VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2023

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER

SPECIALIST MATHEMATICS

Written examination 2

Friday 26 May 2023

Reading time: 2.00 pm to 2.15 pm (15 minutes)
Writing time: 2.15 pm to 4.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20
B	6	6	60
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set squares, aids for curve sketching, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 24 pages
- Formula sheet
- Answer sheet for multiple-choice questions

Instructions
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

LILAC STRIPE

SECTION A – Multiple-choice questions

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

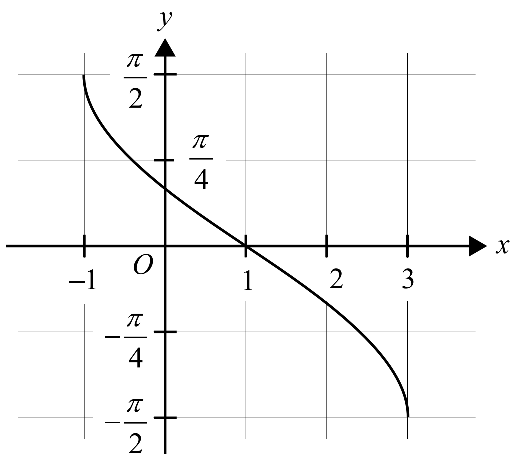
Question 1

The implied domain and range of $f(x)=\sin\left(\cos^{-1}(1-2x)\right)$ are respectively

A. [0, 1] and [0, 1]
B. [−1, 0] and [0, 1]
C. R and [−1, 1]
D. [0, 1] and [−1, 1]
E. R and [0, 1]

Question 2

Let $f(x) = \arcsin(x)$ and $g(x) = ax + b$, where $a, b \in R$.
The graph of $y = f\left(g(x)\right)$ is shown below.



The values of a and b are, respectively

A. $\frac{1}{2}$ and $\frac{1}{2}$
B. $-\frac{1}{2}$ and $-\frac{1}{2}$
C. $-\frac{1}{2}$ and $\frac{1}{2}$
D. $\frac{1}{2}$ and 1
E. $-\frac{1}{2}$ and 1

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3

The graph of the relation $f(x) = \dfrac{|x-1|}{x^2-1}$ does **not** have a

A. domain of $R\backslash\{-1, 1\}$.

B. horizontal asymptote.

C. vertical asymptote.

D. vertical axis intercept.

E. local maximum turning point.

Question 4

If $z = a + bi$, where $a, b \in R$ and $a > b > 0$, then $\text{Arg}\left(z + i\bar{z}\right)$ is equal to

A. $-\dfrac{3\pi}{4}$

B. $-\dfrac{\pi}{4}$

C. $-\dfrac{3\pi}{4}$ or $\dfrac{\pi}{4}$

D. $\dfrac{\pi}{4}$

E. $\dfrac{5\pi}{4}$

Question 5

$z = 2 + 4i$ is a root of the equation $z^3 + pz^2 + qz + 5 = 0$, where $p, q \in R$.

The real root of the equation is

A. -4

B. $-\dfrac{1}{4}$

C. $\dfrac{1}{4}$

D. 4

E. 20

Question 6

The set S consists of all points on the complex plane that are equidistant from the points represented by $1 + 2i$ and $-2 + i$.

The set S expressed in Cartesian form is

A. $y = -\frac{1}{3}x + \frac{4}{3}$

B. $y = \frac{1}{3}x + \frac{5}{3}$

C. $|z - 1 - 2i| = |z + 2 - i|$

D. $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{5}{2}$

E. $y = -3x$

Question 7

The definite integral $\int_0^{\frac{\pi}{2}} \left(\cos^3(x)\sin^2(x)\right) dx$ can be expressed as

A. $\int_0^1 \left(u^4 - u^2\right) du$, where $u = \sin(x)$

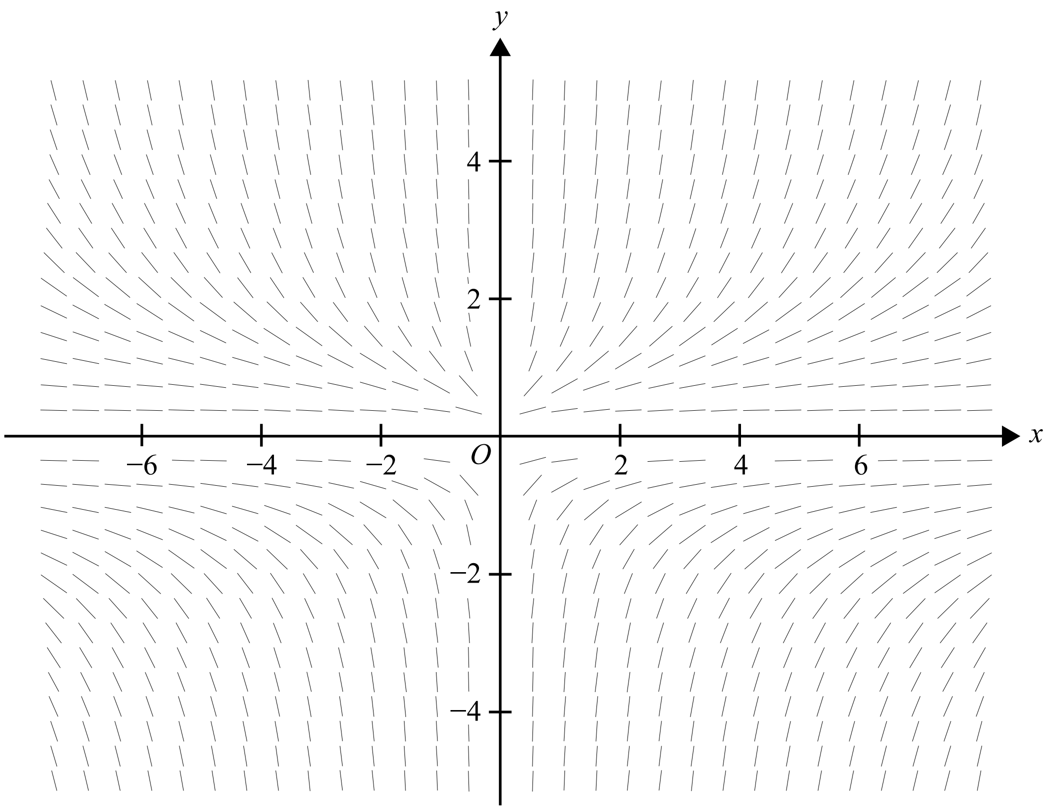
B. $\int_0^{\frac{\pi}{2}} \left(u^2 - u^4\right) du$, where $u = \sin(x)$

C. $\int_0^1 \left(u^2 - u^4\right) du$, where $u = \sin(x)$

D. $\int_1^0 \left(u^3 - u^5\right) du$, where $u = \cos(x)$

E. $\int_0^{\frac{\pi}{2}} \left(u^3 - u^5\right) du$, where $u = \cos(x)$

Question 8



The direction field shown above best represents the differential equation

A. $\dfrac{dy}{dx} = \dfrac{y}{x}$

B. $\dfrac{dy}{dx} = \dfrac{y^2}{x^2}$

C. $\dfrac{dy}{dx} = \dfrac{y^2}{x}$

D. $\dfrac{dy}{dx} = \dfrac{y}{x^2}$

E. $\dfrac{dy}{dx} = -\dfrac{y}{x^2}$

Question 9

The length L of a curve from $x = 1$ to $x = 5$ is given by $\int_1^5 \sqrt{1+\frac{1}{4x^4}}\,dx$.

The equation of the curve could be

A. $y = \frac{1}{2x^2}$

B. $y = -\frac{1}{x^3}$

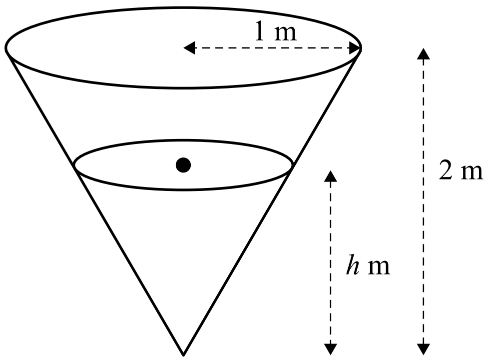
C. $y = -\frac{1}{12x^3}$

D. $y = -\frac{1}{2x}$

E. $y = -\frac{1}{144x^6}$

Question 10

An inverted right circular cone of radius 1 m and height 2 m is initially full of water, as shown below.



Water flows out through a hole at the bottom at a rate of $0.5\sqrt{h}$ m^3 h^{-1}, where h metres is the depth of water remaining in the cone after t hours.

At time t hours, $\frac{dh}{dt}$ is given by

A. $-\frac{\pi h^{2.5}}{8}$

B. $-\frac{\pi h^{1.5}}{2}$

C. $\frac{4}{\pi h^2}$

D. $\frac{2}{\pi h^{1.5}}$

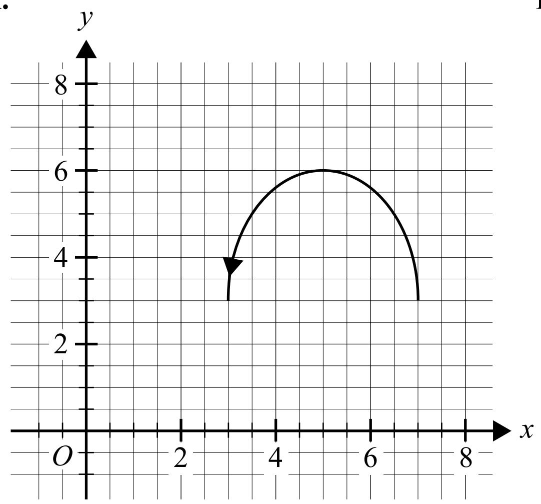
E. $-\frac{2}{\pi h^{1.5}}$

Question 11

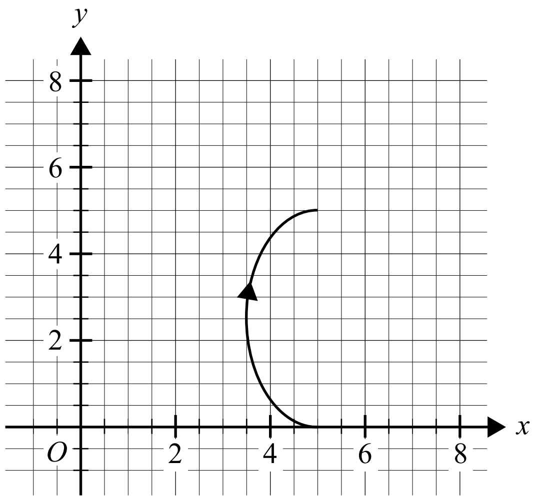
The position of a moving particle at time t is given by $\underset{\sim}{r}(t) = (5 + 2\cos(t))\underset{\sim}{i} + 3(1 + \sin(t))\underset{\sim}{j}$, where $\frac{\pi}{2} \le t \le \frac{3\pi}{2}$.

The graph that shows the path and the direction taken by the moving particle is

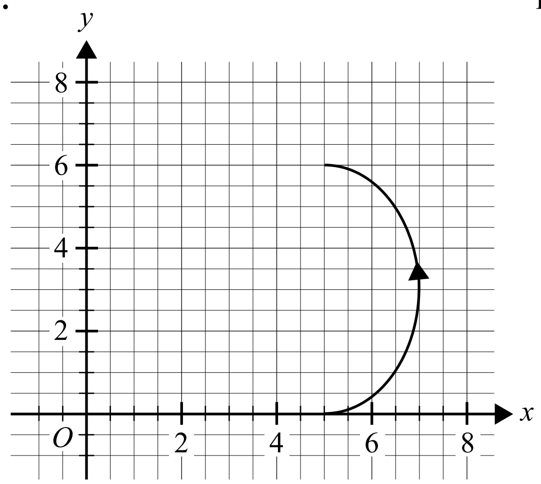
A.



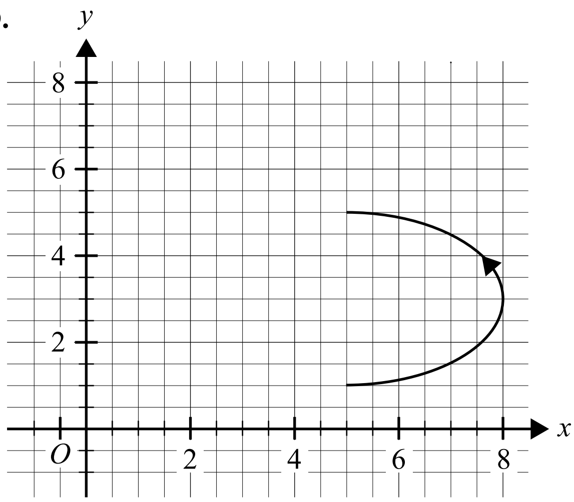
B.



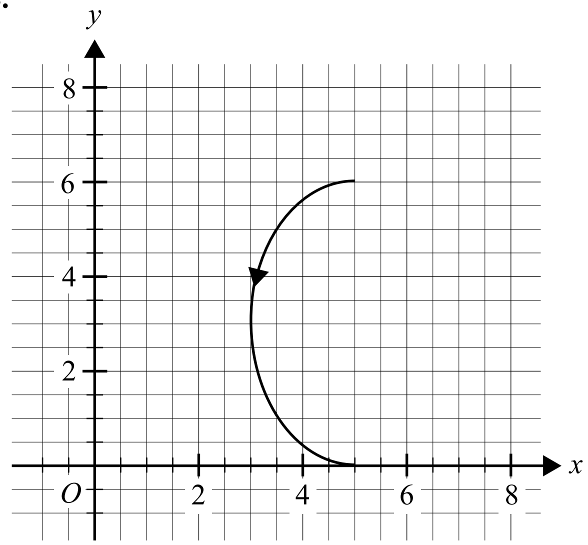
C.



D.



E.



Question 12

The position of a particle at time t seconds is given by $\underset{\sim}{r}(t) = \sin(3t)\underset{\sim}{i} - 2\cos(3t)\underset{\sim}{j}$, where displacement components are measured in metres.

The maximum speed of the particle, in ms^{-1}, is

A. 3

B. $3\sqrt{3}$

C. 6

D. $3\sqrt{5}$

E. 9

Question 13

The vector resolute of $\underset{\sim}{a} = 4\underset{\sim}{i} + 3d\underset{\sim}{j} - d\underset{\sim}{k}$, where $d \in R$, in the direction of a non-zero vector $\underset{\sim}{b}$ is $2\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$.
The value of d is

A. -1

B. $-\frac{1}{5}$

C. $\frac{1}{3}$

D. 1

E. 5

Question 14

The angle between the vector $\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ and the positive direction of the z-axis is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\cos^{-1}\left(\frac{-4}{\sqrt{6}}\right)$

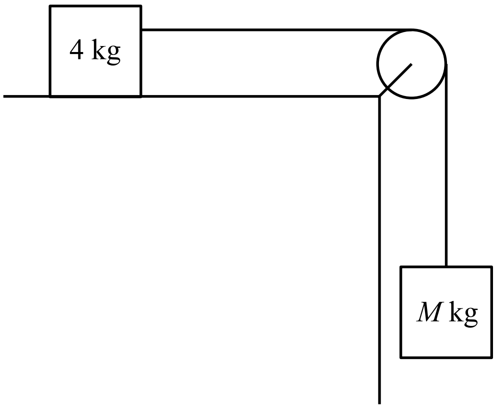
D. $\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$

E. $\cos^{-1}\left(\frac{4}{\sqrt{6}}\right)$

DO NOT WRITE IN THIS AREA

Question 15

Two masses of 4 kg and M kg are connected by a light inextensible string over a smooth pulley. The 4 kg mass rests on a smooth horizontal surface. Initially the system is held at rest but, when released, the 4 kg mass accelerates at 1.5 ms^{-2} to the right.



The value of M is closest to

A. 0.6
B. 0.7
C. 1.2
D. 5.0
E. 6.0

Question 16

A mass of 5 kg sitting at rest on a smooth horizontal surface is acted on by a horizontal force of 10 N.

How far, in metres, will the mass move from its original position in 3 seconds?

A. 2.25
B. 3
C. 4.5
D. 9
E. 45

Question 17

A particle of mass m kilograms moving in a horizontal straight line is acted on by a horizontal variable force.

The velocity of the particle when it is x metres from a fixed point, O, is given by $\sqrt{\tan^{-1}(x)}$ ms^{-1}.

If the force acting on the particle is 1.5 N when it is 1 m from O then, when the force acting is 1 N, the distance of the particle from O would be

A. 1 m
B. $\sqrt{2}$ m
C. $\sqrt{3}$ m
D. 2 m
E. 3 m

Question 18

Each night, a student typically spends 100 minutes on mathematics homework and 40 minutes reading. The standard deviations of the times spent on mathematics and reading are 12 minutes and 5 minutes respectively.

Assuming that the times spent on the two tasks are independent and normally distributed, the probability that the student spends less than a total of 120 minutes on mathematics and reading on a particular night is closest to

A. 0.06

B. 0.12

C. 0.45

D. 0.94

E. 0.99

Question 19

A random sample of 50 passengers flying from Melbourne to Brisbane has a mean mass of 73 kg and a standard deviation of 8 kg.

Assuming that the standard deviation obtained from the sample is a sufficiently accurate estimate of the population standard deviation, an approximate 95% confidence interval for the mean mass of all like passengers is closest to

A. (70.8, 75.2)

B. (71.0, 75.0)

C. (72.2, 73.8)

D. (72.7, 73.3)

E. (72.8, 73.2)

Question 20

X and Y are independent random variables, each of which has a mean, m, and variance, v.

The standard deviation of the random variable Z, where $Z = pX + qY$, is given by

A. $\sqrt{v^2\left(p^2+q^2\right)}$

B. $\sqrt{v\left(p^2+q^2\right)}$

C. $v\sqrt{p+q}$

D. $(p+q)\sqrt{v}$

E. $\sqrt{v(p+q)}$

END OF SECTION A

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

SECTION B

Instructions for Section B

Answer **all** questions in the spaces provided.
Unless otherwise specified, an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$

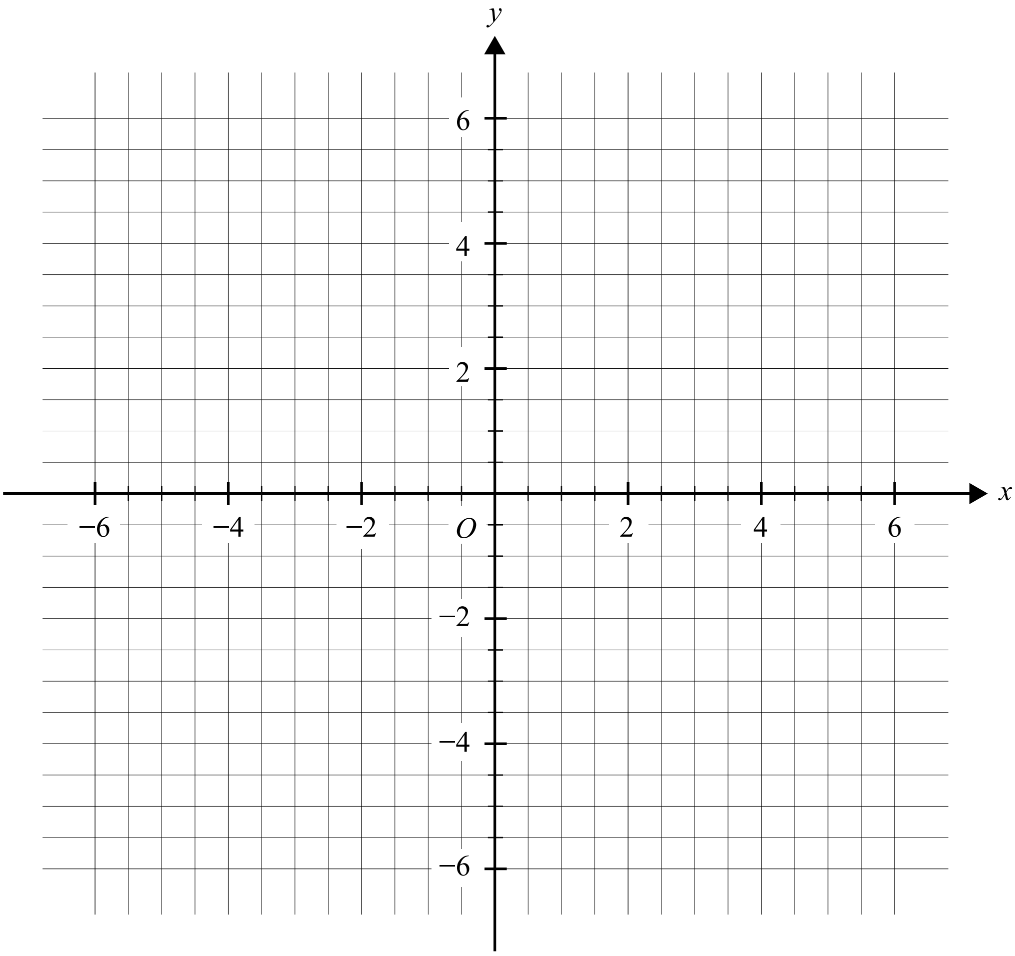
Question 1 (9 marks)

Let $f(x) = \dfrac{4x^2 + 2x + 7}{2x^2 - 3x - 2}$.

a. Express $f(x)$ in the form $A + \dfrac{Bx + C}{2x^2 - 3x - 2}$, where A, B and C are real constants. 1 mark

b. State the equations of the asymptotes of the graph of f. 2 marks

c. Sketch the graph of f on the axes below. Label the asymptotes with their equations and label any stationary and inflection points with their coordinates, correct to two decimal places. Label any intercepts with the coordinate axes. 3 marks



d. Let $g_k(x) = \dfrac{4x^2 + 2x + 7}{2x^2 - 3x + k}$, where k is a real constant.

For what values of k will the graph of g_k have

i. one vertical asymptote 1 mark

ii. no vertical asymptotes? 1 mark

e. The graph of g_2, where $g_2(x) = \dfrac{4x^2 + 2x + 7}{2x^2 - 3x + 2}$, is rotated about the x-axis from $x = 0$ to $x = 5$ to form a solid of revolution.

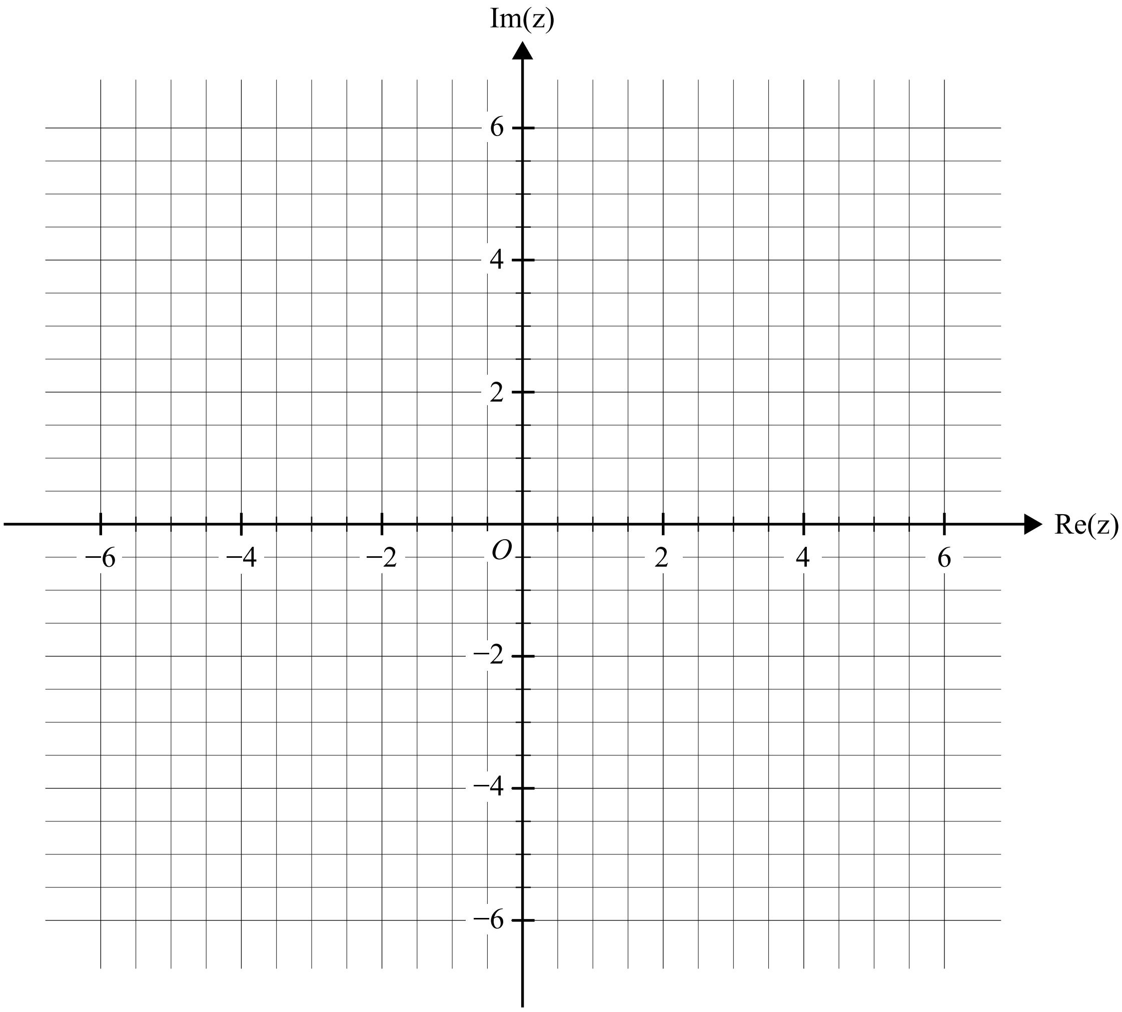
Write down a definite integral that gives the volume of the solid formed. An evaluation of this integral is not required. 1 mark

Question 2 (11 marks)

The circle given by $\left|z+2-i\sqrt{3}\right|=2\sqrt{3}$, where $z \in C$, has its centre at $\left(-2, \sqrt{3}\right)$.

a. Verify by substitution that $z=-5$ is a real axis intercept of the graph of $\left|z+2-i\sqrt{3}\right|=2\sqrt{3}$ and find the other real axis intercept. 2 marks

b. Sketch the circle given by $\left|z+2-i\sqrt{3}\right|=2\sqrt{3}$ on the Argand diagram below. Label the axes intercepts with their values. 3 marks



c. The line given by $\left|z+2-i\sqrt{3}\right|=\left|z-z_0\right|$, where $z, z_0 \in C$, is tangent to the circle given by $\left|z+2-i\sqrt{3}\right|=2\sqrt{3}$ at $z=-5$.

Find z_0 in the form $a+ib$, where $a, b \in R$. 1 mark

d. Find the area of the part of the circle given by $\left|z+2-i\sqrt{3}\right|=2\sqrt{3}$, which lies below the real axis. Give your answer in the form $p\pi+q$, where $p, q \in R$. 3 marks

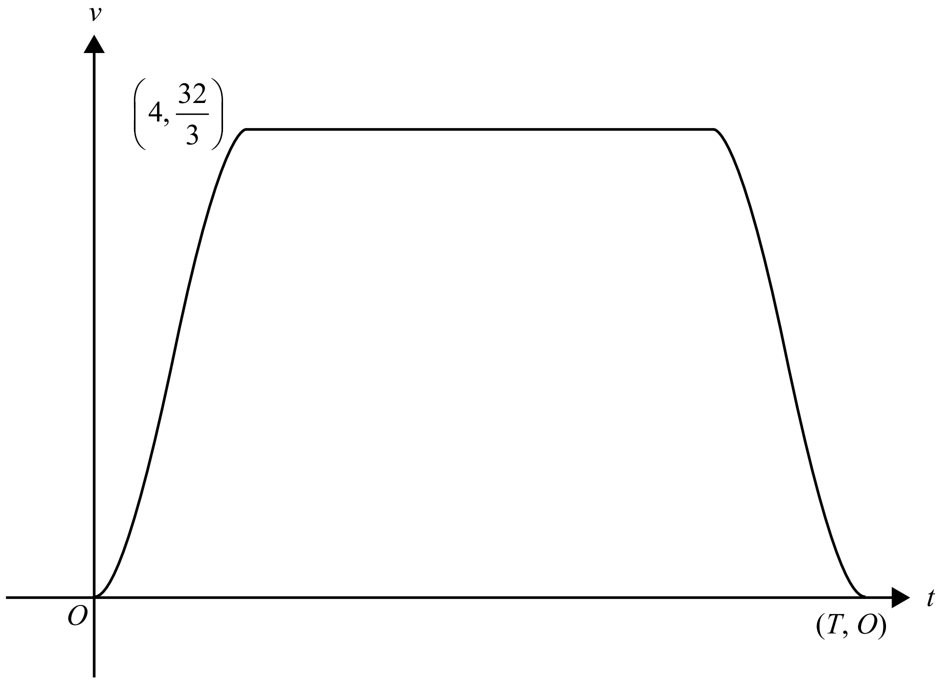
e. A second circle is given by $\left|\overline{z}+2-i\sqrt{3}\right|=2\sqrt{3}$, where $z \in C$.

Sketch this circle on the axes provided in **part b.** You do not need to label the axes intercepts. 2 marks

DO NOT WRITE IN THIS AREA

Question 3 (10 marks)

The velocity–time graph below shows the velocity of an express lift as it travels from the ground floor to the top floor of a city building. The displacement of the lift from the ground floor is measured in metres, and its velocity, v, after travelling for t seconds is measured in ms^{-1}.



For the first 4 seconds of motion, the velocity of the lift is given by $v = 2t^2\left(1 - \frac{t}{6}\right)$.

After the first 4 seconds, the lift travels at a constant velocity of $\frac{32}{3}$ ms^{-1} until it slows down to stop at the top floor, having travelled for T seconds. The velocity–time graph is symmetrical about $t = \frac{T}{2}$.

a. Show that the maximum acceleration of the lift during the first 4 seconds occurs when $t = 2$ and that the magnitude of this acceleration is 4 ms^{-2}. 2 marks

DO NOT WRITE IN THIS AREA

b. Given that the lift travels 96 m from the ground floor to the top floor, find T. 3 marks

c. Find a relation that gives the velocity of the lift, v, in terms of t for the last 4 seconds of motion. 2 marks

d. A 60 kg student stands in the lift.

Find, in newtons, the minimum and maximum reaction forces that the floor of the lift exerts on the student during the journey from the ground floor to the top floor. 3 marks

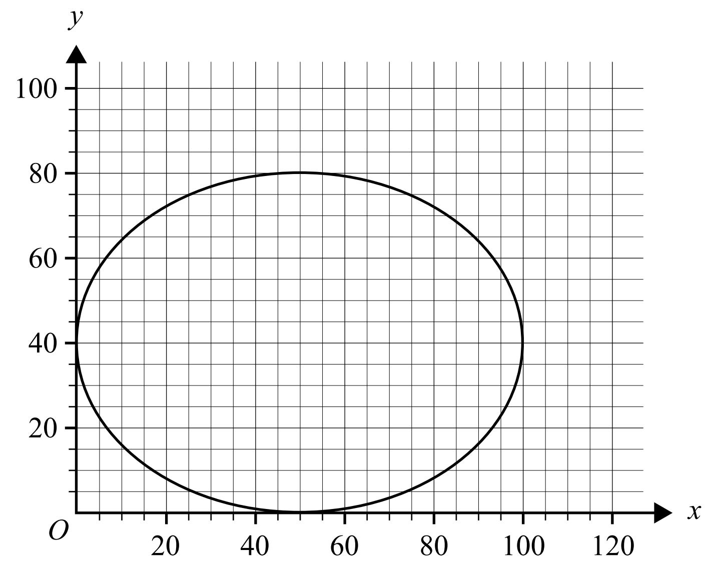
Question 4 (11 marks)

Larry goes for a morning swim in a river that flows into the ocean. He swims once around a closed path. His position vector, $\underset{\sim}{\text{r}}_{\text{L}}(t)$, relative to an origin, O, after t minutes, is given by $\underset{\sim}{\text{r}}_{\text{L}}(t) = \left(50 + 50\sin(t)\right)\underset{\sim}{\text{i}} + \left(40 + 40\cos(t)\right)\underset{\sim}{\text{j}}$, for $0 \le t \le 2\pi$, where the displacement components are measured in metres.

a. Find the Cartesian equation of the closed path using $\underset{\sim}{\text{r}}_{\text{L}}(t)$ above. 1 mark

b. The graph of the closed path is shown on the set of axes below.

Label Larry's starting point with its coordinates and show the direction in which he swims. 2 marks



c. **i.** Find the distance that Larry travels when he swims once around the closed path. Give your answer in metres, correct to two decimal places. 1 mark

ii. Find Larry's average speed when he swims once around the closed path. Give your answer in metres per minute, correct to one decimal place. 1 mark

d. **i.** Express Larry's speed at time t in the form $\sqrt{a + bf(t)}$, where $a, b \in R$. 1 mark

ii. Find Larry's maximum speed, in metres per minute, and give the coordinates of the points at which he reaches this maximum speed. 2 marks

e. On a particular morning, a dolphin is seen in the river.

The position of the dolphin, $\underset{\sim}{r}_D(t)$, at time, t minutes, is given by $\underset{\sim}{r}_D = (120 - at)\underset{\sim}{i} + \left(10 + t^2\right)\underset{\sim}{j}$, relative to the origin, O, where $t \geq 0$, a is a positive real constant and the displacement components are measured in metres.

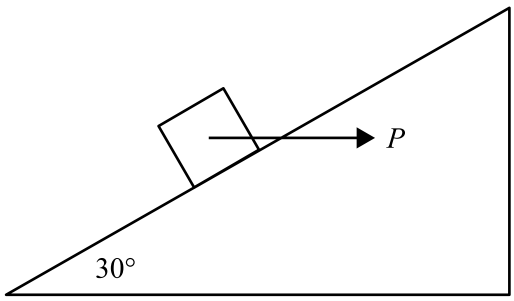
For what values of a will Larry meet the dolphin? Give your answers correct to two decimal places. 3 marks

DO NOT WRITE IN THIS AREA

Question 5 (10 marks)

A mass of 2 kg sits on a smooth plane inclined at 30° to the horizontal. A light inextensible horizontal string is attached to the mass.

a. Initially, the mass is held in equilibrium by the string, which has a tension force of magnitude P newtons. The situation is shown on the diagram below.



i. On the diagram above, show and clearly label any other forces acting on the mass. 1 mark

ii. Find P. 2 marks

The magnitude of the tension in the horizontal string is changed to T newtons. As a result, the mass accelerates up the inclined plane at $\frac{g}{4}$ ms^{-2}.

b. Find T in terms of g. 2 marks

c. After travelling 2 m up the inclined plane, the mass is at the top of the plane. At this instant the mass leaves the inclined plane, the string is released and the mass travels through the air. Assume that air resistance is negligible.

i. Show that the speed of the mass at the instant that it leaves the top of the plane is $\sqrt{g}$ ms^{-1}. 1 mark

ii. The top of the inclined plane is 1.5 m vertically above the horizontal ground.

Find the maximum height above the ground, in metres, reached by the mass as it travels through the air. 2 marks

iii. Find the speed of the mass, in ms^{-1}, when it hits the ground. Give your answer in terms of g. 2 marks

DO NOT WRITE IN THIS AREA

Question 6 (9 marks)

A manufacturer produces bottles of mineral water. The volume of mineral water in each bottle is normally distributed with a claimed population mean of 750 mL and a known standard deviation of 2 mL. Quality control inspectors take random samples of 10 bottles.

a. For samples of 10 bottles, state the mean of the sampling distribution and show that the standard deviation of the sampling distribution is $\frac{\sqrt{10}}{5}$ mL. Assume that the mean volume of mineral water in all bottles produced is 750 mL as claimed. 1 mark

b. The inspectors will query the manufacturer's claim that the mean volume of mineral water in all bottles produced is 750 mL if the mean volume in 10 randomly selected bottles is less than 749 mL.

i. What is the probability that the inspectors will query the manufacturer's claim? Give your answer correct to four decimal places. Assume that the mean volume of mineral water in all bottles produced is 750 mL. 1 mark

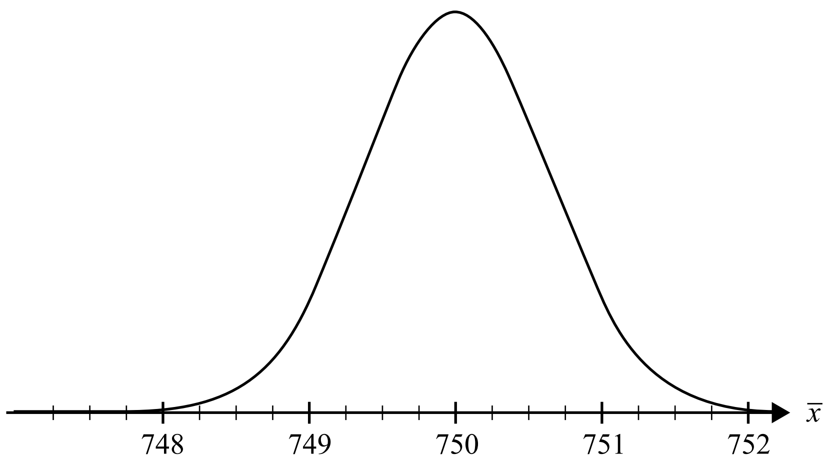
ii. What is the probability that the inspectors will **not** query the manufacturer's claim if the mean volume of mineral water in all bottles produced is in fact 749.5 mL and not the claimed 750 mL? Give your answer correct to four decimal places. 1 mark

DO NOT WRITE IN THIS AREA

c. The graph shown below represents the sampling distribution of the mean for samples of 10 bottles, where the population mean is 750 mL and the standard deviation is 2 mL.

i. On the axes below, for samples of 10 bottles, sketch the graph of the sampling distribution if the population mean is in fact 749.5 mL and the standard deviation is 2 mL. 1 mark

ii. On the axes below, shade the area that represents the probability found in **part b.ii.** 1 mark



The inspectors decide to apply a one-sided statistical test at the 5% level of significance to determine whether to reject or not reject the manufacturer's claimed mean of 750 mL.

d. Write down suitable null and alternative hypotheses for this test. 1 mark

In applying the statistical test, the inspectors find that the mean volume of mineral water in a random sample of 10 bottles is 748.9 mL. Assume that the mean for the population is 750 mL and the standard deviation is 2 mL.

e. **i.** Find the p value for the test, correct to two decimal places. 1 mark

ii. Use the p value found in **part e.i.** to justify an appropriate conclusion about the claim. 1 mark

f. For samples of 10 bottles, find the smallest sample mean above which the inspectors would not reject the claim of 750 mL at the 5% level of significance. Give your answer in millilitres, correct to two decimal places. 1 mark

END OF QUESTION AND ANSWER BOOK

VICTORIAN CURRICULUM
AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2023

SPECIALIST MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Specialist Mathematics formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1, 1]$	$[-1, 1]$	R
Range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r(\cos(\theta) + i\sin(\theta)) = r\,\text{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \text{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$
$z^n = r^n \,\text{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\text{E}(aX + b) = a\text{E}(X) + b$ $\text{E}(aX + bY) = a\text{E}(X) + b\text{E}(Y)$ $\text{var}(aX + b) = a^2\text{var}(X)$
for independent random variables X and Y	$\text{var}(aX + bY) = a^2\text{var}(X) + b^2\text{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\overline{X}$	mean $\text{E}(\overline{X}) = \mu$ variance $\text{var}(\overline{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e \lvert x \rvert + c$
$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$	$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$	$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$	$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c,\ n \neq -1$
	$\int (ax+b)^{-1} dx = \frac{1}{a}\log_e \lvert ax+b \rvert + c$
product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1 + \left(f'(x)\right)^2}\, dx$ or $\int_{t_1}^{t_2} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert \underset{\sim}{r} \rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET