Victorian Curriculum and Assessment Authority

**Victorian Certificate of Education
2023**

FURTHER MATHEMATICS

Written examination 1

Thursday 25 May 2023

Reading time: **2.00 pm to 2.15 pm (15 minutes)**
Writing time: **2.15 pm to 3.45 pm (1 hour 30 minutes)**

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of modules	Number of modules to be answered	Number of marks
A – Core	24	24			24
B – Modules	32	16	4	2	16
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question book of 36 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book and the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Core

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                          Instructions for Section A

Answer all questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is correct for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will not be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.
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Data analysis

Question 1

Data relating to the following five variables was collected for trees on a farm:

- *age* (in years)
- *type of tree* (fruit, not fruit)
- *deciduous tree* (yes, no)
- *height of tree* (short, medium, tall)
- *circumference* at 1.5 m above ground, in metres

The number of ordinal variables in the above list is

A. 0
B. 1
C. 2
D. 3
E. 4

Use the following information to answer Questions 2–4.

The histogram below shows the distribution of the *volume*, in cubic metres, of the trunks of 31 black cherry trees.
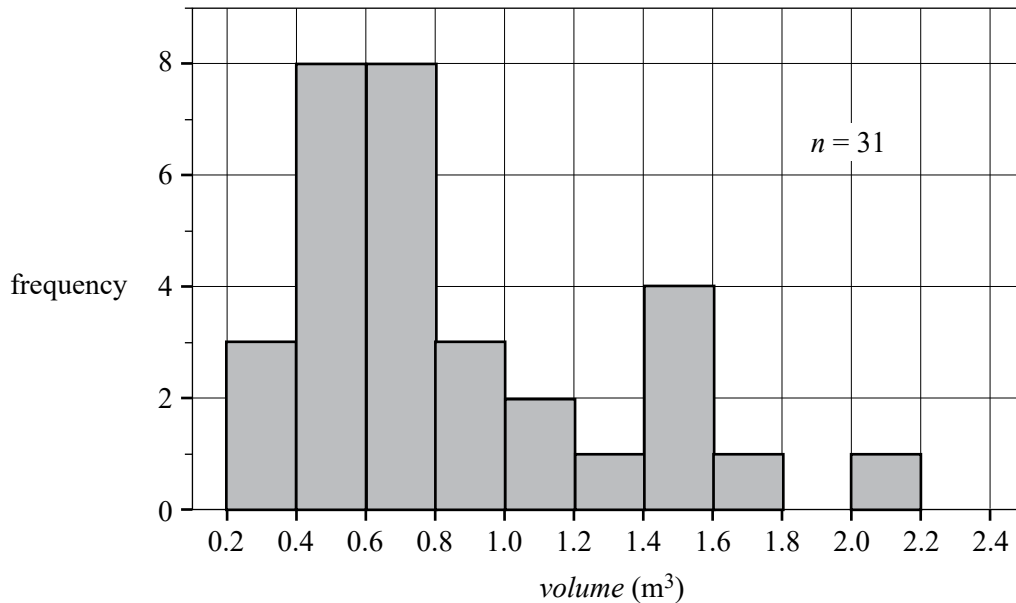
Data: 'Black Cherry Trees', Wolfram Data Repository,
<https://datarepository.wolframcloud.com/resources/Sample-Data-Black-Cherry-Trees>

Question 2

The shape of the distribution is best described as

A. symmetric.
B. negatively skewed.
C. symmetric with an outlier.
D. negatively skewed with an outlier.
E. positively skewed with an outlier.

Question 3

The percentage of trees that have trunks with a *volume* between 1.2 and 2.2 m^3 is closest to

A. 23%
B. 25%
C. 26%
D. 51%
E. 74%

Question 4

The third quartile (Q_3) for this distribution, in m^3, could be

A. 0.8
B. 0.9
C. 1.1
D. 1.3
E. 1.4

Use the following information to answer Questions 5–7.

The scatterplot below shows the *volume* of the trunks of 31 black cherry trees plotted against the *height* of the trees, in metres. A least squares line with *height* as the explanatory variable has also been fitted to the scatterplot.
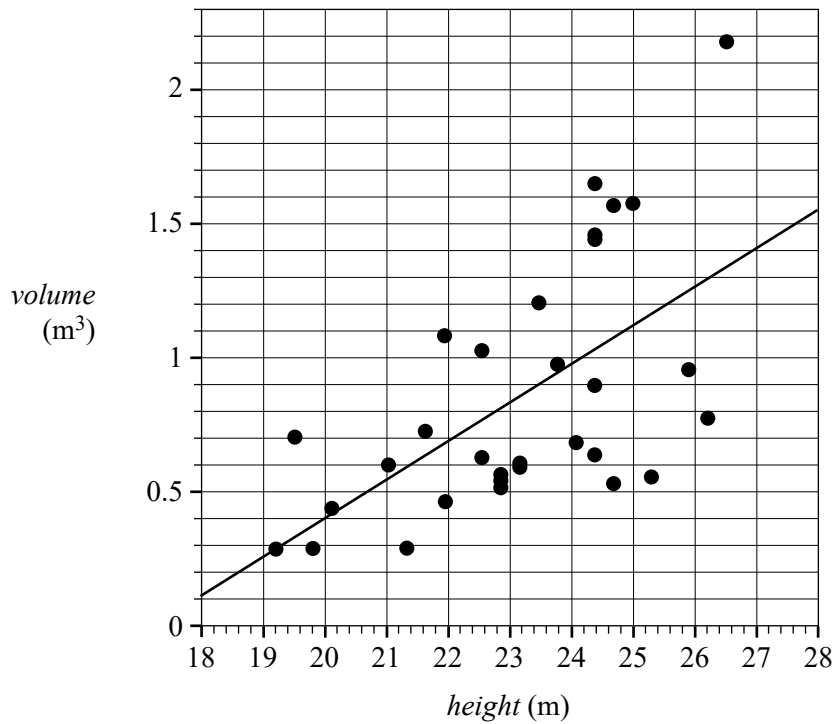
Data: 'Black Cherry Trees', Wolfram Data Repository,
<https://datarepository.wolframcloud.com/resources/Sample-Data-Black-Cherry-Trees>

Question 5

When the least squares line is used to predict the *volume* of a tree with a height of 25 m, the residual, in metres, is closest to

A.   −1.5

B.   −0.5

C.   0.0

D.   0.5

E.   1.5

Question 6

The equation of the least squares line is closest to

A.   *volume* = −2.47 + 0.143 × *height*

B.   *volume* = −0.12 + 0.143 × *height*

C.   *volume* = 0.12 + 1.43 × *height*

D.   *volume* = 2.47 + 7.14 × *height*

E.   *volume* = 17.1 + 7.14 × *height*

Question 7

The residuals for this least squares line have a mean of 0.00 and a standard deviation of 0.37.

The standardised value of one of the residuals is $z = -0.81$.

The value of the residual is closest to

A. −0.5

B. −0.3

C. −0.2

D.   0.0

E.   0.4

Use the following information to answer Questions 8 and 9.

The data in the table below was collected in a study of the association between having *symptoms* of respiratory disease (yes, no) and *level of exposure* to a component of a product (none, limited, high) that is expected to cause respiratory problems to workers in a factory. Five hundred people were surveyed.

Symptoms	*Level of exposure*		
	None	**Limited**	**High**
yes	17	33	185
no	72	73	120

Question 8

The percentage of the workers with *symptoms* of respiratory disease in this study is

A.   0%

B. 31%

C. 44%

D. 47%

E. 61%

Question 9

Which of the following statements supports the contention that increasing exposure of workers to the product is associated with having *symptoms* of respiratory disease?

A. 78.7% of those with *symptoms* had a high level of exposure to the product.

B. 19.2% of those with *symptoms* had a limited level of exposure to the product.

C. 88.9% of those without *symptoms* had no exposure to the product.

D. The percentage of workers with *symptoms* increased from 7.2% to 14.0% to 78.7% as the level of exposure increased from none, to limited, to high.

E. The percentage of workers with *symptoms* increased from 19.1% to 31.1% to 60.7% as the level of exposure increased from none, to limited, to high.

Use the following information to answer Questions 10 and 11.

The time series plot below displays the quarterly sales, in millions of dollars, of a manufacturer during the period 2000 to 2007.
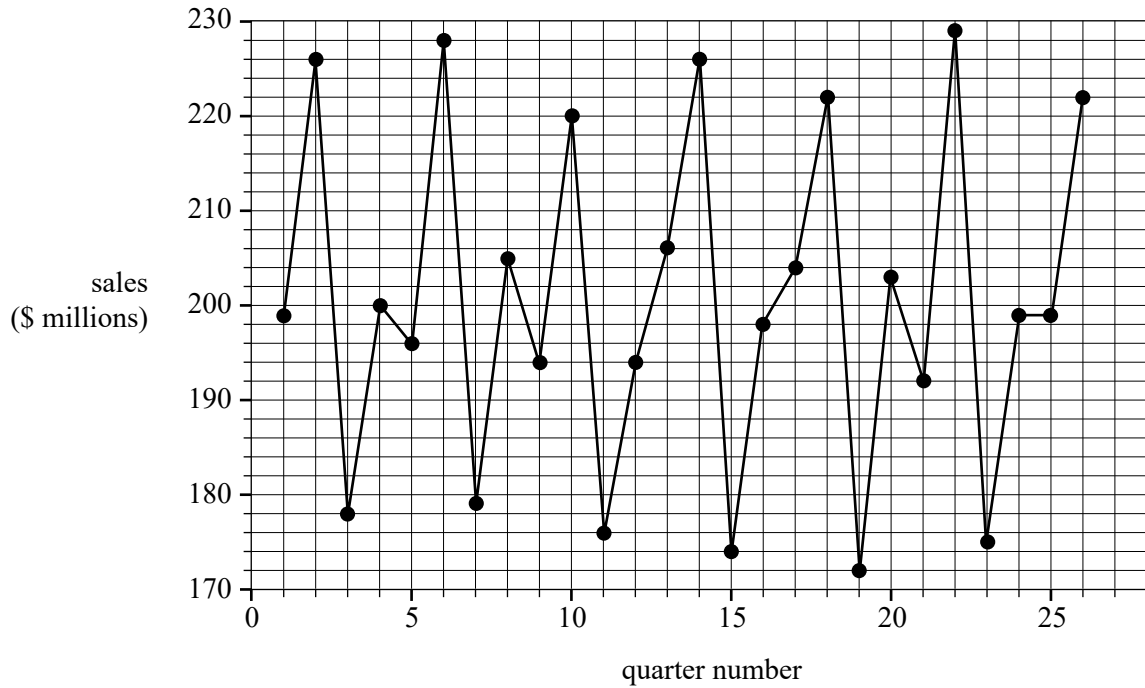
Question 10

The time series plot is best described as having

A.   seasonality only.

B.   irregular fluctuations only.

C.   seasonality with irregular fluctuations.

D.   a decreasing trend with seasonality and irregular fluctuations.

E.   an increasing trend with seasonality and irregular fluctuations.

Question 11

The seven-median smoothed sales, in millions of dollars, for quarter 8 is closest to

A.   194

B.   196

C.   198

D.   200

E.   205

Question 12

The table below shows the number of hours worked each week by casual staff in a coffee shop over a 14-week period.

Week number	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Hours	56	45	58	29	28	32	38	34	48	38	39	41	28	32

The eight-mean smoothed value with centring for week 7 is closest to

A. 35

B. 36

C. 37

D. 38

E. 39

Question 13

The seasonal index for sales in a clothes shop in October is 1.65.

This tells us that October's sales tend to be

A. 35% less than the average monthly sales.

B. 35% of average monthly sales.

C. 61% of average monthly sales.

D. 61% above the average monthly sales.

E. 65% above the average monthly sales.

Use the following information to answer Questions 14 and 15.

The time series plot below is non-linear. The data used to construct this plot is also shown below.
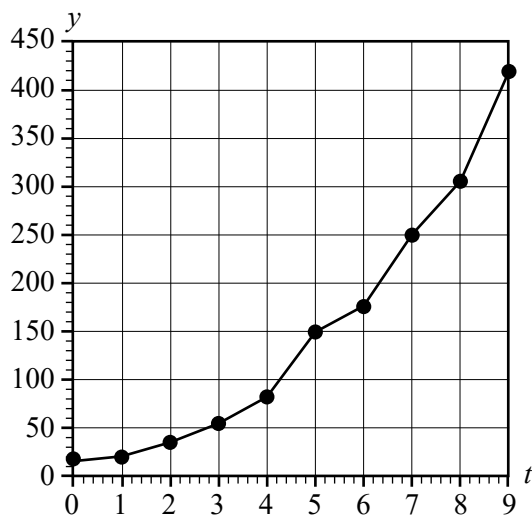
t	y
0	15
1	20
2	35
3	55
4	82
5	150
6	175
7	250
8	305
9	420

Question 14

This time series plot is linearised by applying a squared transformation to the variable *t*. A least squares line is fitted to the transformed data.

The equation of this line is closest to

A.　$y = -44.10 + 43.29 \times t$

B.　$y = -44.10 + 43.29 \times t^2$

C.　$y = 1.325 + 0.002107 \times t^2$

D.　$y = 12.92 + 4.834 \times t$

E.　$y = 12.92 + 4.834 \times t^2$

Question 15

This time series plot can also be linearised by applying a $\log_{10}$ transformation to the variable *y*.

When a least squares line is fitted to the transformed data, the equation of this line is:

$$\log_{10}(y) = 1.214 + 0.1657 \times t$$

This equation is used to predict *y* when *t* = 11. The value of *y* is closest to

A.　　　3

B.　　589

C.　　673

D.　　721

E.　1088

Question 16

A food van operates in an industrial area from Monday to Friday.

The table below shows the seasonal indices for the daily sales for this food van. The seasonal index for Wednesday is missing.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Day number	1	2	3	4	5
Seasonal indices	1.25	1.1	–	0.85	0.9

The following equation can be used to forecast the daily de-seasonalised sales for each day:

de-seasonalised sales = 4207.9 + 135.31 × *day number*

The actual sales for Wednesday are predicted to be closest to

A. $3691
B. $4152
C. $4208
D. $4614
E. $5126

Recursion and financial modelling

Question 17

A sequence of numbers is generated by the recurrence relation shown below.

$$T_0 = 11, \qquad T_{n+1} = T_n - 4$$

The first negative term in this sequence is

A. T_2

B. T_3

C. T_4

D. T_5

E. T_{11}

Use the following information to answer Questions 18 and 19.

Eddie has a reducing balance loan. Interest is calculated monthly, and Eddie makes monthly repayments.

Four lines of the amortisation table for Eddie's loan are shown below.

Payment number	Payment ($)	Interest ($)	Principal reduction ($)	Balance ($)
0	0.00	0.00	0.00	420 000.00
1	2634.51	1540.00	1094.51	418 905.49
2	2634.51	1535.99	1098.52	417 806.97
3	2634.51	1531.96		

Question 18

The balance of Eddie's loan after payment number 3 is

A. $416 275.01

B. $416 704.42

C. $416 708.45

D. $416 712.46

E. $418 909.52

Question 19

The interest rate, per annum, for Eddie's loan is

A. 3.7%

B. 3.8%

C. 4.0%

D. 4.2%

E. 4.4%

Question 20

The value of a computer, purchased for $6000, is depreciated by a flat rate of 8% per annum.

Which one of the following recurrence relations models the year-to-year value, V_n, of the computer?

A. $V_0 = 6000$, $V_{n+1} = V_n - 480$

B. $V_0 = 6000$, $V_{n+1} = V_n - 600$

C. $V_0 = 6000$, $V_{n+1} = 0.08V_n$

D. $V_0 = 6000$, $V_{n+1} = 0.092V_n$

E. $V_0 = 6000$, $V_{n+1} = 0.92V_n$

Question 21

The rule for the future value of an asset after n depreciation periods is $V_n = 4000 - 20n$.

The method used for depreciating the value of the asset could be

A.   reducing balance only.

B.   flat rate only.

C.   unit cost only.

D.   either flat rate or reducing balance.

E.   either flat rate or unit cost.

Question 22

James invested an amount of money in a perpetuity that earns interest at the rate of 3.6% per annum.

For this perpetuity, interest is calculated and paid monthly.

If James receives a regular monthly payment of $1440, then the balance of the perpetuity after one year of monthly payments is

A. $17 280

B. $462 720

C. $480 000

D. $481 440

E. $497 280

Question 23

Mo invested $10 000 into an account that earns interest, compounding fortnightly. The balance, in dollars, after n fortnights, M_n, can be modelled by the recurrence relation shown below.

$$M_n = 10\,000 \qquad M_{n+1} = 1.001M_n$$

The effective annual rate of interest for Mo's investment is closest to

A.   2.57%

B.   2.60%

C.   2.63%

D.   2.66%

E.   2.69%

Question 24

Roula takes out a reducing balance loan of $440 000 with an interest rate of 3.64% per annum, compounding monthly. This loan is to be repaid monthly, with equal payments over 15 years.

In addition to the scheduled repayments, Roula decides to pay an additional amount of $299.52 per month to fully repay the loan sooner.

The total amount saved by Roula by making the additional monthly payments is closest to

A. $15 593

B. $15 810

C. $16 220

D. $16 460

E. $18 220

END OF SECTION A

SECTION B – Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet **and** writing the name of the module in the box provided.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

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Contents   **Page**

Module 1 – Matrices

Before answering these questions, you must **shade** the 'Matrices' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

Which one of the following is **not** an example of a square matrix?

A. a state matrix

B. an inverse matrix

C. a transition matrix

D. a permutation matrix

E. a communication matrix

Question 2

Students can select one elective subject from Woodwork (W), Fashion (F) and Coding (C).

Matrix A shows the number of students in Class A who choose each elective.

$$A = \begin{matrix} W & F & C \end{matrix} \\ \begin{bmatrix} 12 & 6 & 7 \end{bmatrix}$$

Matrix B shows the number of students in Class B who choose each elective.

$$B = \begin{matrix} W & F & C \end{matrix} \\ \begin{bmatrix} 10 & 9 & 6 \end{bmatrix}$$

When Matrix A is added to Matrix B, the resulting matrix shows

A. the total number of students in Class A.

B. the total number of students in Class B.

C. the total number of students in both classes.

D. the total number of students in both Class A and Class B who choose each elective.

E. the difference in the number of students from Class A and Class B who choose each elective.

Question 3

Which one of the following matrix products does **not** produce a matrix with five rows and three columns?

A.

$$\begin{bmatrix} 2 \\ 3 \\ 1 \\ 4 \\ 3 \end{bmatrix} \times \begin{bmatrix} 3 & -1 & 0 \end{bmatrix}$$

B.

$$\begin{bmatrix} 6 & 3 \\ 4 & 2 \\ -8 & 2 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 8 & 5 \\ -4 & 5 & -2 \end{bmatrix}$$

C.

$$\begin{bmatrix} 5 & 0 & 9 & 4 & -3 \\ -5 & 4 & 1 & 9 & 0 \\ 4 & -6 & 4 & -8 & 1 \end{bmatrix} \times \begin{bmatrix} 6 & 7 & 3 \\ 2 & 4 & 5 \\ 3 & -9 & -1 \\ 5 & 4 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

D.

$$\begin{bmatrix} 0 & 8 & -4 & 5 \\ 3 & -9 & 0 & 0 \\ 1 & 1 & 4 & -2 \\ -6 & 5 & 6 & 2 \\ -1 & 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

E.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Question 4

A cinema increased the cost of a child's ticket, an adult's ticket and a senior's ticket by the same scalar factor, a.

The increase is found by completing the following calculation.

$$a \times \begin{bmatrix} \$11.50 \\ \$15.00 \\ \$13.50 \end{bmatrix} = \begin{bmatrix} \$13.80 \\ \$x \\ \$16.20 \end{bmatrix}$$

The value of x is equal to

A. 2.30

B. 2.70

C. 17.30

D. 17.70

E. 18.00

Question 5

The following matrix shows the possible flight routes to and from five towns, named V, W, X, Y and Z.

$$
\begin{array}{c}
 & \textit{to} \\
 & \begin{matrix} V & W & X & Y & Z \end{matrix} \\
\textit{from}\quad
\begin{matrix} V \\ W \\ X \\ Y \\ Z \end{matrix}
&
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

A '1' in the matrix shows that a town in that row is connected by a direct route to a town in that column.

For example, the '1' in row 2, column 4, shows that a direct flight can go from town W to town Y.

How many ways can people travel from town Y to town V without visiting the same town more than once?

A. 1

B. 2

C. 3

D. 4

E. 5

Question 6

Matrices P, Q and R are square matrices.

Matrix P is the inverse of matrix Q.

Matrix R is the transpose of matrix P.

All three matrices are defined if

A. $R = \begin{bmatrix} 2 & 8 \\ 1 & 4 \end{bmatrix}$

B. $R = \begin{bmatrix} 2 & 8 \\ 4 & 1 \end{bmatrix}$

C. $P = \begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix}$

D. $Q = \begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix}$

E. $Q = \begin{bmatrix} 2 & 8 \\ 1 & 4 \end{bmatrix}$

Question 7

Max is a baker who makes and sells different types of cakes. The following table is an example of how Max records the price of each cake and the number of cakes he sells each day.

	Chocolate cake	Cheesecake	Sponge cake	Carrot cake
Sale price ($)	a	b	c	d
Number of cakes sold	e	f	g	h

Max wants to set up a matrix equation that will allow him to determine:
- the total number of cakes sold each week
- the total sales, in dollars, made from selling all of his cakes.

The matrix calculation required by Max is given by

A.
$$\begin{bmatrix} a & b & c & d \end{bmatrix} \times \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix}$$

B.
$$\begin{bmatrix} a & b & c & d \end{bmatrix} \times \begin{bmatrix} 1 & e \\ 1 & f \\ 1 & g \\ 1 & h \end{bmatrix}$$

C.
$$\begin{bmatrix} a & b & c & d \end{bmatrix} \times \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & h \end{bmatrix}$$

D.
$$\begin{bmatrix} e & f & g & h \end{bmatrix} \times \begin{bmatrix} 1 & a \\ 1 & b \\ 1 & c \\ 1 & d \end{bmatrix}$$

E.
$$\begin{bmatrix} e & f & g & h \end{bmatrix} \times \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

Question 8

A group of 400 workers is based at four different locations: J, K, L and M.

The matrix recurrence relation below is used to model the way in which workers change location from week to week.

$$S_{n+1} = T S_n \qquad \text{where } T = \begin{array}{c} \text{This week} \\ \begin{array}{cccc} J & K & L & M \end{array} \\ \begin{bmatrix} 0.2 & 0.6 & 0.4 & 0.1 \\ 0.7 & 0.1 & 0.2 & 0.5 \\ 0 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0.2 \end{bmatrix} \begin{array}{c} J \\ K \\ L \\ M \end{array} \end{array} \text{ Next week} \qquad \text{and } S_1 = \begin{bmatrix} 130 \\ 150 \\ 70 \\ 50 \end{bmatrix} \begin{array}{c} J \\ K \\ L \\ M \end{array}$$

Consider the following five statements about the information above.

- The highest number will always be at K.
- All four numbers in matrix S_0 are equal.
- 30% of workers at L one week will remain at L the following week.
- Of those workers who started at J, 20 will still be at J two weeks later.
- In the long term, the number at K, to the nearest whole number, will be 152.

How many of these statements are true?

A. 1
B. 2
C. 3
D. 4
E. 5

Module 2 – Networks and decision mathematics

Before answering these questions, you must **shade** the 'Networks and decision mathematics' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The graph below represents a friendship network. The vertices represent four students: Khan (*K*), Louis (*L*), Mario (*M*) and Nikos (*N*).

An edge represents the presence of a friendship between a pair of these students. For example, the edge connecting *K* and *L* shows that Khan and Louis are friends.
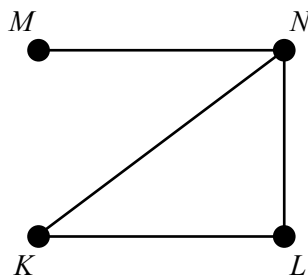
Which one of the following statements is true?

A.   All of the students are friends of Khan.

B.   Khan is a friend of both Louis and Mario.

C.   Louis is a friend of Mario.

D.   Mario is a friend of both Nikos and Khan.

E.   Nikos is a friend of all of the other students.

Question 2

The graph below has eight vertices, labelled *A* to *H*.
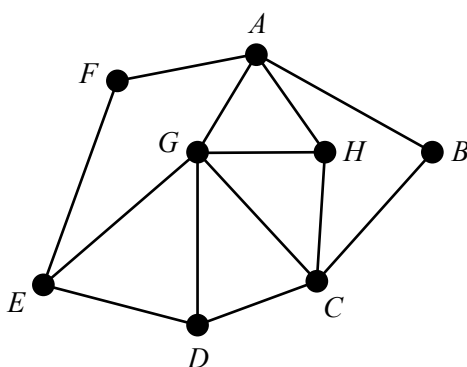
Which one of the following is **not** a Hamiltonian path?

A.   *ABCHGDEF*

B.   *BCHAGDEF*

C.   *CDEFAGHA*

D.   *DEFAHGCB*

E.   *EFABCHGD*

Question 3

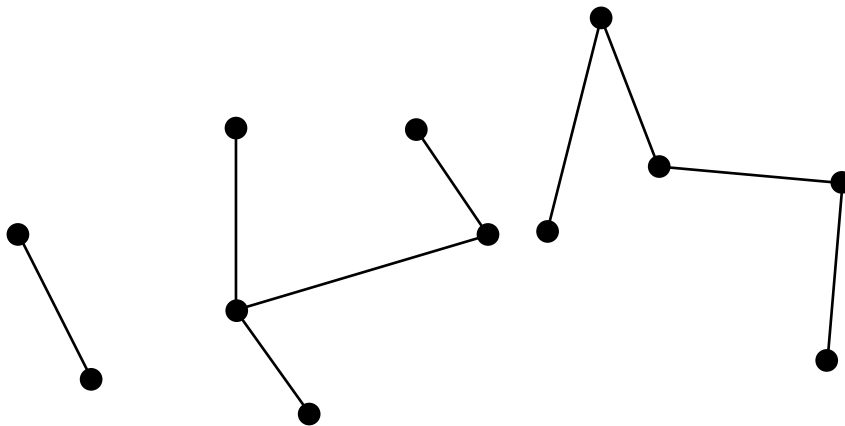A network of tracks connects the entrance of a park to the exit, as shown in the directed graph below.
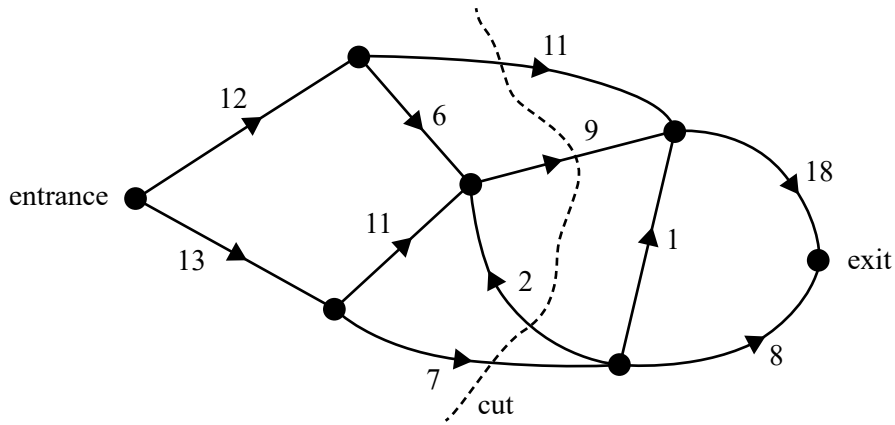
The arrows show the direction that visitors can travel along each of the tracks, and the numbers show each track's capacity in visitors per second.

The capacity of the cut shown above is

A.　25

B.　26

C.　27

D.　28

E.　29

Question 4

Consider the graph below.

A number of edges must be added to create a connected planar graph with 12 vertices and 3 faces.

The number of edges that must be added is

A.　1

B.　2

C.　3

D.　4

E.　5

Question 5

Margorie drives from her home to the office where she works.

The network below shows the distances, in kilometres, along a series of roads connecting Margorie's home to the office.

The vertices *A, B, C, D, E* and *F* represent the intersections of these roads.
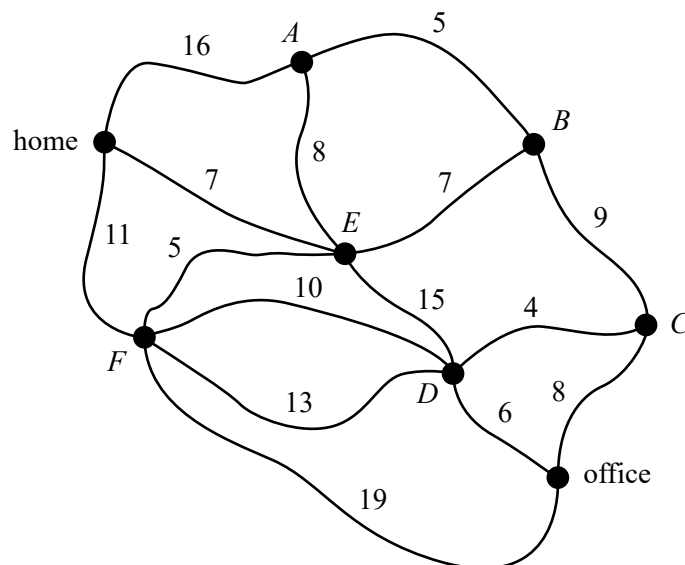
The shortest distance, in kilometres, for Margorie to travel from her home to the office is

A. 26
B. 27
C. 28
D. 29
E. 30

Question 6

$$\begin{array}{c c} & \begin{array}{c c c c c} P & Q & R & S & T \end{array} \\ \begin{array}{c} P \\ Q \\ R \\ S \\ T \end{array} & \left[\begin{array}{c c c c c} 0 & 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{array}\right] \end{array}$$

The adjacency matrix above represents a graph with five vertices: *P, Q, R, S* and *T*.

Which one of the following statements regarding this graph is true?

A. It is planar.
B. It contains two loops.
C. It contains an Eulerian circuit.
D. The sum of the degrees of the vertices equals 21.
E. It contains three vertices of odd degree.

Question 7

Ava, Belle, Chet, Diego and Eden are five workers on a production line. Each worker will perform one task. Each task must be completed in numerical order: 1, 2, 3, 4 and 5. The time for each worker to complete tasks 1, 2, 3, 4 and 5, in minutes, is shown in the table below.

	Task 1	**Task 2**	**Task 3**	**Task 4**	**Task 5**
Ava	11	9	16	15	21
Belle	12	11	15	14	19
Chet	8	9	14	16	18
Diego	9	8	14	17	19
Eden	11	10	16	15	21

The production manager assigns each worker a task that will minimise the total production completion time. Diego and Eden swap their assigned tasks.

This will result in an increase in the total completion time by

A.　one minute.

B.　two minutes.

C.　three minutes.

D.　four minutes.

E.　five minutes.

Question 8

A project has 11 activities, *A* to *K*. The table below shows the duration of each activity, the earliest start time (EST) and the latest start time (LST).

All times are in weeks.

Activity	Duration	EST	LST
A	7	0	1
B	6	0	0
C	9	0	5
D	2	6	6
E	5	6	9
F	6	8	8
G	8	8	10
H	4	14	14
I	3	14	17
J	2	18	18
K	5	20	20

In this project, the number of activities that have exactly two immediate predecessors is

A.   1

B.   2

C.   3

D.   4

E.   5

Module 3 – Geometry and measurement

> Before answering these questions, you must **shade** the 'Geometry and measurement' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

The areas allocated to five categories on a bookshop sale table are shown in the diagram below. The areas allocated to sport, humour, crime and cooking all have equal dimensions. The area for romance is twice that of each of the other four categories.
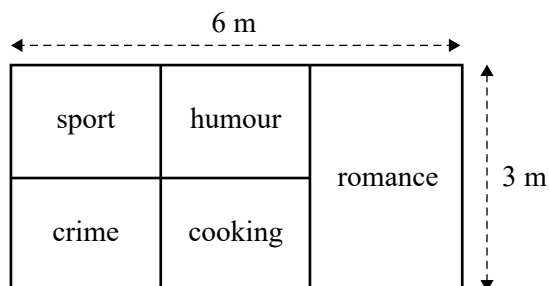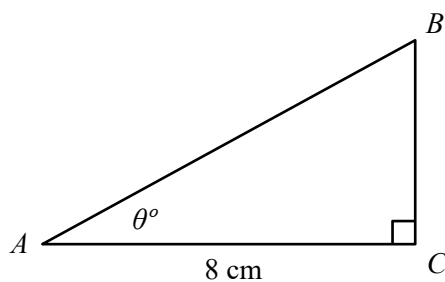

The area allocated to cooking, in square metres, is

A. 2.0

B. 2.5

C. 3.0

D. 3.6

E. 6.0

Question 2

In triangle ABC, shown in the diagram below, $\cos \theta° = 0.4700$.

BC has a length, in centimetres, closest to

A. 9

B. 12

C. 15

D. 17

E. 19

Question 3

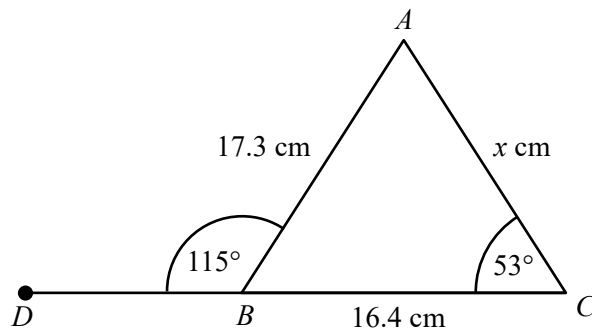With triangle *ABC*, *BC* has been extended to point *D*.

Angle *ABD* is 115°

Angle *ACB* is 53°

BC = 16.4 cm

AB = 17.3 cm

AC = *x* cm
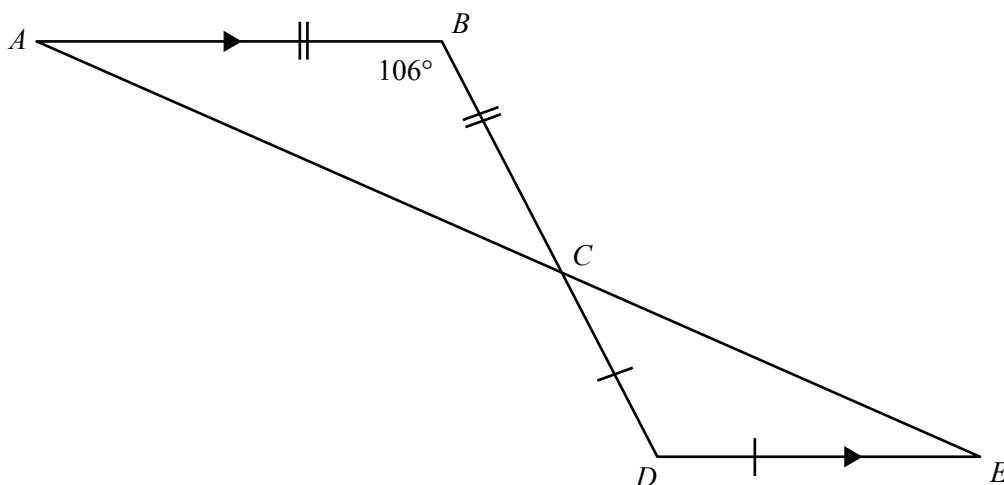
This information is shown in the diagram below.

Which one of the following calculations will correctly determine the value of *x*?

A. $x = \sqrt{17.3^2 + 16.4^2 - 2 \times 17.3 \times 16.4 \times \cos 53°}$

B. $x = \sqrt{17.3^2 + 16.4^2 - 2 \times 17.3 \times 16.4 \times \cos 62°}$

C. $x = \sqrt{17.3^2 + 16.4^2 - 2 \times 17.3 \times 16.4 \times \cos 75°}$

D. $x = \sqrt{17.3^2 + 16.4^2 - 2 \times 17.3 \times 16.4 \times \cos 115°}$

E. $x = \sqrt{17.3^2 + 16.4^2 - 2 \times 17.3 \times 16.4 \times \cos 65°}$

Question 4

Triangle *BAC* is similar to triangle *DEC*, as shown in the diagram below.

Angle *CED* is equal to

A. 27°

B. 37°

C. 47°

D. 53°

E. 74°

Question 5

Two Canadian cities, Vancouver (49° N, 123° W) and Ottawa (45° N, 76° W), are in time zones that have a three-hour time difference.

A plane left Ottawa at 8.30 am and landed in Vancouver five-and-a-half hours later.

At what time did the plane land in Vancouver?

A. 10.00 am

B. 11.00 am

C. 12.00 midday

D. 2.00 pm

E. 5.00 pm

Question 6

The shortest great circle distance between the cities of Boston (North America) and Santiago (South America) is approximately 8378 kilometres along the 71° W meridian of longitude.
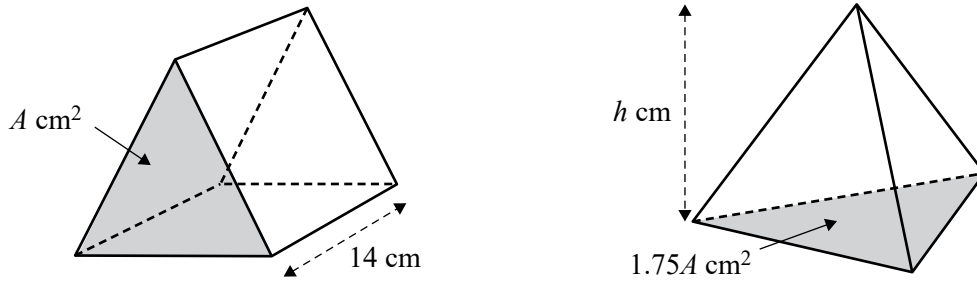
Assume that the radius of Earth is 6400 km. Boston is at latitude 42° N.

The latitude of Santiago is

A. 5° S

B. 33° N

C. 33° S

D. 75° N

E. 75° S

Question 7

A triangular prism has a cross-sectional area (shaded) of A cm^2 and a length of 14 cm. A triangular-based pyramid has a base area (shaded) of $1.75A$ cm^2 and a height of h cm. These two solids are shown below.


If the volume of these two solids is equal, then the value of h, in centimetres, is closest to

A.   8

B.   16

C.   22

D.   24

E.   28

Question 8

A survey team measured the angle of elevation of the top of a tall tower from three points.

The angles were 56° at point A, 45° at point B and 38° at point C, as shown in the diagram below.

All of the measuring sites and the base of the tower were in a straight line and on the same level ground.

Points A and B were 50.0 m apart. This information is shown in the diagram below.
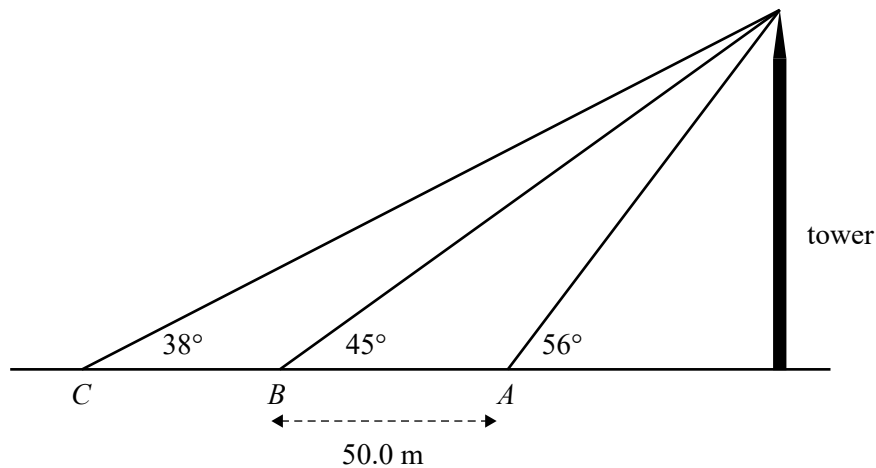
The distance between point B and point C, in metres, was closest to

A.   38

B.   40

C.   43

D.   47

E.   50

End of Module 3 – SECTION B – continued
TURN OVER

Module 4 – Graphs and relations

Before answering these questions, you must **shade** the 'Graphs and relations' box on the answer sheet for multiple-choice questions and write the name of the module in the box provided.

Question 1

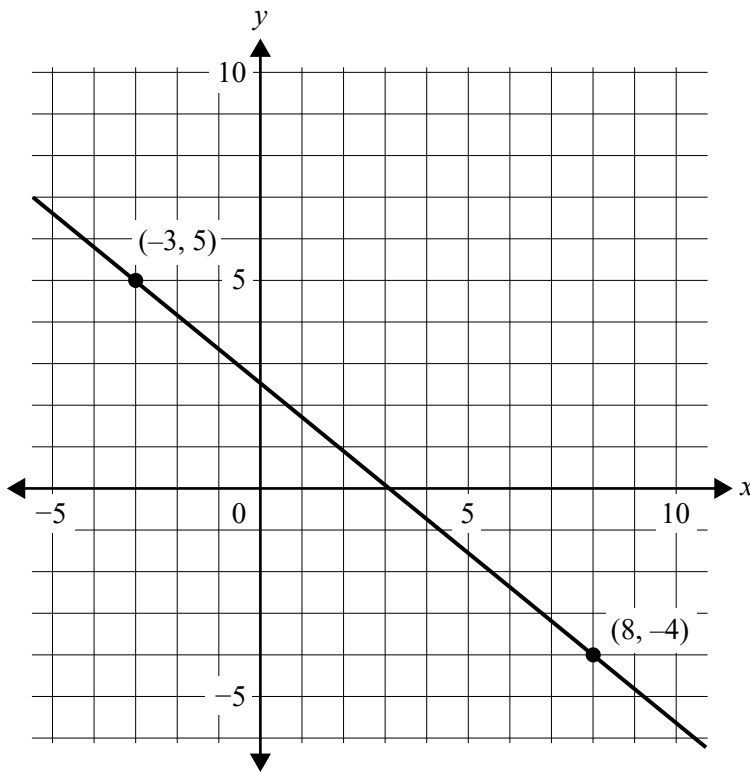The graph below shows the average wind speed for a location, in kilometres per hour, over a 20-day period.
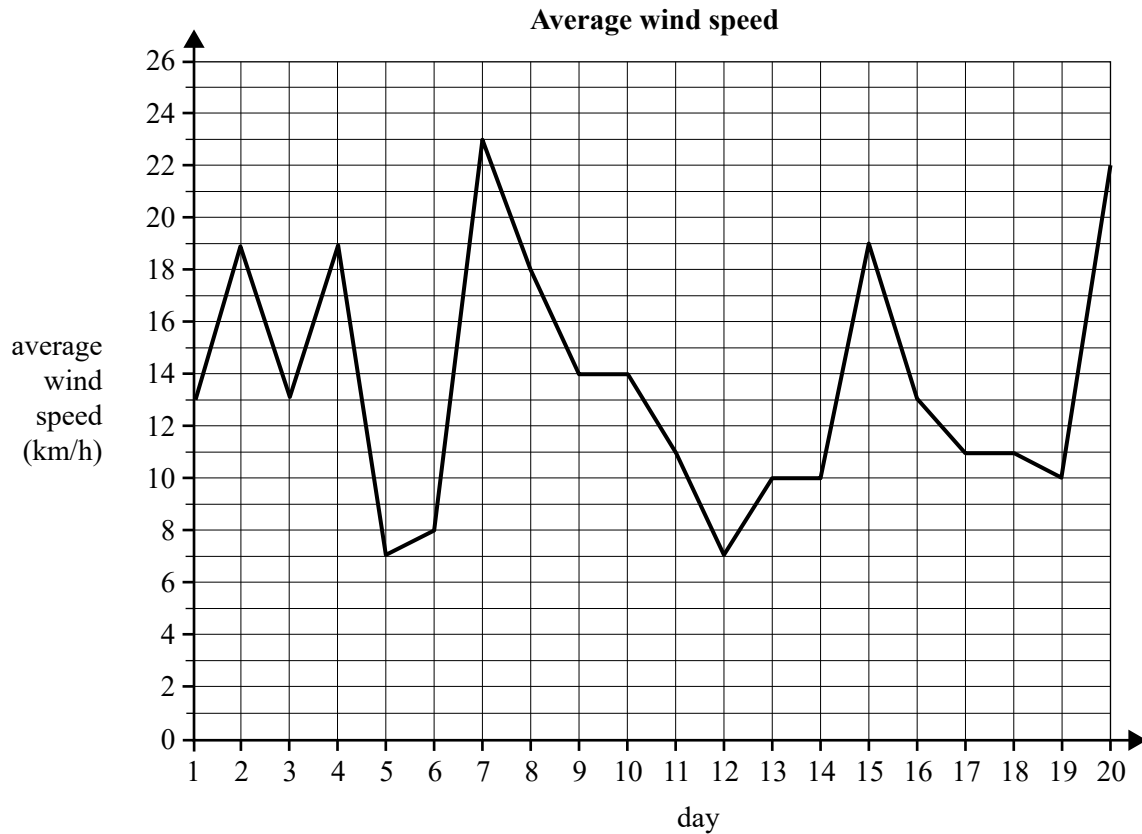
The difference between the highest and lowest average wind speeds, in kilometres per hour, is closest to

A. 15
B. 16
C. 17
D. 23
E. 24

Question 2

The graph below shows a line passing through the points $(8, -4)$ and $(-3, 5)$.

The slope of the line is

A. $-\dfrac{28}{11}$

B. $-\dfrac{5}{2}$

C. $-\dfrac{5}{4}$

D. $-\dfrac{11}{9}$

E. $-\dfrac{9}{11}$

Question 3

At an amusement park, the price for rides is different for people under 1.2 m tall than it is for people 1.2 m tall or more.

Two groups of people, Group A and Group B, went to the park and paid for their rides. The table below shows how many people in each group were under 1.2 m tall, how many were 1.2 m and above, and the total amount each group paid for its rides.
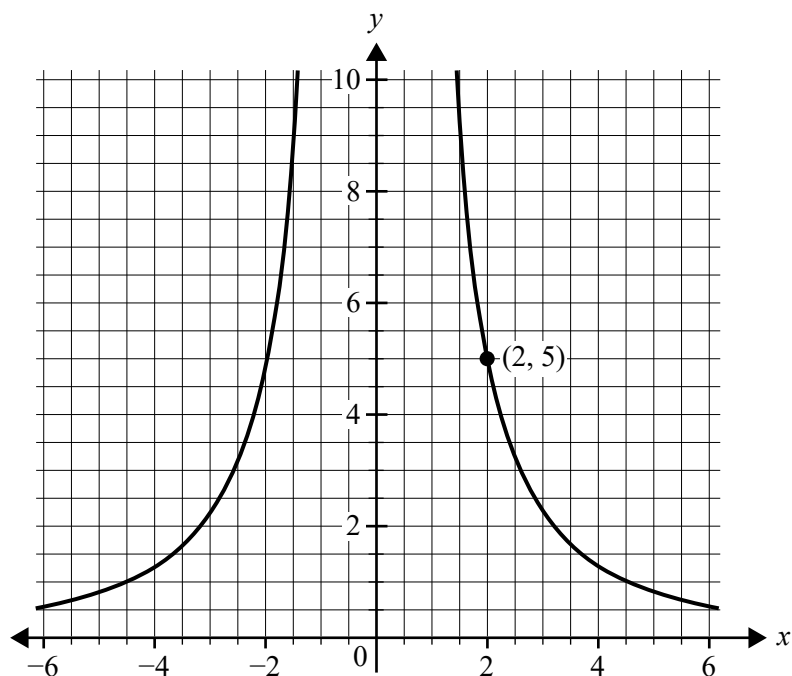
	Group A	Group B
Under 1.2 m	2	4
1.2 m tall and above	4	3
Total amount paid	$86	$92

The price for rides for a person under 1.2 m is

A. $11
B. $13
C. $15
D. $16
E. $17

Question 4

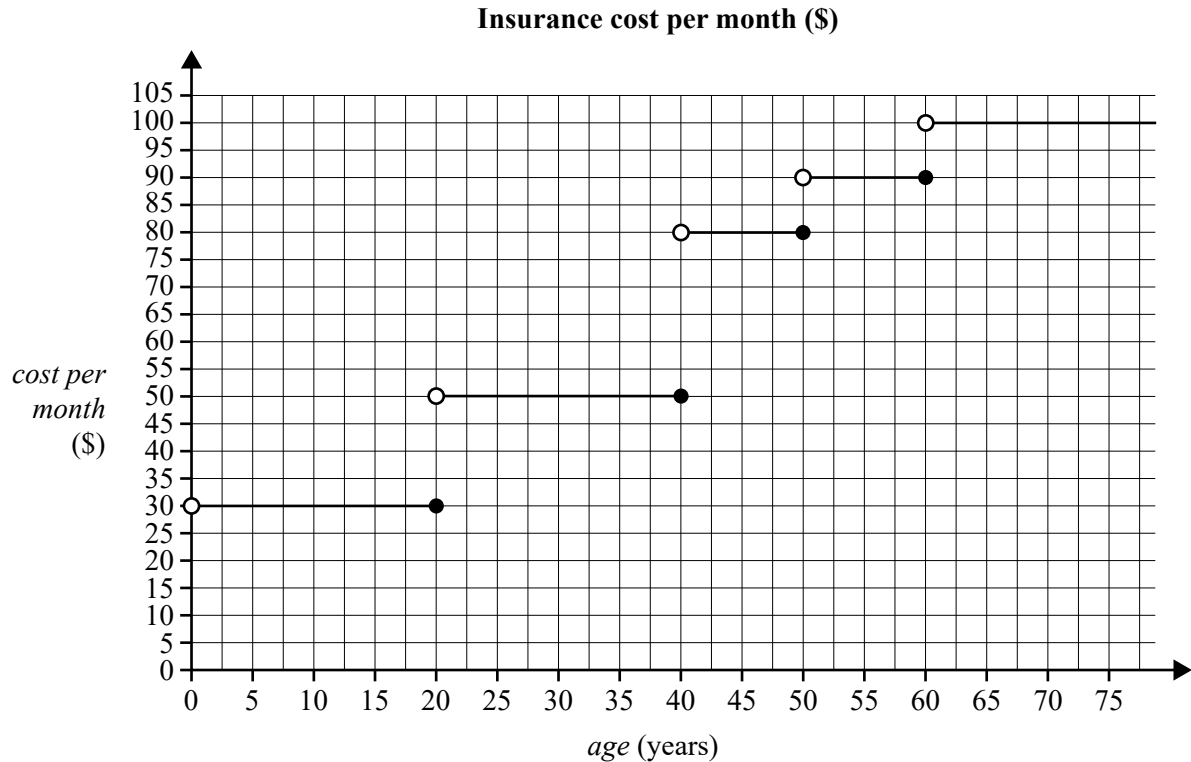The point (2, 5) lies on the graph of $y = \dfrac{k}{x^2}$.

The value of y when $x = 8$ is

A. 0.225
B. 0.25
C. 0.275
D. 0.3125
E. 0.375

Question 5

An insurance company bases the total cost of health insurance per month on the age of the applicants. This is shown in the step graph below.

Insurance cost per month ($)

The Mayver and Lenoc families have health insurance policies with this company.

The ages of the Mayver family members are as follows: Jim, 61, June, 50, Alix, 25 and Alto, 21.

The ages of the Lenoc family members are as follows: Alan, 49, Jordan, 40, Kennedy, 20, Austin, 11 and Drew, 1.

Which family pays more for health insurance, and by how much?

A.   The Lenoc family, by $10

B.   The Lenoc family, by $40

C.   The Mayver family, by $40

D.   The Mayver family, by $60

E.   The Mayver family, by $80

Question 6

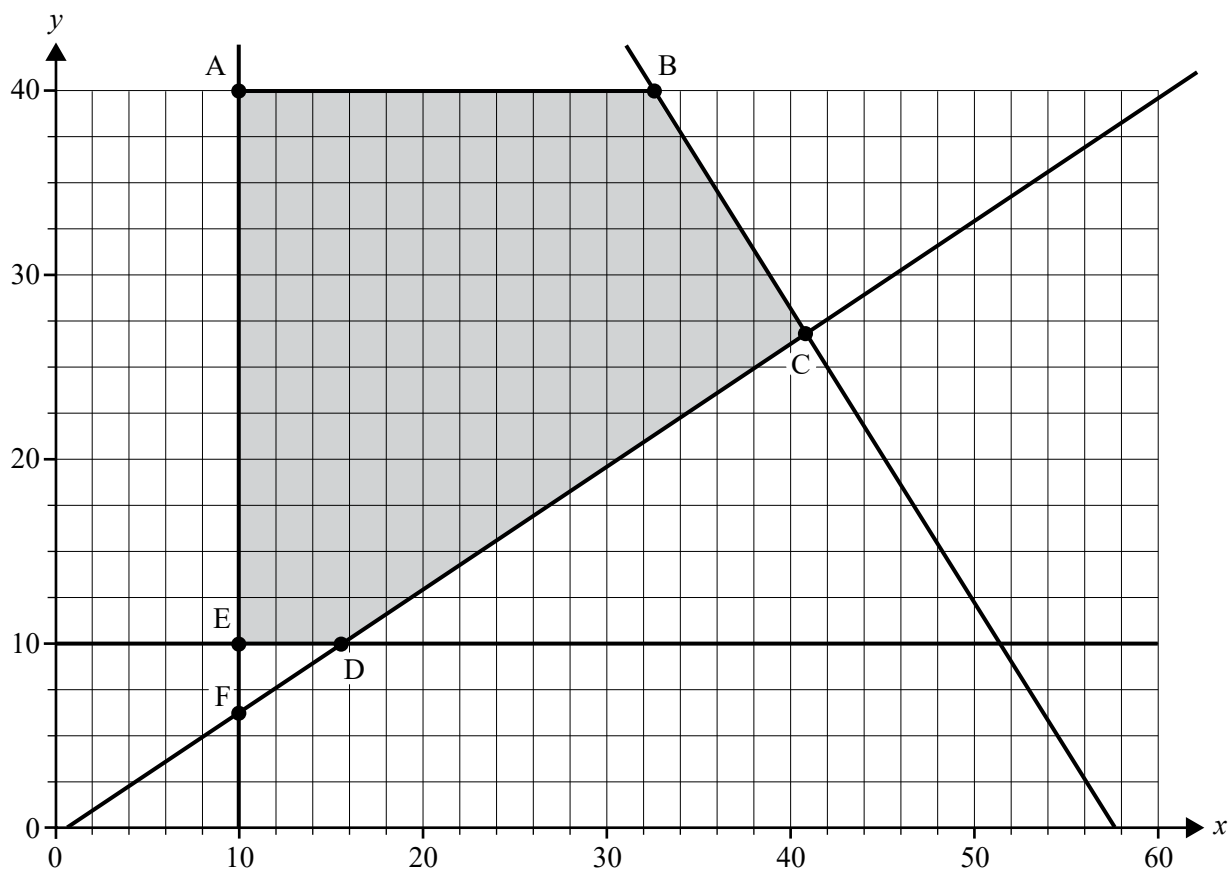The shaded region on the graph below shows a feasible region defined by four inequalities.
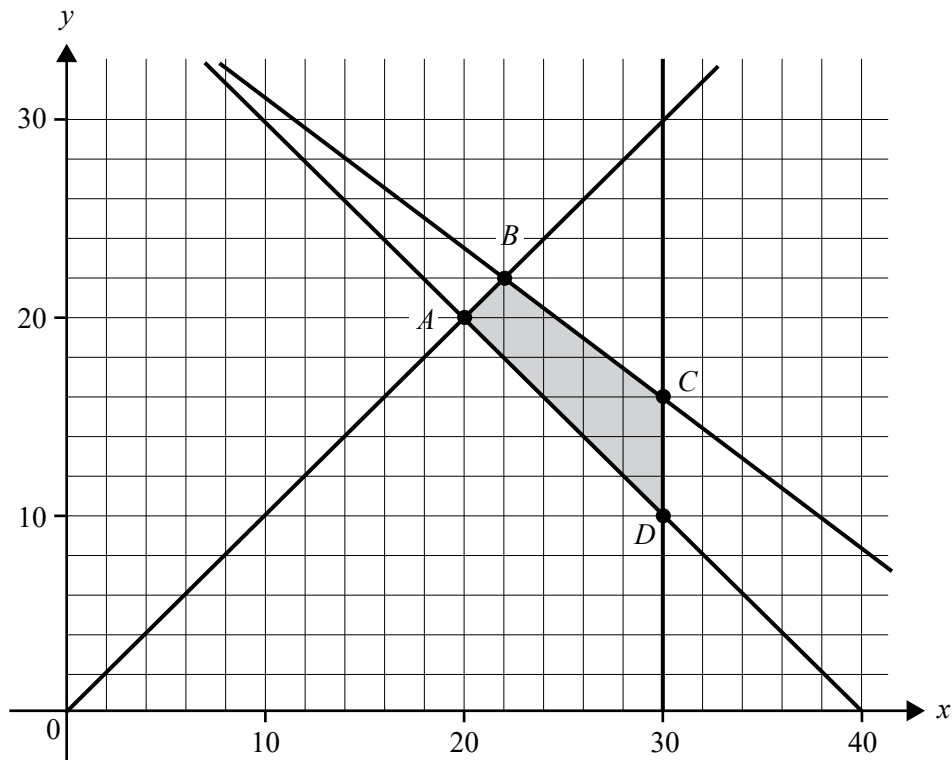
Point *A* is (20, 20), Point *B* is (22, 22), Point *C* is (30, 16) and Point *D* is (30, 10).

One of the inequalities used to define this feasible region is

A. $y - x \geq 0$

B. $3x + 4y \leq 171$

C. $4x + 3y \leq 154$

D. $4x + 3y \leq 171$

E. $3x + 4y \leq 154$

Question 7

The feasible region for a linear programming problem is shaded in the diagram below.

The equation of the objective function for this problem is of the form

$$P = mx + ny, \qquad \text{where } m > 0 \text{ and } n > 0$$

How many of the following options could describe the location of the maximum value of the objective function?

- Value at point A
- Value at point B
- Value at point C
- Any value along the line AB
- Any value along the line BC
- Any value along the line AE

A. 1

B. 2

C. 3

D. 4

E. 5

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Question 8

A bandicoot is moving away from its burrow. It moves in a straight line along the ground.

The *distance*, in metres, of the bandicoot from the burrow t seconds after it begins moving away is given by the equation

$$distance = \begin{cases} 2t, & 0 \le t \le 5 \\ 3t + b, & 5 < t \le 9 \\ 4t + c, & 9 < t \le a \end{cases}$$

The bandicoot sees an owl a seconds after leaving the burrow, and returns safely to the burrow at 5 m/s in a straight line. The total time for the bandicoot's journey, both moving away from the burrow and returning, is 53 seconds.

The value of a is

A. 29
B. 30
C. 31
D. 32
E. 33

END OF MULTIPLE-CHOICE QUESTION BOOK

**Victorian Certificate of Education
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FURTHER MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A multiple-choice question book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Further Mathematics formulas

Core – Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $\quad Q_1 - 1.5 \times IQR \qquad$ upper $\quad Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx,$ where $\quad b = r\dfrac{s_y}{s_x}$ and $\quad a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \qquad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n} \right)^n - 1 \right] \times 100\%$

Module 1 – Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$ where $\quad \det A \neq 0$
recurrence relation	$S_0 = $ initial state, $\qquad S_{n+1} = T S_n + B$

Module 2 – Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta°)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta°$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta°}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base × height
volume of a pyramid	$\frac{1}{3}$ × area of base × height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET