2018 VCE Mathematical Methods 1 (NHT) examination report

Specific information

This report provides sample answers or an indication of what answers may have been included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

Question 1a.

$$f'(x) = \frac{\left(x^2-3\right)e^x - e^x(2x)}{\left(x^2-3\right)^2}$$

$$= \frac{e^x\left(x^2-2x-3\right)}{\left(x^2-3\right)^2}$$

Use of the quotient rule was the most straightforward method.

Question 1b.

$$\frac{dy}{dx} = \log_e(x) + \frac{x+5}{x}$$

$$\text{At } x = 5,\quad \frac{dy}{dx} = \log_e(5) + 2$$

Students are reminded to take care with notation when dealing with logarithms.

Question 2a.

$$f(3) = -2$$
$$g(f(3)) = 2$$

Question 2b.

$$f(g(x)) = -\left(x^2-2\right)^2 + \left(x^2-2\right) + 4$$
$$= -x^4 + 5x^2 - 2$$

Question 3

$$\int_0^1 \left(e^x - e^{-x}\right)dx = \left[e^x + e^{-x}\right]_0^1$$

$$= e + \frac{1}{e} - 2$$

Question 4

$\log_3\left(\frac{t}{t^2-4}\right)=-1$

$\left(\frac{t}{t^2-4}\right)=\frac{1}{3}$

$t^2-3t-4=0$

$t=4,\quad \text{reject } t=-1 \quad \text{because} \quad t>0$

Question 5a.

Range: $\left(-3,\frac{1}{2}\right]$

Question 5b.

For inverse: $x=\frac{7}{h^{-1}(x)+2}-3$

Thus $h^{-1}(x)=\frac{7}{x+3}-2$

Question 6a.

$k+4k+6k+k=1$

$12k=1$

$k=\frac{1}{12}$

Question 6b.

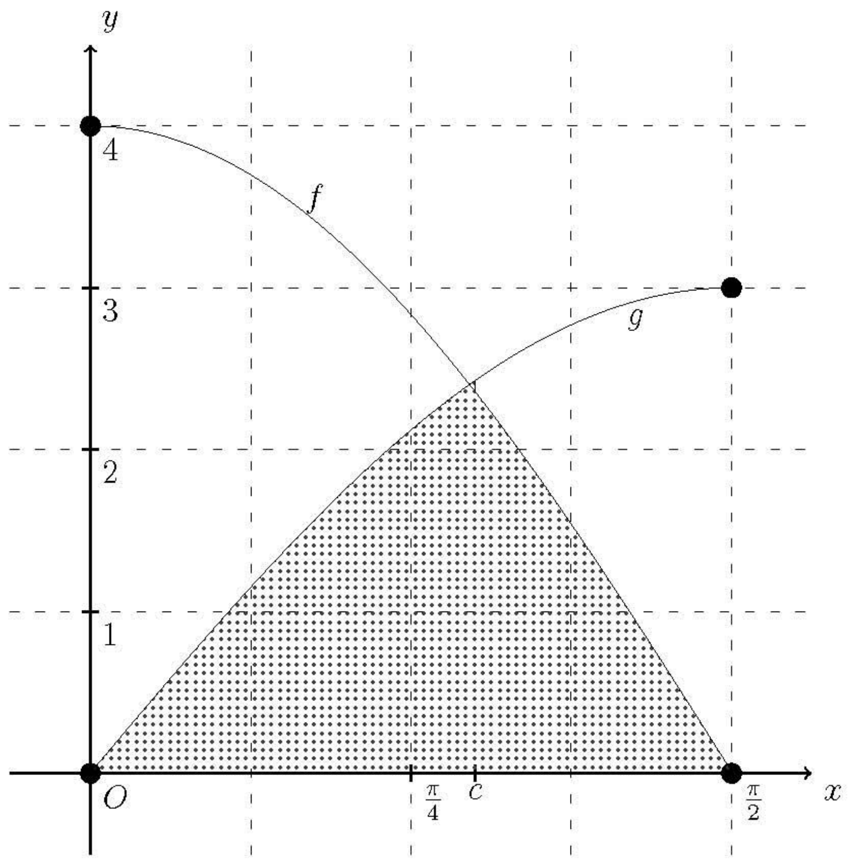
$E(X)=1\times\frac{1}{12}+4\times\frac{4}{12}+\ 6\times\frac{6}{12}+3\times\ \frac{1}{12}=\frac{14}{3}$

Question 6c.

$$\Pr\left(X\geq 3\,|\,X\geq 2\right)=\frac{\Pr(X\geq 3\cap X\geq 2)}{\Pr\left(X\geq 2\right)}$$

$$=1$$

Question 7a.

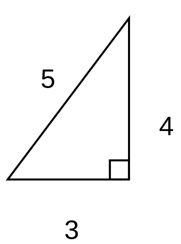


The curves required needed to be smooth, continuous and drawn over the given restricted domain.

Question 7b.

$4\cos(c) = 3\sin(c)$

$\tan(c) = \dfrac{4}{3}$



$\sin(c) = \dfrac{4}{5}, \quad \cos(c) = \dfrac{3}{5}$

Question 7ci.

See diagram given in Question 7a.

Question 7cii.

$$\text{Area} = \int_0^c 3\sin(x)\,dx + \int_c^{\frac{\pi}{2}} 4\cos(x)\,dx$$

$$= \left[-3\cos(x)\right]_0^c + \left[4\sin(x)\right]_c^{\frac{\pi}{2}}$$

$$= -3\cos(c) - 4\sin(c) + 7$$

$$= -3\times\frac{3}{5} - 4\times\frac{4}{5} + 7$$

$$= 2$$

Question 8a.

Using symmetry about the mean $2\hat{p} = \frac{10\ 000}{50\ 000}$

$$\hat{p} = \frac{1}{10}$$

Question 8b.

$$\frac{1}{10} + \frac{49}{25}\sqrt{\frac{\left(\frac{1}{10}\times\frac{9}{10}\right)}{n}} = \frac{5147}{50\ 000}$$

$$\frac{49}{25}\sqrt{\frac{9}{100n}} = \frac{147}{50\ 000}$$

$$n = 40\ 000$$

Question 9a.

$$\frac{dy}{dx} = a(x-b)^2 + 2ax(x-b)$$

Solve $\frac{dy}{dx} = 0$ for x

For local maximum $x = \frac{b}{3}$

Using the fact that local maximum occurs at $y = b$

$$b = a\times\frac{b}{3}\left(\frac{-2b}{3}\right)^2$$

$$a = \frac{27}{4b^2}$$

Question 9b.

Shaded area = area of rectangle – area under cubic polynomial graph – area of triangle

Shaded area = $(3\times 2)-a\int_0^b \left(x^3-2bx^2+b^2x\right)dx-\left(\frac{1}{2}\times(3-b)\times 2\right)$

$$=6-\left(a\times\frac{b^4}{12}\right)-(3-b)$$

$$=b+3-\left(\frac{27}{4b^2}\times\frac{b^4}{12}\right)$$

$$=b+3-\frac{9b^2}{16}$$

There were several valid methods to find the required area. Students are advised to set out their work in a logically sequenced manner. Students are also reminded that this was a 'show that' question and that the final answer, although given, had to be obtained from correct and relevant mathematical working.

Question 9c.

$$\frac{dA}{db}=0, \quad 1-\frac{9b}{8}=0$$

$$b=\frac{8}{9}$$

$$\text{Maximum area} = \frac{8}{9}+3-\left(\frac{9}{16}\times\left(\frac{8}{9}\right)^2\right)=\frac{31}{9}$$

This question could also be answered using the information given in the previous part of the question, independently of part b.