VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

**Victorian Certificate of Education 2015**

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

**STUDENT NUMBER**

# FURTHER MATHEMATICS

## Written examination 2

**Monday 2 November 2015**

**Reading time: 9.00 am to 9.15 am (15 minutes)**
**Writing time: 9.15 am to 10.45 am (1 hour 30 minutes)**

## QUESTION AND ANSWER BOOK

**Structure of book**

| Core | | |
|---|---|---|
| *Number of questions* | *Number of questions to be answered* | *Number of marks* |
| 5 | 5 | 15 |
| Module | | |
| *Number of modules* | *Number of modules to be answered* | *Number of marks* |
| 6 | 3 | 45 |
| | | Total 60 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

**Materials supplied**
- Question and answer book of 43 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

**Instructions**
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

## Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, $\pi$, surds or fractions.

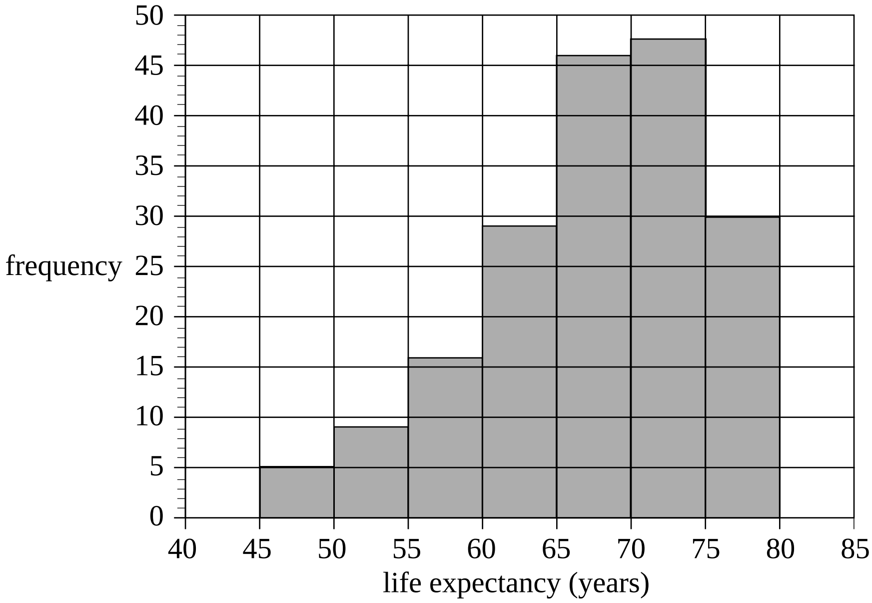
Diagrams are not to scale unless specified otherwise.

**Page**

## Core

**Question 1** (3 marks)

The histogram below shows the distribution of life expectancy of people for 183 countries.




**a.** For this distribution, the modal interval is ☐. 1 mark

**b.** In how many of these countries is life expectancy less than 55 years? 1 mark

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**c.** In what percentage of these 183 countries is life expectancy between 75 and 80 years?
Write your answer correct to one decimal place. 1 mark

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**Question 2** (3 marks)

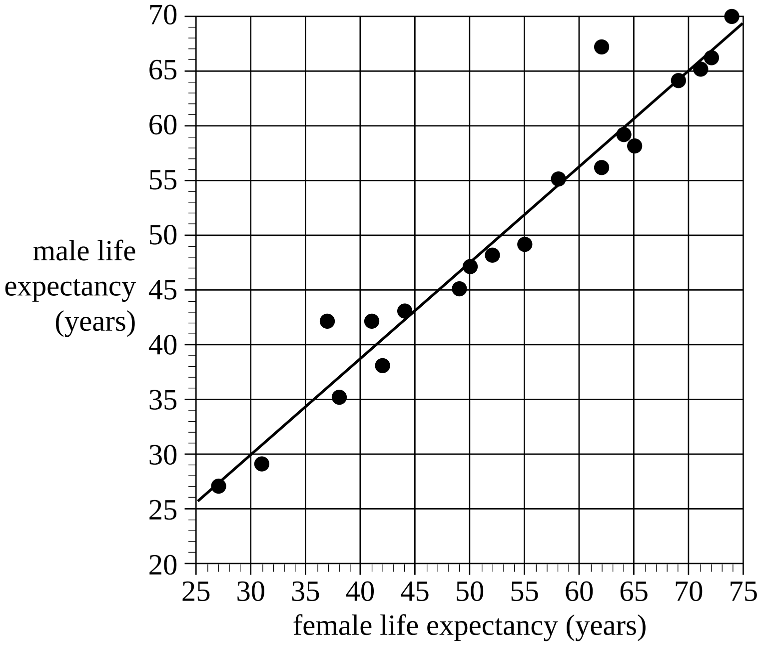
The parallel boxplots below compare the distribution of life expectancy for 183 countries for the years 1953, 1973 and 1993.


**a.** Describe the shape of the distribution of life expectancy for 1973. 1 mark

**b.** Explain why life expectancy for these countries is associated with the year. Refer to specific statistical values in your answer. 2 marks

**Question 3** (3 marks)

The scatterplot below plots male life expectancy (*male*) against female life expectancy (*female*) in 1950 for a number of countries. A least squares regression line has been fitted to the scatterplot as shown.




The slope of this least squares regression line is 0.88

**a.** Interpret the slope in terms of the variables *male* life expectancy and *female* life expectancy. 1 mark

The equation of this least squares regression line is

$$male = 3.6 + 0.88 \times female$$

**b.** In a particular country in 1950, *female* life expectancy was 35 years.

Use the equation to predict *male* life expectancy for that country. 1 mark

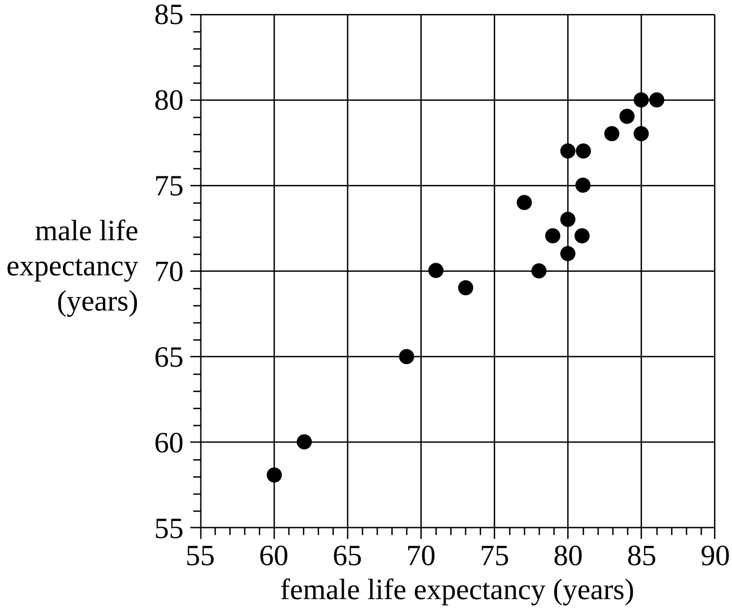
**c.** The coefficient of determination is 0.95

Interpret the coefficient of determination in terms of male life expectancy and female life expectancy. 1 mark

**Question 4** (2 marks)

The table below shows male life expectancy (*male*) and female life expectancy (*female*) for a number of countries in 2013. The scatterplot has been constructed from this data.

| **Life expectancy (in years) in 2013** | |
|---|---|
| ***male*** | ***female*** |
| 80 | 85 |
| 60 | 62 |
| 73 | 80 |
| 70 | 71 |
| 70 | 78 |
| 78 | 83 |
| 77 | 80 |
| 65 | 69 |
| 74 | 77 |
| 70 | 78 |
| 75 | 81 |
| 58 | 60 |
| 80 | 86 |
| 69 | 73 |
| 79 | 84 |
| 72 | 81 |
| 78 | 85 |
| 72 | 79 |
| 77 | 81 |
| 71 | 80 |




**a.** Use the scatterplot to describe the association between *male* life expectancy and *female* life expectancy in terms of strength, direction and form. 1 mark

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**b.** Determine the equation of a least squares regression line that can be used to predict *male* life expectancy from *female* life expectancy for the year 2013.

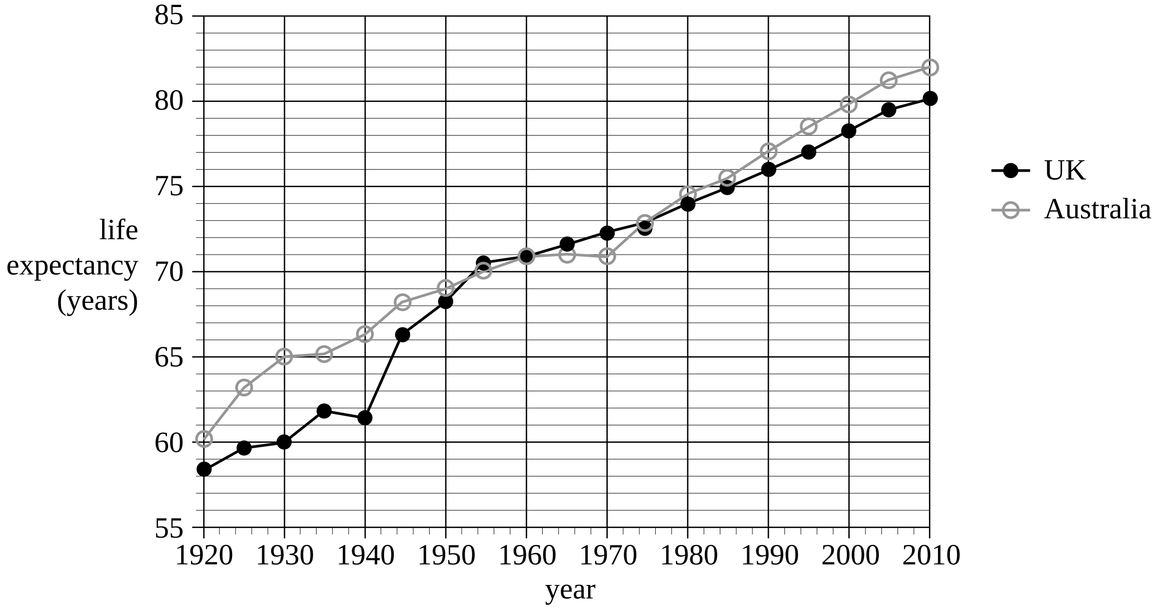
Complete the equation for the least squares regression line below by writing the intercept and slope in the boxes provided.

Write these values correct to two decimal places. 1 mark

*male* = ☐ + ☐ × *female*

**Question 5** (4 marks)

The time series plot below displays the *life expectancy*, in years, of people living in Australia and the United Kingdom (UK) for each *year* from 1920 to 2010.



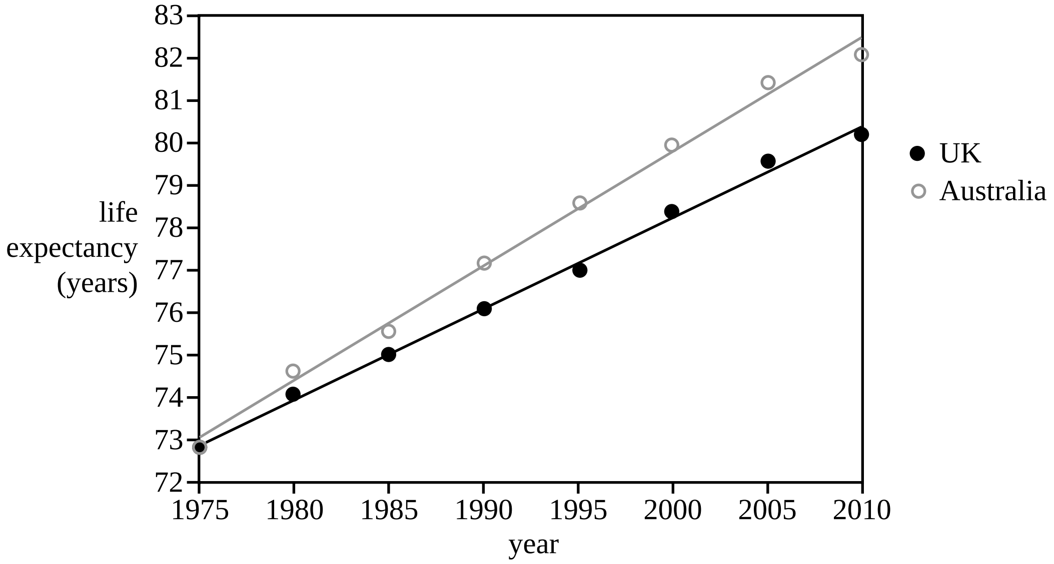

**a.** By how much did *life expectancy* in Australia increase during the period 1920 to 2010?
Write your answer correct to the nearest year. 1 mark

**b.** In 1975, the life expectancies in Australia and the UK were very similar.

From 1975, the gap between the life expectancies in the two countries increased, with people in Australia having a longer life expectancy than people in the UK.

To investigate the difference in life expectancies, least squares regression lines were fitted to the data for both Australia and the UK for the period 1975 to 2010.

The results are shown below.




The equations of the least squares regression lines are as follows.

Australia: *life expectancy* = –451.7 + 0.2657 × *year*

UK: *life expectancy* = –350.4 + 0.2143 × *year*

**i.** Use these equations to predict the difference between the life expectancies of Australia and the UK in 2030.

Give your answer correct to the nearest year. 2 marks

**ii.** Explain why this prediction may be of limited reliability. 1 mark

**END OF CORE**
**TURN OVER**

## Module 1: Number patterns

**Question 1** (5 marks)

A crop of capsicums is being harvested from a field on a large farm.

In week 1 of the harvest, 2000 kg of capsicums were picked.

In week 2 of the harvest, 2150 kg of capsicums were picked.

In week 3 of the harvest, 2300 kg of capsicums were picked.

The weight of the capsicums, in kilograms, picked each week continues in this pattern for the first eight weeks of the harvest.

The weight of the capsicums, in kilograms, picked each week forms the terms of an arithmetic sequence, as shown below.

2000, 2150, 2300 …

**a.** Write down a calculation that shows that the common difference for this sequence is 150. 1 mark

**b.** How many kilograms of capsicums will be picked in week 5? 1 mark

**c.** How many kilograms of capsicums in total will have been picked in the first eight weeks of the harvest? 1 mark

**d.** In which week will the total weight of the harvest first exceed 14 000 kg? 1 mark

**e.** The weight of the capsicums, $C_n$, picked in week $n$, is modelled by a difference equation.

Write down the rule for this difference equation in the box provided below. 1 mark

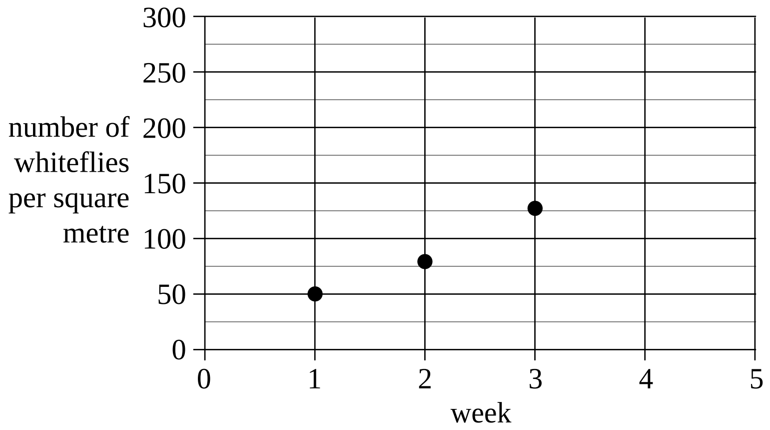
$C_{n+1} =$ ☐ $C_1 = 2000$

**Question 2** (7 marks)

Whiteflies are a pest that affect capsicums.

The number of whiteflies per square metre in another capsicum field was recorded at the beginning of each week for three consecutive weeks.

This information is displayed on the graph below.




At the beginning of week 1, 50 whiteflies per square metre were recorded.

The number of whiteflies per square metre is expected to increase following a geometric sequence with common ratio $r = 1.6$

**a.** On the **graph above**, use a cross (×) to plot the number of whiteflies per square metre that are expected at the beginning of week 4. 1 mark

(*Answer on the graph above.*)

**b.** At the beginning of which week will the number of whiteflies first be expected to exceed 500 per square metre? 1 mark

**c.** What is the percentage increase in the number of whiteflies per square metre from the beginning of any week to the beginning of the next week? 1 mark

The farmer implements a pest control treatment from the beginning of week 3. The treatment kills an average of $k$ whiteflies per square metre of the field each week.

Let $W_n$ be the average number of whiteflies per square metre of the field at the beginning of week $n$.

The change in the average number of whiteflies per square metre of the field, from week to week, is modelled by the difference equation

$$W_{n+1} = 1.6 \times W_n - k \qquad W_3 = 128$$

**d.** **i.** Find the value of $k$ if the number of whiteflies per square metre is to remain constant. 1 mark

**ii.** If $k = 50$, how many whiteflies per square metre would be expected to be in the field at the beginning of week 4? 1 mark

**iii.** Find the value of $k$ if, at the beginning of week 5, there will be 30 fewer whiteflies per square metre than at the beginning of week 3.

Write your answer correct to one decimal place. 2 marks

**Question 3** (3 marks)

Ladybirds are a natural predator of whiteflies.

The farmer is planning to breed ladybirds in a greenhouse. She plans to release the ladybirds into the field to reduce the whitefly population.

Let $G_n$ be the number of ladybirds in the greenhouse at the beginning of the *n*th week.

The change in the number of ladybirds in the greenhouse, from week to week, is modelled by the difference equation

$$G_{n+1} = 1.25 \times G_n - m \qquad G_1 = 1200 \qquad (m \geq 0)$$

Let $F_n$ be the number of ladybirds in the field at the beginning of the *n*th week.

The change in the number of ladybirds in the field, from week to week, is modelled by the difference equation

$$F_{n+1} = 0.65 \times F_n + m \qquad F_1 = f \qquad (m \geq 0)$$

where $f$ is the number of ladybirds in the field at the beginning of the first week.

**a.** Both difference equations above include $m$.

Explain what $m$ represents, referring to the difference equations above in your answer. 1 mark

**b.** Find the value of $f$ that will allow the farmer to maintain a constant number of ladybirds in both the greenhouse and the field.

Write your answer correct to the nearest number of ladybirds. 2 marks

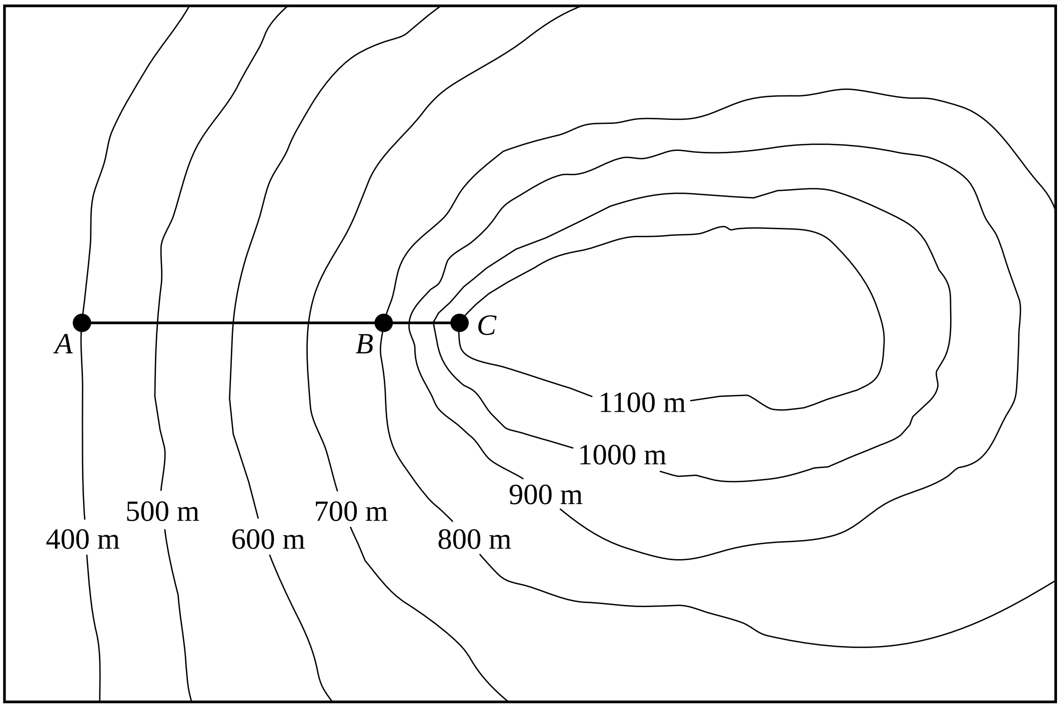
## Module 2: Geometry and trigonometry

**Question 1** (6 marks)

The contour map below represents a mountain with a flat top.

The flat top of the mountain is 1100 m above sea level.

The first section of a cable car travels from a visitors' centre at *A* to a cable car station at *B*.




**a.** What is the difference in height above sea level between the visitors' centre at *A* and the cable car station at *B*? 1 mark

The second section of the cable car travels from the cable car station at *B* to a cable car station on top of the mountain at *C*.

The horizontal distance from the station at *B* to the station at *C* is 500 m.

The vertical distance from the station at *B* to the station at *C* is 300 m.

**b.** What is the average slope from *B* to *C*? 1 mark

The cable car travels along cables that are supported by pylons.
The horizontal distance between the visitors' centre at *A* and the first pylon is 400 m.
The scale factor used on the contour map is 1:50 000.

**c.** What is the distance **on the map** between the visitors' centre at *A* and the first pylon?
Write your answer in metres. 1 mark

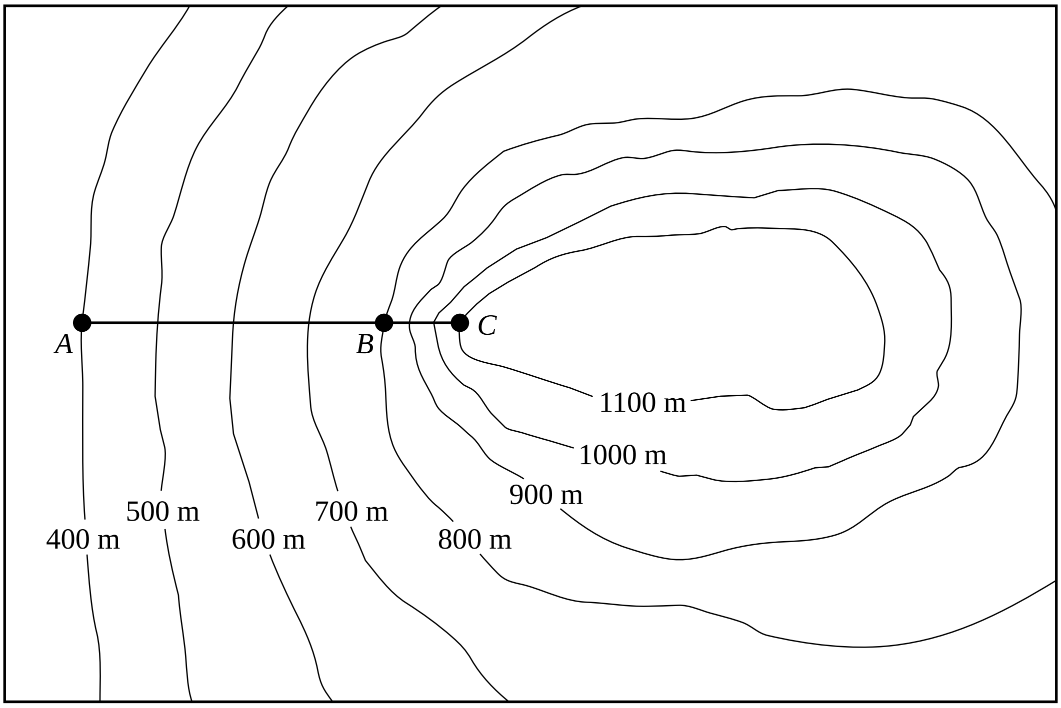
The horizontal distance from *A* to *C* is 2500 m.
The straight-line distance from *A* to *C* is 2596 m.

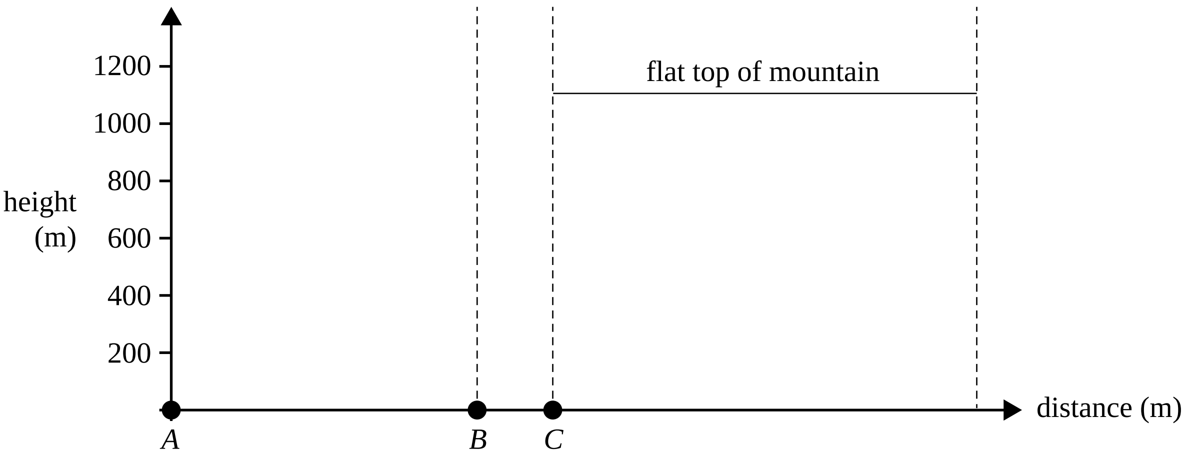
**d.** What is the angle of elevation from *A* to *C*?
Write your answer correct to the nearest degree. 1 mark

**e.** The contour map is shown below. Underneath the contour map is a graph of the height of the mountain against the horizontal distance from the visitors' centre.

The flat top of the mountain is shown on the graph.

Draw a cross-section of the mountain from *A* to *C* on the graph. 2 marks

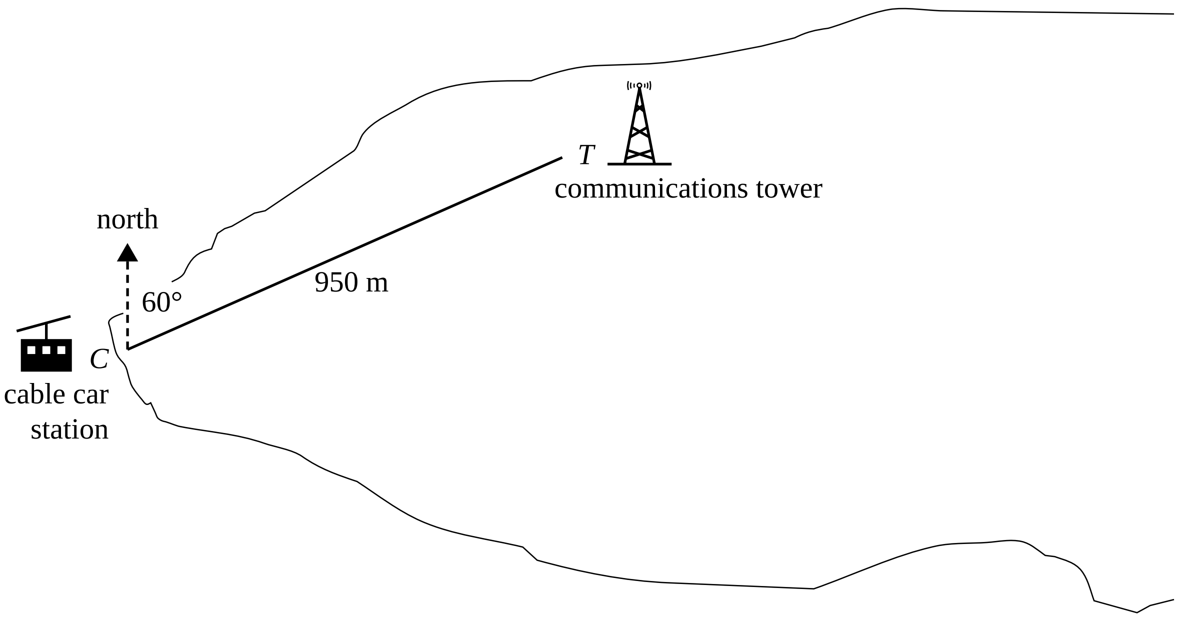


**Question 2** (4 marks)

There are plans to construct a series of straight paths on the flat top of the mountain.

A straight path will connect the cable car station at *C* to a communications tower at *T*, as shown in the diagram below.

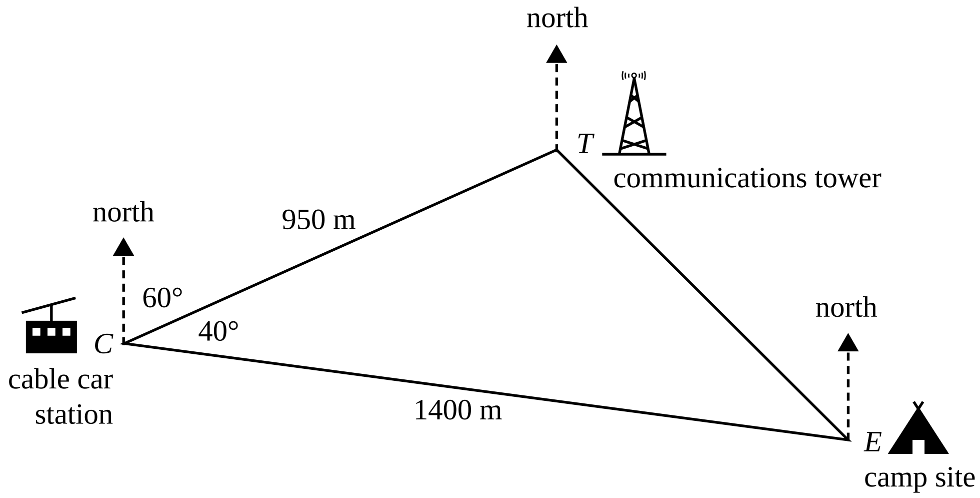



The bearing of the communications tower from the cable car station is 060°.

The length of the straight path between the communications tower and the cable car station is 950 m.

**a.** How far north of the cable car station is the communications tower? 1 mark

Paths will also connect the cable car station and the communications tower to a camp site at *E*, as shown below.




The length of the straight path between the cable car station and the camp site is 1400 m.

The angle *TCE* is 40°.

**b.** **i.** What will be the length of the straight path between the communications tower and the camp site?

Write your answer correct to the nearest metre. 1 mark

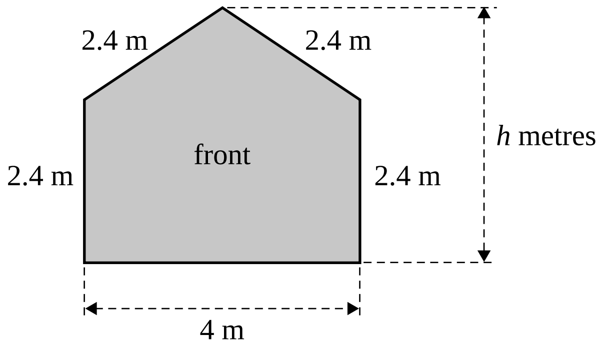
**ii.** Use the cosine rule to find the bearing of the camp site from the communications tower.

Write your answer correct to the nearest degree. 2 marks

**Question 3** (3 marks)

Cabins are being built at the camp site.

The dimensions of the front of each cabin are shown in the diagram below.




The walls of each cabin are 2.4 m high.

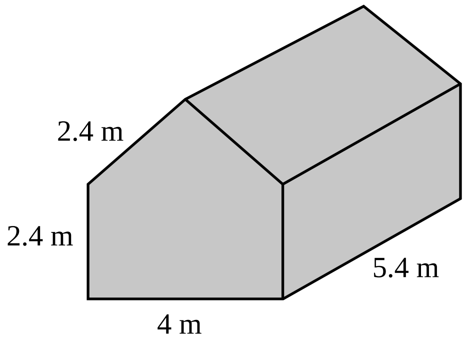
The sloping edges of the roof of each cabin are 2.4 m long.

The front of each cabin is 4 m wide.

The overall height of each cabin is $h$ metres.

**a.** Show that the value of $h$ is 3.73, correct to two decimal places. 1 mark

Each cabin is in the shape of a prism, as shown in the diagram below.

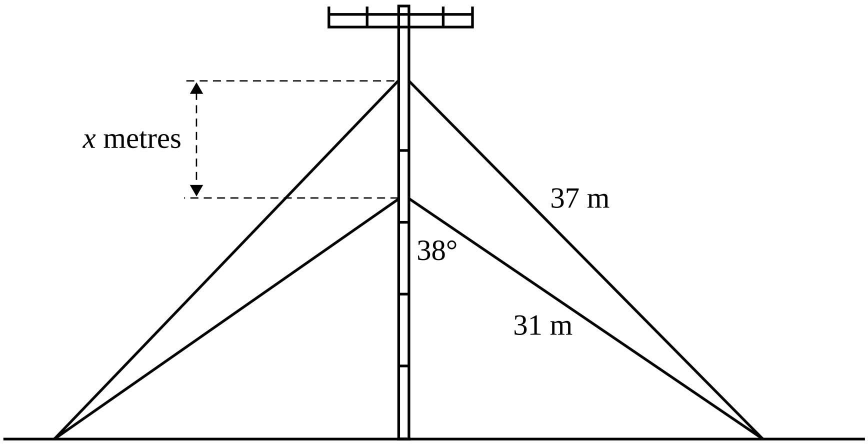



**b.** All external surfaces of one cabin are to be painted, excluding the base.

What is the total area of the surface to be painted?
Write your answer correct to the nearest square metre. 2 marks

**Question 4** (2 marks)

Wires support the communications tower, as shown in the diagram below.




The shortest wire is 31 m long.
The shortest wire makes an angle of 38° with the communications tower.
The longest wire is 37 m long.
The longest wire is attached to the communications tower *x* metres above the shortest wire.

What is the value of *x*?
Write your answer in metres, correct to one decimal place.

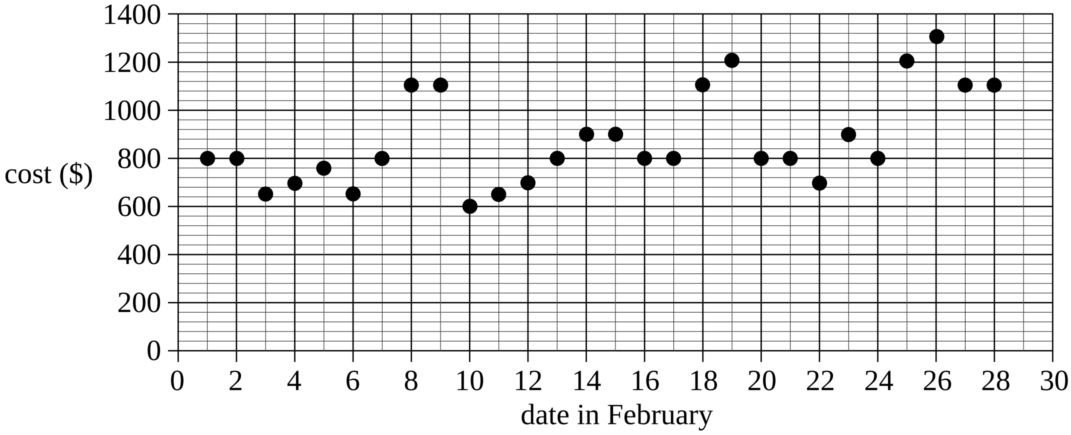
**END OF MODULE 2**
**TURN OVER**

## Module 3: Graphs and relations

**Question 1** (2 marks)

Ben is flying to Japan for a school cultural exchange program.

The graph below shows the cost of a particular flight to Japan, in dollars, on each day in February.




**a.** What is the cost, in dollars, of this flight to Japan on 19 February? 1 mark

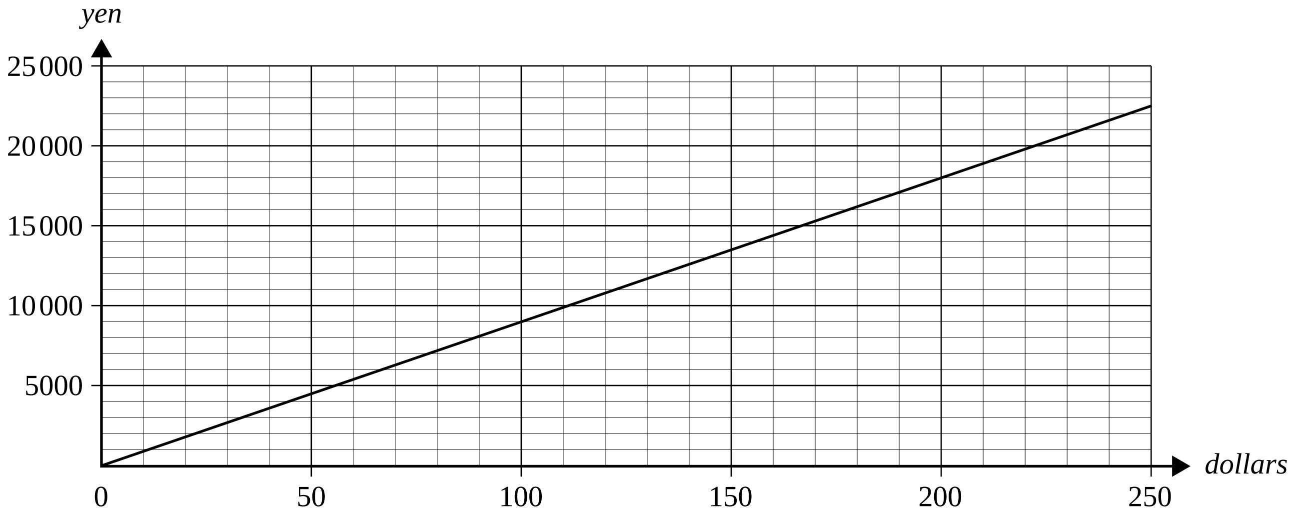
**b.** On how many days in February is the cost of this flight to Japan more than $1000? 1 mark

**Question 2** (2 marks)

Ben will use a currency exchange agency to buy some Japanese yen (the Japanese currency unit).

The graph below shows the relationship between Japanese *yen* and Australian *dollars* on a particular day.

This graph can be used to calculate a conversion between dollars and yen on that day.




**a.** Ben converts his dollars into yen using this graph.

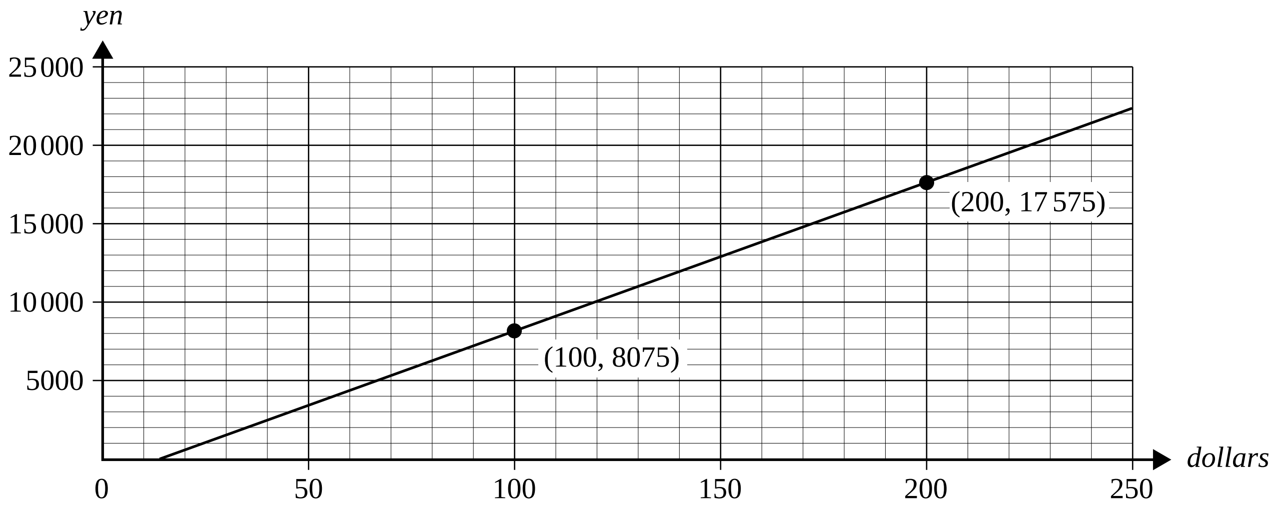
How many yen does he receive for $200? 1 mark

**b.** The slope of this graph is the exchange rate for converting dollars into yen on that particular day.

How many yen will Ben receive for each dollar? 1 mark

**Question 3** (3 marks)

The graph below shows the relationship between the *yen* and the *dollar* on the same day at a different currency exchange agency.




The points (100, 8075) and (200, 17 575) are labelled.

The equation for the relationship between the *yen* and the *dollar* is

$$yen = 95 \times dollars - k$$

**a.** Use the point (100, 8075) to show that the value of $k$ is 1425. 1 mark

**b.** **i.** Determine the intercept on the horizontal axis. 1 mark

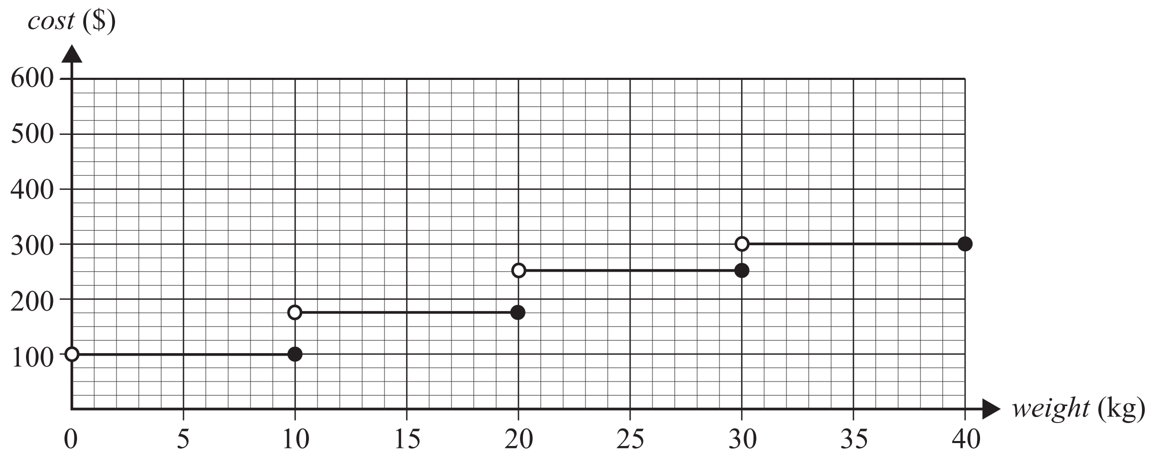
**ii.** Interpret the intercept on the horizontal axis in the context of converting dollars to yen. 1 mark

**Question 4** (5 marks)

The airline that Ben uses to travel to Japan charges for the seat and luggage separately.

The charge for luggage is based on the *weight*, in kilograms, of the luggage.

If the luggage is paid for at the airport, the graph below can be used to determine the *cost*, in dollars, of luggage of a certain *weight*, in kilograms.




**a.** Find the cost at the airport for 23 kg of luggage. 1 mark

If the luggage is paid for online prior to arriving at the airport, the equation for the relationship between the *online cost*, in dollars, and the *weight*, in kilograms, would be

$$\text{online cost} = \begin{cases} 75 & 0 < \text{weight} \le 20 \\ 22.5 \times \text{weight} - 375 & 20 < \text{weight} \le 40 \end{cases}$$

**b.** Find the online cost for 30 kg of luggage. 1 mark

**c.** On the **graph above**, sketch a graph of the *online cost* of luggage for $0 < \textit{weight} \le 40$. Include the end points. 2 marks

(*Answer on the graph above.*)

**d.** Determine the weight of luggage for which the airport cost and online cost are the same.
Write your answer correct to one decimal place. 1 mark

**Question 5** (3 marks)

When Ben is in Japan, he will study at a Japanese school.

Some of his lessons will be in English and some of his lessons will be in Japanese.

Let $x$ be the number of lessons in English that he will attend each week.

Let $y$ be the number of lessons in Japanese that he will attend each week.

There are 35 lessons each week.

It is a condition of his exchange that Ben attends at least 24 lessons each week.

It is also a condition that Ben attends no more than two lessons in English for every lesson in Japanese.

This information can be represented by Inequalities 1, 2 and 3.

Inequality 1 $x + y \leq 35$

Inequality 2 $x + y \geq 24$

Inequality 3 $y \geq \frac{x}{2}$

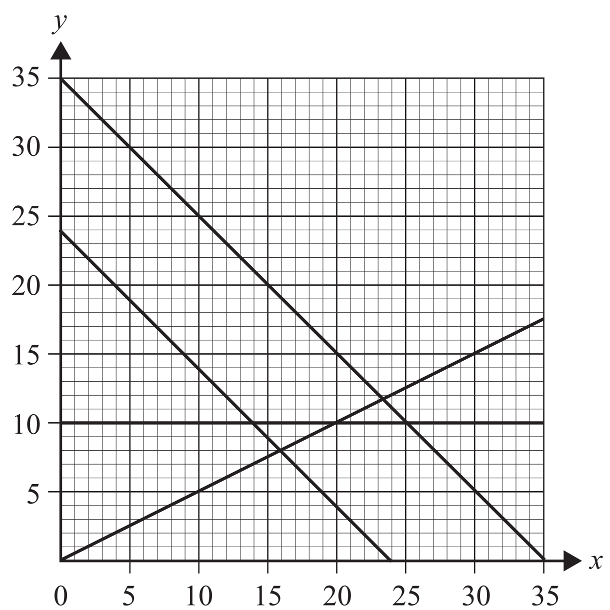
There is another constraint given by

Inequality 4 $y \geq 10$

**a.** Describe Inequality 4 in terms of the lessons that Ben must attend. 1 mark

**b.** The graph below shows the lines that represent the boundaries of Inequalities 1 to 4.

On the graph below, shade the region that contains the points that satisfy these inequalities. 1 mark




**c.** Determine the maximum number of lessons in English that Ben can attend. 1 mark

**END OF MODULE 3**

## Module 4: Business-related mathematics

**Question 1** (3 marks)

Jane and Michael have started a business that provides music at parties.

The business charges customers \$88 per hour.

The \$88 per hour includes a 10% goods and services tax (GST).

**a.** Calculate the amount of GST included in the \$88 hourly rate. 1 mark

Jane and Michael's first booking was a party where they provided music for four hours.

**b.** Calculate the total amount they were paid for this booking. 1 mark

After six months of regular work, Jane and Michael decided to increase the hourly rate they charge by 12.5%.

**c.** Calculate the new hourly rate (including GST). 1 mark

**Question 2** (3 marks)

The sound system used by the business was initially purchased at a cost of $3800.

After two years, the value of the sound system had depreciated to $3150.

**a.** Assuming the flat rate method of depreciation was used, show that the value of the sound system was depreciated by $325 each year. 1 mark

**b.** The value of the sound system will continue to depreciate by $325 each year.

How many years will it take, after the initial purchase, for the sound system to have a value of $550? 1 mark

**c.** The recording equipment used by the business was initially purchased at a cost of $2100.

After five years, the value of the recording equipment had depreciated to $1040 using the reducing balance method.

Find the annual percentage rate by which the value of this recording equipment depreciated.

Write your answer correct to two decimal places. 1 mark

**Question 3** (2 marks)

Jane and Michael decide to set up an annual music scholarship.

To fund the scholarship, they invest in a perpetuity that pays interest at a rate of 3.68% per annum.
The interest from this perpetuity is used to provide an annual $460 scholarship.

**a.** Determine the minimum amount they must invest in the perpetuity to fund the scholarship. 1 mark

**b.** For how many years will they be able to provide the scholarship? 1 mark

**Question 4** (3 marks)

As their business grows, Jane and Michael decide to invest some of their earnings.

They each choose a different investment strategy.

Jane opens an account with Red Bank, with an initial deposit of $4000.

Interest is calculated at a rate of 3.6% per annum, compounding monthly.

**a.** Determine the amount in Jane's account at the end of six months.

Write your answer correct to the nearest cent. 1 mark

Michael decides to open an account with Blue Bank, with an initial deposit of $2000.

At the end of each quarter, he adds an additional $200 to his account.

Interest is compounded at the end of each quarter.

The equation below can be used to determine the balance of Michael's account at the end of the first quarter.

$$\text{account balance} = 2000 \times (1 + 0.008) + 200$$

**b.** Show that the annual compounding rate of interest is 3.2%. 1 mark

**c.** Determine the amount in Michael's account, after the $200 has been added, at the end of five years.

Write your answer correct to the nearest cent. 1 mark

**Question 5** (4 marks)

Jane and Michael borrow $50 000 to expand their business.

Interest on the unpaid balance is charged to the loan account monthly.

The $50 000 is to be fully repaid in equal monthly repayments of $485.60 for 12 years.

**a.** Determine the annual compounding rate of interest.
Write your answer correct to two decimal places. 1 mark

**b.** Calculate the amount that will be paid off the principal at the end of the first year.
Write your answer correct to the nearest dollar. 1 mark

**c.** Halfway through the term of the loan, at the end of the sixth year, Jane and Michael make an additional one-off payment of $3500.

Assume no other changes are made to their loan conditions.

Determine how much time Jane and Michael will save in repaying their loan.

Give your answer correct to the nearest number of months. 2 marks

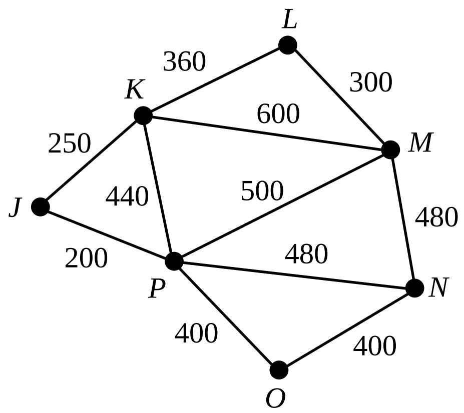
**END OF MODULE 4**

## Module 5: Networks and decision mathematics

**Question 1** (5 marks)

A factory requires seven computer servers to communicate with each other through a connected network of cables.

The servers, *J*, *K*, *L*, *M*, *N*, *O* and *P*, are shown as vertices on the graph below.




The edges on the graph represent the cables that could connect adjacent computer servers.

The numbers on the edges show the cost, in dollars, of installing each cable.

**a.** What is the cost, in dollars, of installing the cable between server *L* and server *M*? 1 mark

**b.** What is the cheapest cost, in dollars, of installing cables between server *K* and server *N*? 1 mark

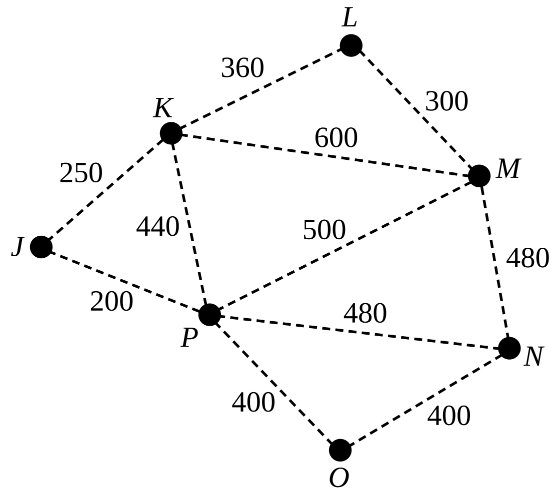
**c.** An inspector checks the cables by walking along the length of each cable in one continuous path.

To avoid walking along any of the cables more than once, at which vertex should the inspector start and where would the inspector finish? 1 mark

**d.** The computer servers will be able to communicate with all the other servers as long as each server is connected by cable to at least one other server.

**i.** The cheapest installation that will join the seven computer servers by cable in a connected network follows a minimum spanning tree.

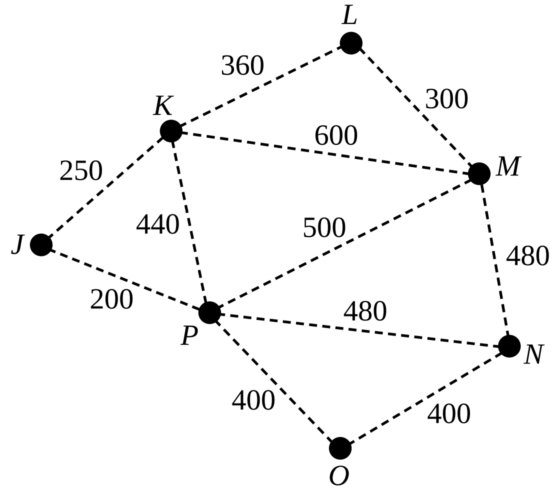
Draw the minimum spanning tree on the plan below. 1 mark




**ii.** The factory's manager has decided that only six connected computer servers will be needed, rather than seven.

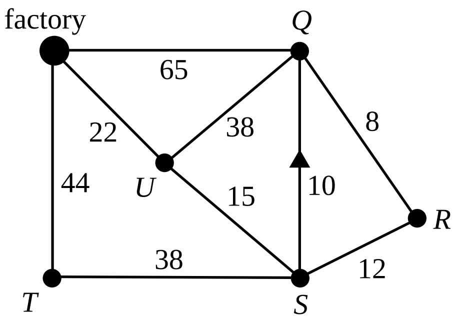
How much would be saved in installation costs if the factory removed computer server *P* from its minimum spanning tree network?

A copy of the graph above is provided below to assist with your working. 1 mark



**Question 2** (3 marks)

The factory supplies groceries to stores in five towns, *Q*, *R*, *S*, *T* and *U*, represented by vertices on the graph below.




The edges of the graph represent roads that connect the towns and the factory.

The numbers on the edges indicate the distance, in kilometres, along the roads.

Vehicles may only travel along the road between towns *S* and *Q* in the direction of the arrow due to temporary roadworks.

Each day, a van must deliver groceries from the factory to the five towns.

The first delivery must be to town *T*, after which the van will continue on to the other four towns before returning to the factory.

**a.** **i.** The shortest possible circuit from the factory for this delivery run, starting with town *T*, is not Hamiltonian.

Complete the order in which these deliveries would follow this shortest possible circuit. 1 mark

factory – *T* – ______________________________ – factory

**ii.** With reference to the town names in your answer to **part a.i.**, explain why this shortest circuit is not a Hamiltonian circuit. 1 mark

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**b.** Determine the length, in kilometres, of a delivery run that follows a Hamiltonian circuit from the factory to these stores if the first delivery is to town *T*. 1 mark

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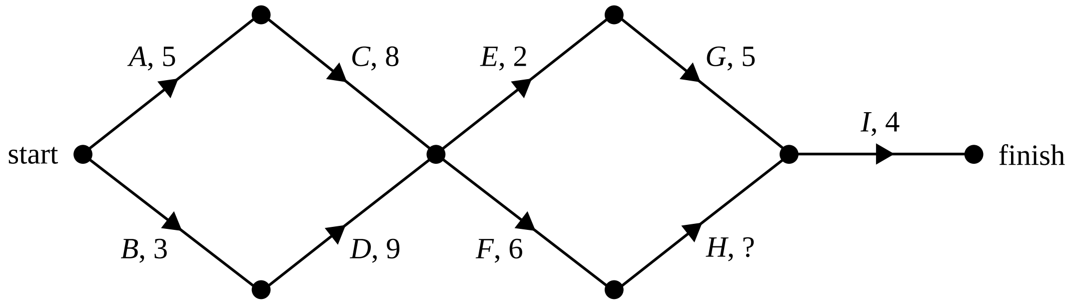
**Question 3** (7 marks)

Nine activities are needed to prepare a daily delivery of groceries from the factory to the towns. The duration, in minutes, earliest starting time (EST) and immediate predecessors for these activities are shown in the table below.

| Activity | Duration | EST | Predecessor(s) |
|---|---|---|---|
| *A* | 5 | 0 | – |
| *B* | 3 | 0 | – |
| *C* | 8 | 5 | *A* |
| *D* | 9 | | *B* |
| *E* | 2 | 13 | *C*, *D* |
| *F* | 6 | 13 | *C*, *D* |
| *G* | 5 | 15 | *E* |
| *H* | | 19 | *F* |
| *I* | 4 | 22 | *G*, *H* |

The directed network that shows these activities is shown below.




All nine of these activities can be completed in a minimum time of 26 minutes.

**a.** What is the EST of activity *D*? 1 mark

**b.** What is the latest starting time (LST) of activity *D*? 1 mark

**c.** Given that the EST of activity *I* is 22 minutes, what is the duration of activity *H*? 1 mark

**d.** Write down, in order, the activities on the critical path. 1 mark

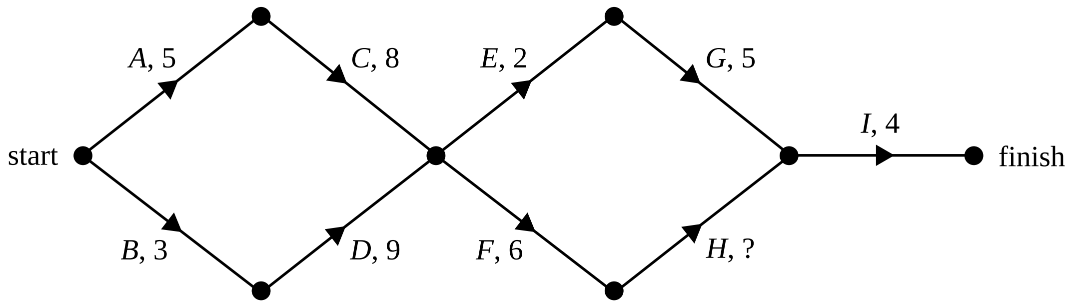
**e.** Activities *C* and *D* can only be completed by either Cassie or Donna.

One Monday, Donna is sick and both activities *C* and *D* must be completed by Cassie. Cassie must complete one of these activities before starting the other.

What is the least effect of this on the usual minimum preparation time for the delivery of groceries from the factory to the five towns? 1 mark

**f.** Every Friday, a special delivery to the five towns includes fresh seafood. This causes a slight change to activity *G*, which then cannot start until activity *F* has been completed.

**i.** On the directed graph below, show this change without duplicating any activity. 1 mark




**ii.** What effect does the inclusion of seafood on Fridays have on the usual minimum preparation time for deliveries from the factory to the five towns? 1 mark

**END OF MODULE 5**
**TURN OVER**

## Module 6: Matrices

**Question 1** (5 marks)

Students in a music school are classified according to three ability levels: beginner ($B$), intermediate ($I$) or advanced ($A$).

Matrix $S_0$, shown below, lists the number of students at each level in the school for a particular week.

$$S_0 = \begin{bmatrix} 20 \\ 60 \\ 40 \end{bmatrix} \begin{matrix} B \\ I \\ A \end{matrix}$$

**a.** How many students in total are in the music school that week? 1 mark

The music school has four teachers, David ($D$), Edith ($E$), Flavio ($F$) and Geoff ($G$).

Each teacher will teach a proportion of the students from each level, as shown in matrix $P$ below.

$$\begin{matrix} & D & E & F & G \\ P = & [0.25 & 0.5 & 0.15 & 0.1] \end{matrix}$$

The matrix product, $Q = S_0P$, can be used to find the number of students from each level taught by each teacher.

**b.** **i.** Complete matrix $Q$, shown below, by writing the missing elements in the shaded boxes. 1 mark

$$Q = \begin{bmatrix} 5 & \square & 3 & 2 \\ 15 & 30 & \square & 6 \\ 10 & 20 & 6 & 4 \end{bmatrix}$$

**ii.** How many intermediate students does Edith teach? 1 mark

The music school pays the teachers \$15 per week for each beginner student, \$25 per week for each intermediate student and \$40 per week for each advanced student.

These amounts are shown in matrix *C* below.

$$\begin{array}{cc} & \begin{array}{ccc} B & I & A \end{array} \\ C = & \begin{bmatrix} 15 & 25 & 40 \end{bmatrix} \end{array}$$

The amount paid to each teacher each week can be found using a matrix calculation.

**c.** **i.** Write down a matrix calculation in terms of *Q* and *C* that results in a matrix that lists the amount paid to each teacher each week. 1 mark

**ii.** How much is paid to Geoff each week? 1 mark

**Question 2** (3 marks)

The ability level of the students is assessed regularly and classified as beginner ($B$), intermediate ($I$) or advanced ($A$).

After each assessment, students either stay at their current level or progress to a higher level.

Students cannot be assessed at a level that is lower than their current level.

The expected number of students at each level after each assessment can be determined using the transition matrix, $T_1$, shown below.

$$\begin{array}{c} \text{before assessment} \\ \begin{array}{ccc} B & I & A \end{array} \end{array}$$

$$T_1 = \begin{bmatrix} 0.50 & 0 & 0 \\ 0.48 & 0.80 & 0 \\ 0.02 & 0.20 & 1 \end{bmatrix} \begin{matrix} B \\ I \\ A \end{matrix} \quad \text{after assessment}$$

**a.** The element in the third row and third column of matrix $T_1$ is the number 1.

Explain what this tells you about the advanced-level students. 1 mark

Let matrix $S_n$ be a state matrix that lists the number of students at beginner, intermediate and advanced levels after $n$ assessments.

The number of students in the school, immediately before the first assessment of the year, is shown in matrix $S_0$ below.

$$S_0 = \begin{bmatrix} 20 \\ 60 \\ 40 \end{bmatrix} \begin{matrix} B \\ I \\ A \end{matrix}$$

**b.** **i.** Write down the matrix $S_1$ that contains the expected number of students at each level after one assessment.

Write the elements of this matrix correct to the nearest whole number. 1 mark

**ii.** How many intermediate-level students have become advanced-level students after one assessment? 1 mark

**Question 3** (7 marks)

A new model for the number of students in the school after each assessment takes into account the number of students who are expected to leave the school after each assessment.

After each assessment, students are classified as beginner (*B*), intermediate (*I*), advanced (*A*) or left the school (*L*).

Let matrix $T_2$ be the transition matrix for this new model.

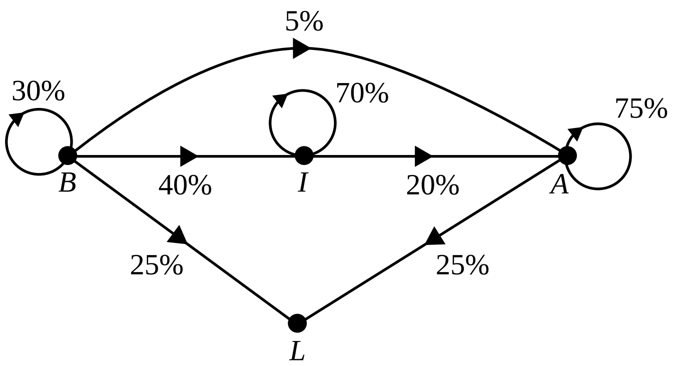
Matrix $T_2$, shown below, contains the percentages of students who are expected to change their ability level or leave the school after each assessment.

$$\begin{array}{c} \text{before assessment} \\ \begin{array}{cccc} B & I & A & L \end{array} \end{array}$$

$$T_2 = \begin{bmatrix} 0.30 & 0 & 0 & 0 \\ 0.40 & 0.70 & 0 & 0 \\ 0.05 & 0.20 & 0.75 & 0 \\ 0.25 & 0.10 & 0.25 & 1 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix} \quad \text{after assessment}$$

**a.** An incomplete transition diagram for matrix $T_2$ is shown below.

Complete the transition diagram by adding the missing information. 2 marks



The number of students at each level, immediately before the first assessment of the year, is shown in matrix $R_0$ below.

$$R_0 = \begin{bmatrix} 20 \\ 60 \\ 40 \\ 0 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$$

Matrix $T_2$, repeated below, contains the percentages of students who are expected to change their ability level or leave the school after each assessment.

$$\begin{matrix} & \text{before assessment} \\ & \begin{matrix} B \quad & I \quad & A \quad & L \end{matrix} \\ T_2 = & \begin{bmatrix} 0.30 & 0 & 0 & 0 \\ 0.40 & 0.70 & 0 & 0 \\ 0.05 & 0.20 & 0.75 & 0 \\ 0.25 & 0.10 & 0.25 & 1 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix} \quad \text{after assessment} \end{matrix}$$

**b.** What percentage of students is expected to leave the school after the first assessment? 1 mark

**c.** How many advanced-level students are expected to be in the school after two assessments? Write your answer correct to the nearest whole number. 1 mark

**d.** After how many assessments is the number of students in the school, correct to the nearest whole number, first expected to drop below 50? 1 mark

Another model for the number of students in the school after each assessment takes into account the number of students who are expected to join the school after each assessment.

Let $R_n$ be the state matrix that contains the number of students in the school immediately after $n$ assessments.

Let $V$ be the matrix that contains the number of students who join the school after each assessment.

Matrix $V$ is shown below.

$$V = \begin{bmatrix} 4 \\ 2 \\ 3 \\ 0 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$$

The expected number of students in the school after $n$ assessments can be determined using the matrix equation

$$R_{n+1} = T_2 \times R_n + V$$

where

$$R_0 = \begin{bmatrix} 20 \\ 60 \\ 40 \\ 0 \end{bmatrix} \begin{matrix} B \\ I \\ A \\ L \end{matrix}$$

**e.** Consider the intermediate-level students expected to be in the school after three assessments.

How many are expected to become advanced-level students after the next assessment?

Write your answer correct to the nearest whole number. 2 marks

**END OF QUESTION AND ANSWER BOOK**

# FURTHER MATHEMATICS

## Written examinations 1 and 2

### FORMULA SHEET

**Instructions**

| |
|---|
| Detach this formula sheet during reading time.<br>This formula sheet is provided for your reference. |

# Further Mathematics formulas

## Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares regression line: $y = a + bx$, where $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

## Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1 - r}, |r| < 1$

## Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: $\pi r^2$

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\frac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

## Module 3: Graphs and relations

### Straight-line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

## Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$, where $R = 1 + \dfrac{r}{100}$

hire-purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

## Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

## Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

**END OF FORMULA SHEET**