2019 VCE Further Mathematics 2 examination report

General comments

Students were required to complete:

- a compulsory Core section of Data Analysis (worth 24 marks)
- a compulsory Core section of Recursion and financial modelling (worth 12 marks)
- two selected modules (each worth 12 marks).

Scanned images are used for assessing and students should ensure their answers are clearly written and can be easily read. Students were asked to write in blue or black pen. Students are urged to take care with the presentation of their responses.

It was clear that many students did not read the questions carefully enough and often gave answers that appeared to be based on a similar kind of question from their studies and revision. Students are strongly encouraged to make good use of the fifteen-minute reading time, and check their response to a question after they have completed their answer to ensure that they are answering the question suitably. For example, in Core Question 3 students were asked to refer to the values of *an* appropriate statistic (the median being an appropriate choice) in their response. Many students went further and compromised their answer by referring to the minimum and maximum values as well.

Many questions on the examination were worth one mark only, and with these questions the mark was awarded for a correct answer. Generally there was no need to put the answer into a sentence. For example in Networks Question 3c. the required answer was simply activity J. Some responses went on to erroneously give the float time of this activity and therefore could not be awarded the mark.

For all questions worth more than one mark, students are strongly advised to show working. An incorrect answer on its own will not be awarded any marks in a two-mark question; however, often a method mark can be awarded for appropriate working.

Some questions required the further application of a previous answer.

When descriptive answers are required to a question, students are advised to keep answers brief. A response in point form is also acceptable.

Some questions asked students to show that a particular answer could be obtained. Students must work towards the given result with relevant steps shown. The answer the student is asked to show may be relevant in a subsequent part of the question.

The following examples illustrate what is required to **show that** a given result can be obtained.

In Core question 7a. students were asked to use recursion to show that the value of the tools after two years, V_2 = \$48 600. This means students must show each iteration by writing down the relevant calculation:

$$V_1 = 60\,000 \times 0.9 = 54\,000$$

V_2 = 54 000 x 0.9 = 48 600

To state V_1 = 54 000 without showing how this was obtained does not address the instruction to 'show'.

Using V_2 = 60 000 x 0.9^2 = 48 600 does not address the **use recursion** instruction.

In the Recursion and financial modelling module Question 8b. students were asked to show that the annual interest rate is 4.2% from the given recurrence relation.
Stating that $1 + \frac{r}{1200} = 1.0035\%$ and r = 4.2% amounts to recognising the well-known interest rate rule and then stating the conclusion. To show that the result is the case, some calculation must be provided:

$\frac{r}{1200} = 0.0035\%$
r = 1200 x 0.0035% = 4.2%

In the Geometry and measurement module, Question 3ai. students were asked to show that the distance AC is 20 m.

This required more than setting up the equation attributed to Pythagoras, $AC^2 + 15^2 = 25^2$.
Writing $AC^2 = 25^2 - 15^2$ is also insufficient as the solution to this could be positive or negative.
A positive square root is the requirement for this task: $AC = \sqrt{25^2 - 15^2} = \sqrt{400} = 20$

In each of these cases the result must be the conclusion of the reasoning process. Showing the process is the basis of achieving the mark.

Rounding of answers to a specified level of accuracy as applicable is an important skill that students must be able to demonstrate. When completing the equation of a least squares line, some students did not understand the difference between decimal places rounding and significant figures rounding.

In the Recursion and financial modelling module students sometimes further engaged from a correct answer to an incorrect answer because they rounded to the nearest 5 c or dollar. They needed to treat the answer to the nearest cent as if it were electronic money where no further rounding occurs. For example, in Core Question 9a. the required answer was $1704.03 and some students incorrectly rounded further to $1704.05 or $1704.

In questions where no instruction to round is given, an exact answer is required. In the Geometry and measurement module, Question 1b, the volume is $6 \times 2.4 \times 2.6 = 37.44$ m^3 . This is an exact answer and as there is no instruction to round, is to be left in this form.

Students are advised to use a ruler to draw straight lines. These are often required in the Core Data analysis and the Graphs and relations module. The use of a ruler enables more accurate lines and related working.

Specific information

This report provides sample answers or an indication of what answers may have included. Unless otherwise stated, these are not intended to be exemplary or complete responses.

The statistics in this report may be subject to rounding resulting in a total more or less than 100 per cent.

Section A – Core

Data analysis

Question 1a.

Marks	0	1	Average
%	17	83	**0.9**

The answer is *day number*.

Most responses given to this question were correct. A small number of students answered 'neither'.

Question 1b.

Marks	0	1	Average
%	17	83	**0.9**

The answer is:

4	1	8	
5			
6	0	7	
7	0	**5**	**7**
8	**0**	**6**	
9	0	2	**8**
10	7		
11	8		
12	7		

Some students entered only the values for day 11 and day 15 rather than the five days from day 11 to day 15.

Question 1ci.

Marks	0	1	Average
%	20	80	**0.8**

The answer is 12.2.

This question was generally well done.

Question 1cii.

Marks	0	1	Average
%	15	85	**0.9**

The answer is 20%.

This question was well done.

Question 2ai.

Marks	0	1	Average
%	11	89	**0.9**

The answer is 5.

This question was well done.

Question 2aii.

Marks	0	1	Average
%	13	87	**0.9**

The answer is 10.

This question was well done.

Question 2aiii.

Marks	0	1	Average
%	21	79	**0.8**

The answer is 1.

This question was generally well done.

Question 2b.

Marks	0	1	Average
%	42	58	**0.6**

The answer is 15.

Some gave the percentage of 50% rather than the number of days.

Question 3.

Marks	0	1	2	Average
%	39	25	36	**1.0**

Two marks were available for this question.

A statement that a **decrease** or **change** in median (or IQR) signals an association was required for the first mark to be awarded. Median (or IQR) values for all three months needed to be quoted correctly for the second mark to be awarded.

Successful responses focused on one statistic only (usually the median) and quoted the values from the table rather than estimating from the boxplots.

Incorrect answers included using the word ‘averages’ or ‘means’ rather than medians and quoting only two medians rather than all three.

Some students went on to comment on the minimums and maximums; this additional information compromised an otherwise correct answer. Comments about the shape of the boxplots were also not appropriate.

Questions of this type require reasoning rather than comments about which season each month falls in.

Question 4a.

Marks	0	1	Average
%	16	84	**0.9**

The answer is *humidity 9 am.*

This question was well done.

Question 4b.

Marks	0	1	Average
%	41	59	**0.6**

The answer is:

humidity 3 pm = | –1.26 | + | 0.765 | × *humidity 9 am*

The responses showed reasonable understanding of significant figures. Some responses gave each value correct to three (or two) decimal places.

Question 4c.

Marks	0	1	Average
%	24	76	**0.8**

The answer is $r = 0.871$.
This question was generally well done.

Question 5a.

Marks	0	1	Average
%	56	44	**0.5**

On average, pressure at 3 pm increases by 0.8894 hPa for each one unit (hPa) **increase** in pressure at 9 am.

Students had to be careful when answering this interpretative question. Many gave a response that was almost correct but failed to reference the one-unit increase in *pressure 9 am.*

Interpreting the slope in terms of the given variables required identifying the correct constant and then describing it.

Describing both constants was to ignore the first step; specific knowledge was required, not various statements provided in the hope of including something relevant.

Question 5b.

Marks	0	1	Average
%	16	84	**0.9**

The answer is 1023.

This question was well done.

Question 5c.

Marks	0	1	Average
%	13	87	**0.9**

The answer is interpolation.

This question was well done.

Question 5d.

Marks	0	1	Average
%	60	40	**0.4**

The answer is 3.

$1015 - (111.4 + 0.8894 \times 1013) = 2.637\ldots$

This question was poorly done. Many found the predicted value correctly but then subtracted from 1013 rather than from the *pressure 3 pm* value of 1015.

Question 5ei.

Marks	0	1	Average
%	66	34	**0.4**

The answer is:

$$0.8894 = r \times \frac{4.1884}{4.5477} \Rightarrow r = 0.8894 \times \frac{4.5477}{4.1884}$$

A reasonable proportion of responses substituted correctly.

Some responses attempted to use the formula:

$$r = \frac{1}{n-1} \Sigma \left(\frac{x - \bar{x}}{s_x}\right)\left(\frac{y - \bar{y}}{s_y}\right)$$

This is not part of the current study design.

Question 5eii.

Marks	0	1	Average
%	54	46	**0.5**

The answer is 93.3%.

This was quite often given as 96.6% by students who did not square the *r* value.

Question 5fi.

Marks	0	1	Average
%	59	41	**0.4**

The answer is *linearity*.

A one-word answer was all that was required. Some gave long answers that contradicted what was expected of them in Question 5fii.

Question 5fii.

Marks	0	1	Average
%	64	36	**0.4**

The answer is that the residuals have a clear pattern.

This question was poorly done. A common mistake was to ignore the given statement that the residuals did not support the assumption and to state that linearity was confirmed.

Many students had difficulty with responses to Questions 5fi. and 5fii., often contradicting themselves or writing definitions directly from a bound reference without any link to the question. Some confused residuals and the correlation coefficient.

Question 6a.

Marks	0	1	2	Average
%	42	26	32	**0.9**

The answer is:

Summer 0.89 Autumn 1.00 Spring 0.70

Student calculations were generally reasonable, but rounding was not. In particular, the calculated value for autumn of 0.9975 was often rounded to the nearest whole number rather than to two decimal places.

Question 6b.

Marks	0	1	Average
%	42	58	**0.6**

The answer is 186.

This question was quite well done. Many students recognised that they could still answer this part even if unsuccessful in Question 6a.

Recursion and financial modelling

Question 7a.

Marks	0	1	Average
%	26	74	**0.8**

$V_1 = 0.9 \times 60\,000 = 54\,000$

$V_2 = 0.9 \times 54\,000 = 48\,600$

This was a straightforward question.

Question 7b.

Marks	0	1	Average
%	34	66	**0.7**

The answer is 10%.

Incorrect answers included 0.9% and 90%.

Question 7c.

Marks	0	1	Average
%	21	79	**0.8**

The answer is 11 years.

This question was generally well done.

Question 7d.

Marks	0	1	Average
%	48	52	**0.5**

The answer is:

$V_0 = 60\,000, \quad V_{n+1} = V_n - 4800$

Responses generally used the notation well, but many wrote a recurrence relation reflective of reducing balance depreciation rather than flat rate.

A few wrote both a recurrence relation and a rule with V_n in terms of n which could not be accepted.

Question 8a.

Marks	0	1	Average
%	28	72	**0.7**

The answer is $3700.

The question was straightforward but often misunderstood. Some gave the answer to Question 8d. while others gave $3000 after calculating A_1.

Question 8b.

Marks	0	1	Average
%	70	30	**0.3**

The answer is:

$(1.0035 - 1) \times 12 \times 100$

Responses needed to show the calculation using basic arithmetic that resulted in an answer of 4.2%, not use the 4.2% and attempt to verify it.

Question 8c.

Marks	0	1	Average
%	64	36	**0.4**

The answer is $92.15.

Rounding of the required answer was problematic for some.

Question 8d.

Marks	0	1	Average
%	60	40	**0.4**

The answer is $700.

The answer to Question 8a. was sometimes given.

Question 9a.

Marks	0	1	Average
%	73	27	**0.3**

The answer is $1704.03.

Finance solver entries:

N = 78

I% = 4.9

PV = 350 000

PMT = – **1704.030033**

FV = – 262 332.33

P/Y & C/Y = 26

$7954.54 was a common incorrect answer obtained by entering the FV in technology as a positive rather than as – 262 332.33. This very large fortnightly repayment should have been a signal that something had been entered incorrectly.

Question 9b.

Marks	0	1	Average
%	82	19	**0.2**

The answer is $45 246.67.

Paid = 78 × $1704.03 = $132 914.34

The principal reduced by:

$350 000 – $262 332.33 = $87 667.67

Interest = $132 914.34 – $87 667.67

Some students followed through with the incorrect response they obtained for Question 9a. but did not show the working required for the consequential mark.

Question 9c.

Marks	0	1	2	Average
%	54	22	24	**0.7**

The answer is:

$B_0 = 262\,332.33, \quad B_{n+1} = 1.004B_n - 3517.28$

The question was done reasonably well, but some responses gave B_0 as 350 000, or R as 1.048, and added rather than subtracted the 3517.28.

Section B – Modules

The selection of modules by students in 2019 is shown in the table below.

Module	%
Matrices	45
Networks and decision mathematics	27
Geometry and measurement	14
Graphs and relations	15

Module 1 – Matrices

Question 1a.

Marks	0	1	Average
%	5	95	**1.0**

The answer is 3×2.

This question was very well done.

Question 1b.

Marks	0	1	Average
%	23	77	**0.8**

The answer is 50 + 20 + 40.

Although a simple arithmetic expression was all that was required, a matrix calculation

$$\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}50\\20\\40\end{bmatrix}$$

was also accepted. This question was generally well done.

Question 1c.

Marks	0	1	Average
%	36	64	**0.7**

The answer is:

$$\begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

This question was mostly well done.

Question 1d.

Marks	0	1	Average
%	24	76	**0.8**

The answer is:

$$\begin{bmatrix} 3 & 6 & 22 & 19 \\ 1 & 10 & 7 & 2 \\ 1 & 3 & 10 & 26 \end{bmatrix}$$

Some responses had transcription errors.

Question 1e.

Marks	0	1	Average
%	47	53	**0.6**

The answer is the number of cars parked in area C for two hours.

Some students recognised area C and 2 hours but did not refer to the number of cars parked.

Question 2a.

Marks	0	1	Average
%	9	91	**0.9**

The answer is 20%.

This question was very well done.

Question 2b.

Marks	0	1	Average
%	16	84	**0.9**

The answer is as follows.

$$\begin{bmatrix} 300 \\ 780 \\ \\ 620 \end{bmatrix}$$

This question was well done.

Question 2c.

Marks	0	1	Average
%	74	26	**0.3**

The answer is 60%.

$$\frac{0.1\times 600 + 0.2\times 600}{300} = 0.6$$

Many responses were incorrect. A few correctly worked out the individual percentages from A or F but did not give an overall percentage.

Question 2d.

Marks	0	1	Average
%	79	21	**0.2**

The answer is:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Many responses were incorrect. Some gave the identity matrix; others gave various assortments of 1s and 0s.

Question 3a.

Marks	0	1	Average
%	25	75	**0.8**

The answer is Air World.

This question was well done.

Question 3bi.

Marks	0	1	Average
%	75	25	**0.3**

The answer is as follows.

$$\begin{bmatrix} -210 \\ 0 \\ 210 \\ 0 \end{bmatrix}$$

$V \times R_0 = \begin{bmatrix} 810 \\ 300 \\ 310 \\ 580 \end{bmatrix}$ Must take 210 from A and add to G

This question was poorly done, with many students apparently unable to identify what was required.

Question 3bii.

Marks	0	1	Average
%	88	12	**0.1**

The answer is:

$$\begin{bmatrix} 600 \\ 288 \\ 512 \\ 600 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 810 \\ 300 \\ 310 \\ 580 \end{bmatrix} + \begin{bmatrix} -210 \\ 0 \\ 210 \\ 0 \end{bmatrix} = \begin{bmatrix} 600 \\ 300 \\ 520 \\ 580 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 786 \\ 288 \\ 282 \\ 644 \end{bmatrix} + \begin{bmatrix} -186 \\ 0 \\ 230 \\ -44 \end{bmatrix} = \begin{bmatrix} 600 \\ 288 \\ 512 \\ 600 \end{bmatrix}$$

Many students did not attempt this question. A few responses gave the matrix B_2 rather than the state matrix R_2.

Module 2 – Networks and decision mathematics

Question 1a.

Marks	0	1	Average
%	4	96	**1.0**

The correct answer is Office.

This question was very well done.

Question 1bi.

Marks	0	1	Average
%	30	70	**0.7**

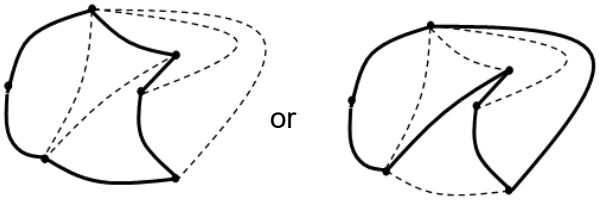
The answer is Hamiltonian cycle.

A few responses erroneously named the route as a path or circuit.

Question 1bii.

Marks	0	1	Average
%	7	93	**1.0**

The answer is as follows.



This question was very well done.

Question 2a.

Marks	0	1	Average
%	7	93	**1.0**

The answer is as follows.

Tennis

Football

Basketball

Athletics

This was well done, with very few incorrect responses.

Question 2b.

Marks	0	1	2	Average
%	28	18	54	**1.3**

The answer is as follows.

400

200

100

300

Some responses were awarded one mark for two correct allocations.

Question 3a.

Marks	0	1	Average
%	40	60	**0.6**

The answer is 8.

This question was reasonably well done. Some responses attempted to give the critical path rather than state the number of activities.

Question 3b.

Marks	0	1	Average
%	55	45	**0.5**

The answer is 12.

Answers from 7 to 15 were observed.

Question 3c.

Marks	0	1	Average
%	55	45	**0.5**

The answer is J.

Students should be aware that if they did not leave their answer as J but further engaged with the question and then stated an incorrect float time, they could not be awarded marks.

Question 3d.

Marks	0	1	Average
%	76	24	**0.3**

The answer is 29.

Many failed to take two weeks off each of the critical activities.

Question 3e.

Marks	0	1	2	Average
%	89	9	3	**0.2**

The answer is as follows.

0

1

2

1

1

A method mark was available for one of the following answers that also reduced the overall time by four weeks and did not involve unnecessary wastage.

	REDUCTION				
C	0	0	1	1	1
D	1	0	2	2	2
G	1	2	0	1	2
H	0	1	0	0	1
K	2	2	2	1	0

Many did not attempt this question.

Module 3 – Geometry and measurement

Question 1a.

Marks	0	1	Average
%	14	86	**0.9**

The answer is 6700 m^2.

This question was well done.

Question 1b.

Marks	0	1	Average
%	12	88	**0.9**

The answer is 37.44 m^3.

The exact value is the correct answer as no rounding was specified.

Question 1c.

Marks	0	1	Average
%	30	70	**0.7**

The answer is 72.48 m^2.

As in Question 1b. no rounding was specified so the exact answer was required. The question was mostly done well but some responses incorrectly answered 74.88, by calculating

$4(6 \times 2.6) + 2(2.6 \times 2.4)$.

Question 1d.

Marks	0	1	Average
%	78	22	**0.2**

The answer is 48

Many incorrect answers were given. The most common incorrect answer was 71.

Question 2a.

Marks	0	1	Average
%	41	59	**0.6**

The equator is at 0° latitude. Sydney's latitude of 34° is a smaller angle from the equator than Magadan's latitude of 60°.

Responses that simply said that 34° is less than 60° failed to identify the key point that the equator is a line of 0° latitude. Careful reading was required. Responses needed to **explain** using the information given, not **calculate** using the information given.

Question 2b.

Marks	0	1	Average
%	51	49	**0.5**

The answer is 10 500 km.

$\frac{94}{360} \times 2\pi \times 6400 = 10\ 499.900...$

Question 2c.

Marks	0	1	Average
%	80	20	**0.2**

The answer is 246 hours (10 days, 4 hours plus the 2-hour time difference).

Many incorrect answers were given. Some responses subtracted the two-hour time difference rather than adding it. Many ignored the number of days, which resulted in a travel time of two hours.

Question 3ai.

Marks	0	1	Average
%	40	60	**0.6**

The answer is:

$\sqrt{25^2 - 15^2}$

The question asked for a calculation to **show** the value was 20.

$25^2 - 15^2 = \sqrt{400} = 20$ was not acceptable as it is not a correct mathematical expression,

whereas

$25^2 - 15^2 = 400 \quad \sqrt{400} = 20$ was acceptable.

Using the equation $AC^2 = 25^2 - 15^2$, then solving with technology does not show the mathematics to obtain the result. The solution of such an equation gives both a positive and negative solution, and the positive root needs to be identified as the required solution.

Question 3aii.

Marks	0	1	Average
%	38	62	**0.6**

The answer is 37°.

This question was reasonably well done.

Question 3b.

Marks	0	1	Average
%	44	56	**0.6**

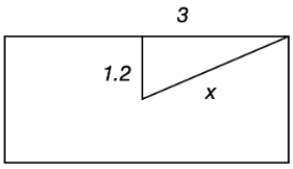
The answer is 51 m.

This question was reasonably well done.

Question 3c.

Marks	0	1	2	Average
%	49	2	50	**1.0**

The answer is 3 m.



$$x = \sqrt{3^2 + 1.2^2} \approx 3.23$$

$$h = \sqrt{4.4^2 - 3.23^2}$$
$$= 2.9866\ldots$$

Module 4 – Graphs and relations

Question 1a.

Marks	0	1	Average
%	2	98	**1.0**

The answer is 60.

This question was very well done.

Question 1bi.

Marks	0	1	Average
%	64	36	**0.4**

The answer is:

$$\frac{32-62}{2018-2013}$$

Some responses gave a slope calculation leading to positive 6 and then described it as a decreasing trend. Some misread the scale on the vertical axis and used values of 31 and 61.

Question 1bii.

Marks	0	1	Average
%	40	60	**0.6**

The answer is 14″.

Question 2a.

Marks	0	1	Average
%	15	85	**0.9**

The answer is 3.

This question was well done.

Question 2b.

Marks	0	1	Average
%	53	47	**0.5**

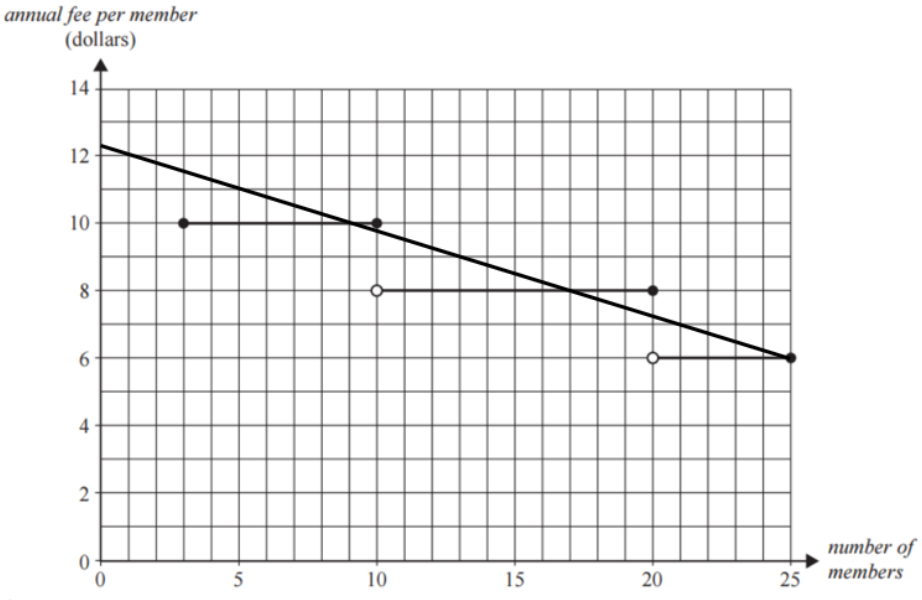
The answer is ≤ 10.

Most students identified the number 10 required but some had difficulty with the inequality.

Question 2c.

Marks	0	1	Average
%	50	50	**0.5**

The answer is as follows.



Many responses were successful by using a ruler, often writing the coordinates of the two endpoints as (0,12.25) and (25,6).

Question 2d.

Marks	0	1	Average
%	61	39	**0.4**

The answer is 9,17,25.

Some students did not seem to know what to identify.

Question 3a.

Marks	0	1	Average
%	37	63	**0.7**

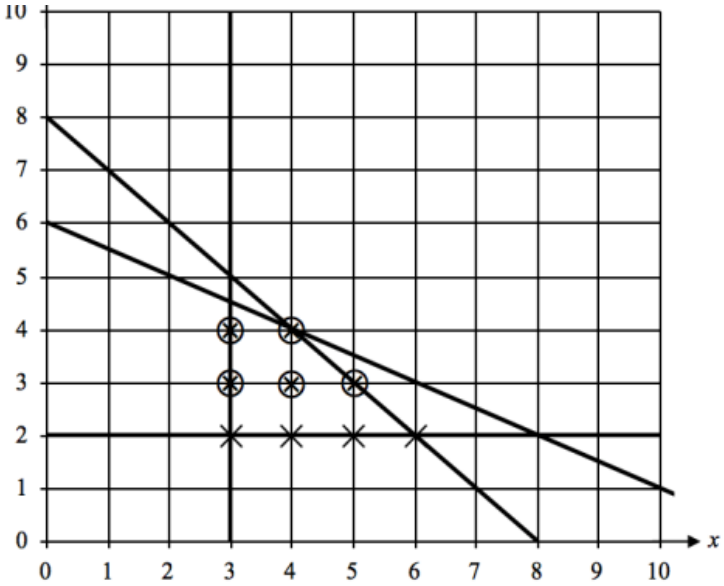
The answer is $5x + 10y \leq 60$.

This question was reasonably well done.

Question 3b.

Marks	0	1	Average
%	58	42	**0.4**

The answer is as follows.



Many responses only identified two or three of the five required points.

Question 3c.

Marks	0	1	Average
%	52	48	**0.5**

The answer is $680.

Some correctly recognised $(4,4)$ as the only integer point on $5x + 10y = 60$.

Question 3d.

Marks	0	1	Average
%	66	34	**0.4**

The answer is $610.

$650 was a common incorrect response as the other integer point for which 55 members could travel.

Question 3e.

Marks	0	1	Average
%	93	7	**0.1**

The answer is 30.

Cost equation becomes $C = (70+k)x + 100y$ with a gradient of $-\frac{70+k}{100}$

Will touch only (6, 2) if gradient < −1

∴ the increase in cost > k if $k = 30$

Only a small number of responses were correct.