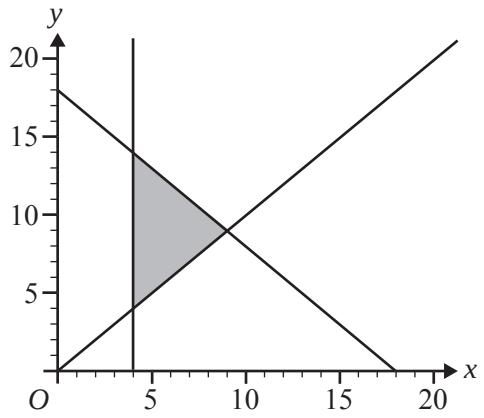
B.



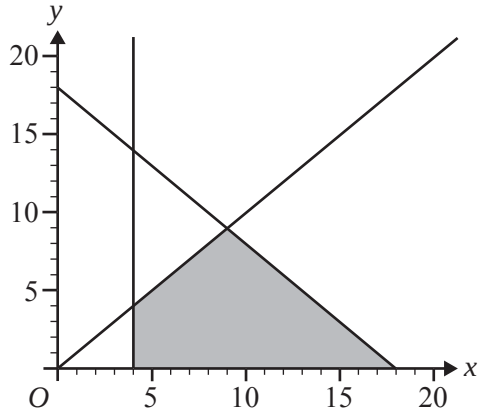
C.



D.



E.

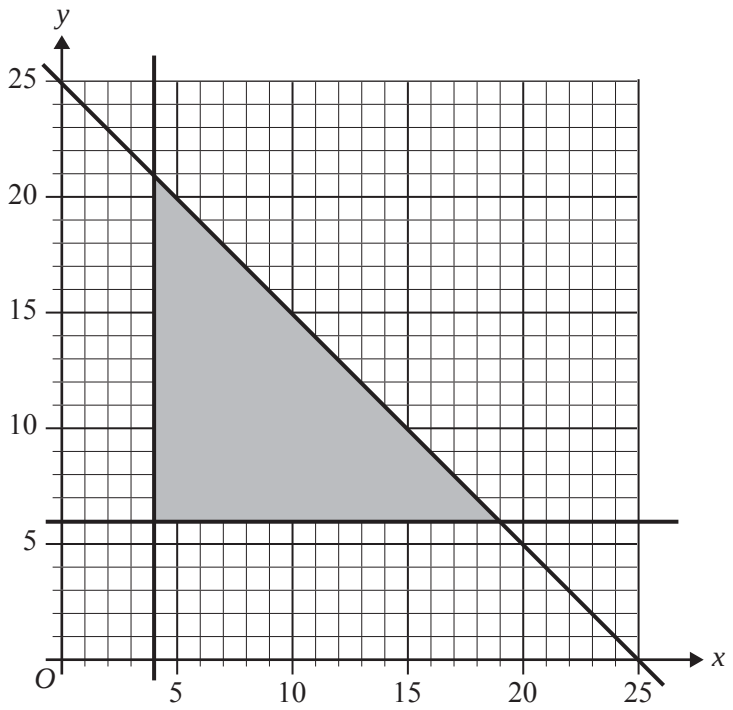


Question 7

A school's squash and volleyball teams plan to enter a sports competition.
A squash team requires at least 4 players.
A volleyball team requires at least 6 players.
No more than 25 students from any one school can enter the competition.

Let x be the number of squash players sent by the school to the competition.
Let y be the number of volleyball players sent by the school to the competition.

The constraints above define the feasible region shaded in the graph below.



A fee is charged for all players entering the competition. Squash players are charged \$5 and volleyball players are charged \$4.

Given the above constraints, the maximum cost for the school's squash and volleyball teams to enter the competition is

A. \$44
B. \$104
C. \$119
D. \$121
E. \$144

Question 8

Brian, a landscaping contractor, charges by the hour for his company's services.

To complete a particular job, he will have to use three workers and pay each of them $20 per hour. The fixed costs for the job are $150 and it will take four hours to complete the job.

To break even on this job, his hourly charge to the client should be

A. $38.25

B. $57.50

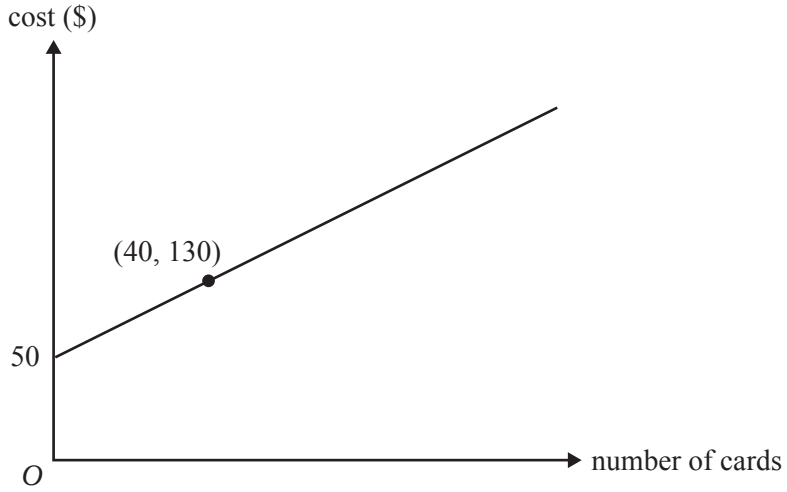
C. $97.50

D. $127.50

E. $132.50

Question 9

The graph below shows the cost (in dollars) of producing birthday cards.



If the profit from the sale of 150 birthday cards is $175, the selling price of one card is

A. $0.30

B. $1.60

C. $3.10

D. $3.50

E. $4.40

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

An amount of $800 is invested for two years at a simple interest rate of 4% per annum.

The total amount of interest earned by the investment is

A. $32

B. $64

C. $160

D. $320

E. $640

Question 2

An amount of $6500 is borrowed at a simple interest rate of 3.5% per annum.

The total interest paid over the period of the loan is $910.

The period of the loan is closest to

A. 2.5 years.

B. 3.5 years.

C. 3.8 years.

D. 4 years.

E. 4.9 years.

Question 3

Jamie bought a $500 games console on a hire-purchase plan.

He paid $50 deposit and monthly instalments of $25 for two years.

The flat interest rate charged per annum is closest to

A. 15.0%

B. 16.7%

C. 30.0%

D. 33.3%

E. 66.7%

Question 4

A delivery truck when new was valued at $65 000.

The truck's value depreciates at a rate of 22 cents per kilometre travelled.

After it has travelled a total distance of 132 600 km, the value of the truck will be

A. $14 300

B. $22 100

C. $22 516

D. $29 172

E. $35 828

Question 5

A new air-conditioning unit was purchased for $5000 on 1 January 2009.

On 1 January of each year after 2009 its value is depreciated by 20% using the reducing balance method.

The value of the air conditioner will be below $1500 for the first time on 1 January

A. 2012

B. 2013

C. 2014

D. 2015

E. 2016

Question 6

In 2005, Peter's annual salary was $35 000.

At the start of each subsequent year, his annual salary increases by 4.75%.

In 2010 his salary will be closest to

A. $36 663

B. $42 140

C. $43 310

D. $44 140

E. $55 670

Question 7

A loan of $17 500 is to be paid back over four years at an interest rate of 6.25% per annum on a reducing monthly balance.

The monthly repayment, correct to the nearest cent, will be

A. $364.58

B. $413.00

C. $802.08

D. $1156.77

E. $5079.29

Question 8

Robin takes out a reducing balance loan of $100 000 with quarterly repayments of $2150.

After seven years of quarterly repayments, Robin still owes $80 000.

Correct to one decimal place, the interest rate per annum for this loan is

A. 6.3%

B. 8.2%

C. 12.9%

D. 18.9%

E. 24.7%

Question 9

To purchase a house Sam has borrowed $250 000 at an interest rate of 4.45% per annum, fixed for ten years.

Interest is calculated monthly on the reducing balance of the loan. Monthly repayments are set at $1382.50.

After 10 years, Sam renegotiates the conditions for the balance of his loan. The new interest rate will be 4.25% per annum. He will pay $1750 per month.

The total time it will take him to pay out the loan fully is closest to

A. 17 years.

B. 20 years.

C. 21 years.

D. 22 years.

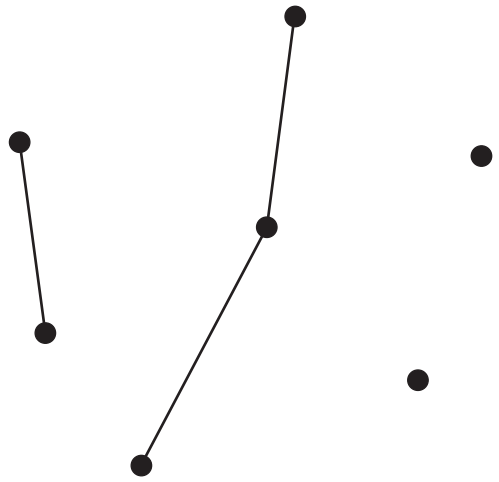
E. 23 years.

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Consider the following graph.

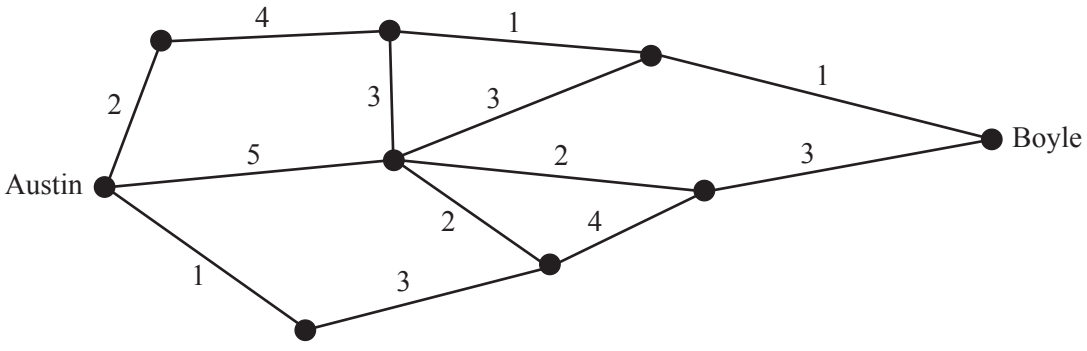


The smallest number of edges that need to be added to make this a connected graph is

A. 1
B. 2
C. 3
D. 4
E. 5

Question 2

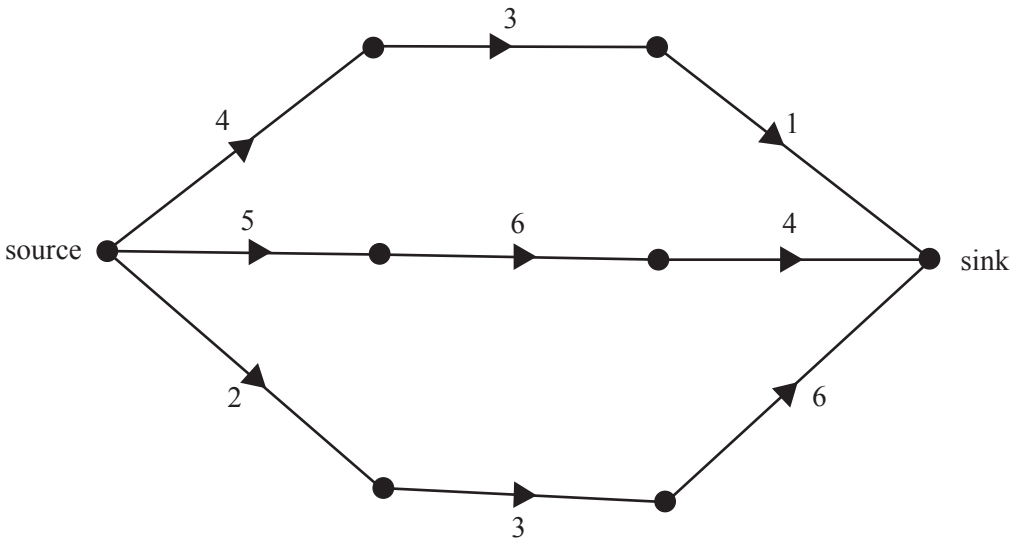
The network shows the distances, in kilometres, along roads that connect the cities of Austin and Boyle.



The shortest distance, in kilometres, from Austin to Boyle is

A. 7
B. 8
C. 9
D. 10
E. 11

Question 3



The maximum flow from source to sink through the network shown above is

A. 6
B. 7
C. 8
D. 11
E. 16

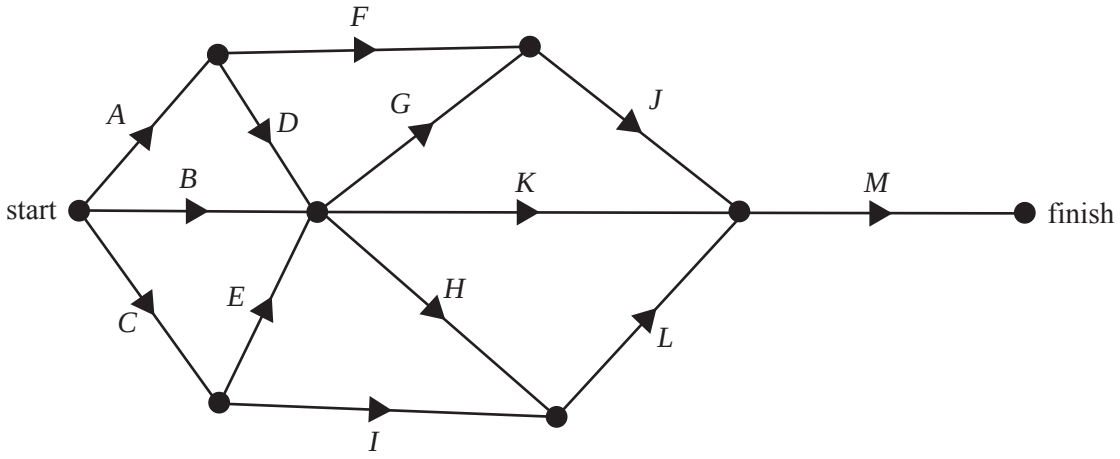
Question 4

A connected planar graph has 10 edges and 10 faces.
The number of vertices for this graph is

A. 2
B. 5
C. 8
D. 12
E. 20

The following information relates to Questions 5 and 6.

The network shows the activities that are needed to complete a particular project.



Question 5

The total number of activities that need to be completed before activity *L* may begin is

A. 2

B. 4

C. 6

D. 7

E. 8

Question 6

The duration of every activity is initially 5 hours. For an extra cost, the completion times of both activity *F* and activity *K* can be reduced to 3 hours each.

If this is done, the completion time for the project will be

A. decreased by 2 hours.

B. decreased by 3 hours.

C. decreased by 4 hours.

D. decreased by 6 hours.

E. unchanged.

Question 7

Four workers, Anna, Bill, Caitlin and David, are each to be assigned a different task.

The table below gives the time, in minutes, that each worker takes to complete each of the four tasks.

	task 1	**task 2**	**task 3**	**task 4**
Anna	7	5	15	9
Bill	8	5	18	10
Caitlin	4	6	22	4
David	7	11	16	10

The tasks are allocated so as to minimise the total time taken to complete the four tasks.

This total time, in minutes, is

A. 21

B. 28

C. 31

D. 34

E. 38

Question 8

An undirected connected graph has five vertices.

Three of these vertices are of even degree and two of these vertices are of odd degree.

One extra edge is added. It joins two of the existing vertices.

In the resulting graph, it is **not** possible to have five vertices that are

A. all of even degree.

B. all of equal degree.

C. one of even degree and four of odd degree.

D. three of even degree and two of odd degree.

E. four of even degree and one of odd degree.

Question 9

Five soccer teams played each other once in a tournament. In each game there was a winner and a loser.

A table of one-step and two-step dominances was prepared to summarise the results.

team	**one-step dominances**	**two-step dominances**
Aardvarks	1	2
Bears	3	5
Chimps	2	4
Donkeys	3	4
Elephants	1	1

One result in the tournament that must have occurred is that

A. Elephants defeated Bears.

B. Elephants defeated Aardvarks.

C. Aardvarks defeated Donkeys.

D. Donkeys defeated Bears.

E. Bears defeated Chimps.

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

$3\begin{bmatrix}2 & 1\\ 0 & 3\end{bmatrix}+2\begin{bmatrix}-1 & 0\\ 2 & -7\end{bmatrix}$ equals

A. $\begin{bmatrix}4 & 3\\ 4 & -5\end{bmatrix}$

B. $6\begin{bmatrix}1 & 1\\ 2 & -4\end{bmatrix}$

C. $\begin{bmatrix}4 & 3\\ 4 & 2\end{bmatrix}$

D. $5\begin{bmatrix}1 & 1\\ 2 & -4\end{bmatrix}$

E. $\begin{bmatrix}3 & 6\\ 7 & 4\end{bmatrix}$

Question 2

The matrix $\begin{bmatrix}12 & 15 & 3\\ -6 & 0 & 24\end{bmatrix}$ can also be written as

A. [12 15 3] + [–6 0 24]

B. $\begin{bmatrix}12\\ -6\end{bmatrix}+\begin{bmatrix}15\\ 0\end{bmatrix}+\begin{bmatrix}3\\ 24\end{bmatrix}$

C. $\begin{bmatrix}3\\ 6\end{bmatrix}\begin{bmatrix}4 & 5 & 1\\ -1 & 0 & 4\end{bmatrix}$

D. $\frac{1}{3}\times\begin{bmatrix}4 & 5 & 1\\ -2 & 0 & 8\end{bmatrix}$

E. $3\times\begin{bmatrix}4 & 5 & 1\\ -2 & 0 & 8\end{bmatrix}$

Question 3

The number of people attending the morning, afternoon and evening sessions at a cinema is given in the table below. The admission charges (in dollars) for each session are also shown in the table.

	session		
	morning	**afternoon**	**evening**
number of people attending	25	56	124
admission charge ($)	12	15	20

A column matrix that can be used to list the number of people attending each of the three sessions is

A. [25 56 124]

B. $\begin{bmatrix} 25 \\ 56 \\ 124 \end{bmatrix}$

C. [12 15 20]

D. $\begin{bmatrix} 12 \\ 15 \\ 20 \end{bmatrix}$

E. $\begin{bmatrix} 25 & 56 & 124 \\ 12 & 15 & 20 \end{bmatrix}$

Question 4

The matrix equation $\begin{bmatrix} 4 & 2 & 8 \\ 2 & 0 & 3 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 6 \end{bmatrix}$ can be used to solve the system of simultaneous linear equations

A. $4x + 2y + 8z = 7$
$2x + 3y = 2$
$3x - y = 6$

B. $4x + 2y + 8z = 7$
$2x + 3y = 2$
$3y - z = 6$

C. $4x + 2y + 8z = 7$
$2y + 3z = 2$
$3x - z = 6$

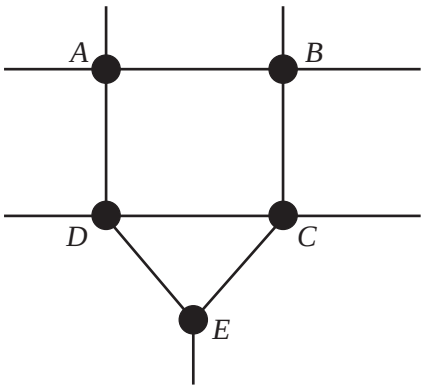
D. $4x + 2y + 8z = 7$
$2x + 3z = 2$
$3y - z = 6$

E. $4x + 2y + 8z = 7$
$2x + 3z = 2$
$3x - z = 6$

Question 5

A, *B*, *C*, *D* and *E* are five intersections joined by roads as shown in the diagram below.

Some of these roads are one-way only.



The matrix below indicates the direction that cars can travel along each of these roads.

In this matrix

- 1 in column *A* and row *B* indicates that cars can travel directly from *A* to *B*
- 0 in column *B* and row *A* indicates that cars cannot travel directly from *B* to *A* (either it is a one-way road or no road exists).

from intersection

A	*B*	*C*	*D*	*E*	
0	0	0	0	0	*A*
1	0	0	0	0	*B*
0	1	0	1	1	*C*
1	0	0	0	0	*D*
0	0	1	1	0	*E*

to intersection

Cars can travel in both directions between intersections

A. *A* and *D*

B. *B* and *C*

C. *C* and *D*

D. *D* and *E*

E. *C* and *E*

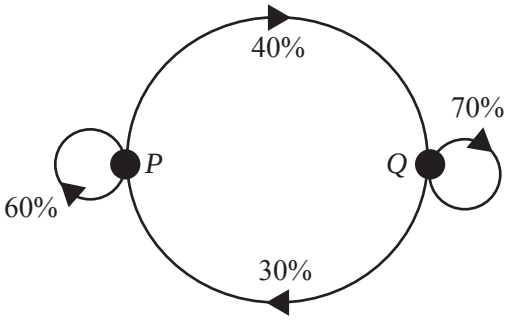
Question 6

T is a transition matrix, where

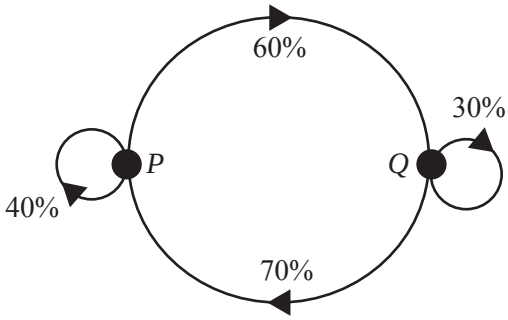
$$T = \begin{array}{c} \text{from} \\ \begin{matrix} P & Q \end{matrix} \\ \begin{bmatrix} 0.6 & 0.7 \\ 0.4 & 0.3 \end{bmatrix} \end{array} \begin{matrix} P \\ Q \end{matrix} \text{ to}$$

An equivalent transition diagram, with proportions expressed as percentages, is

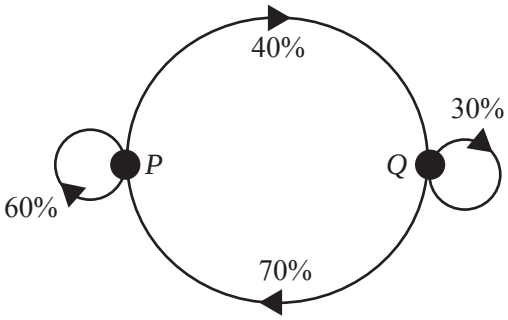
A.



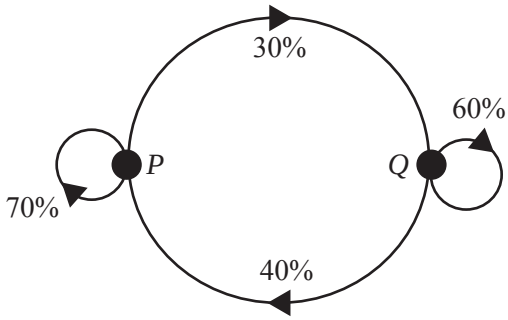
B.



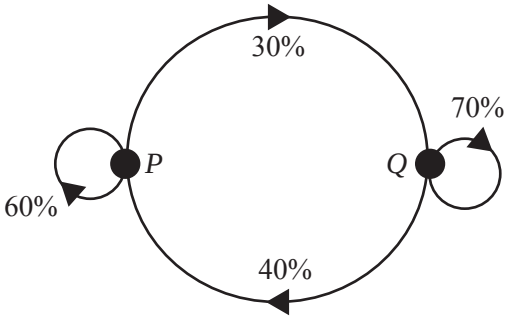
C.



D.



E.



The following information relates to Questions 7 and 8.

In a country town, people only have the choice of doing their food shopping at a store called Marks (M) or at a newly opened store called Foodies (F).

In the first week that Foodies opened, only 300 of the town's 800 shoppers did their food shopping at Marks. The remainder did their food shopping at Foodies.

Question 7

A state matrix S_1 that can be used to represent this situation is

A. $S_1 = \begin{bmatrix} 300 \\ 800 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$

B. $S_1 = \begin{bmatrix} 500 \\ 300 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$

C. $S_1 = \begin{bmatrix} 800 \\ 300 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$

D. $S_1 = \begin{bmatrix} 300 \\ 500 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$

E. $S_1 = \begin{bmatrix} 800 \\ 500 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$

Question 8

A market researcher predicts that

- of those who do their food shopping at Marks this week, 70% will shop at Marks next week and 30% will shop at Foodies
- of those who do their food shopping at Foodies this week, 90% will shop at Foodies next week and 10% will shop at Marks.

A transition matrix that can be used to represent this situation is

A. $T = \begin{matrix} \text{this week} \\ M \quad F \end{matrix} \begin{bmatrix} 0.7 & 0.9 \\ 0.3 & 0.1 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$ next week

B. $T = \begin{matrix} \text{this week} \\ M \quad F \end{matrix} \begin{bmatrix} 0.7 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$ next week

C. $T = \begin{matrix} \text{this week} \\ M \quad F \end{matrix} \begin{bmatrix} 0.3 & 0.7 \\ 0.9 & 0.1 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$ next week

D. $T = \begin{matrix} \text{this week} \\ M \quad F \end{matrix} \begin{bmatrix} 0.7 & 0.3 \\ 0.1 & 0.9 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$ next week

E. $T = \begin{matrix} \text{this week} \\ M \quad F \end{matrix} \begin{bmatrix} 0.3 & 0.9 \\ 0.7 & 0.1 \end{bmatrix} \begin{matrix} M \\ F \end{matrix}$ next week

Question 9

$T = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$ is a transition matrix.

$S_3 = \begin{bmatrix} 1150 \\ 850 \end{bmatrix}$ is a state matrix.

If $S_3 = TS_2$, then S_2 equals

A. $\begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$

B. $\begin{bmatrix} 1090 \\ 940 \end{bmatrix}$

C. $\begin{bmatrix} 1100 \\ 900 \end{bmatrix}$

D. $\begin{bmatrix} 1150 \\ 850 \end{bmatrix}$

E. $\begin{bmatrix} 1175 \\ 825 \end{bmatrix}$

END OF MULTIPLE-CHOICE QUESTION BOOK


Victoria
The Place To Be

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET