2007 Further Mathematics GA 2: Written examination 1

GENERAL COMMENTS

As in 2006, the majority of Further Mathematics students appeared to be well prepared for examination 1 in 2007, with the average marks for the Core – Data analysis and each of the modules exceeding 50 per cent. The number of students who sat for Further Mathematics Examination 1 in 2007 was 25,644 compared to 24,472 in 2006 (note that the figure given in the 2006 assessment report was incorrect).

SPECIFIC INFORMATION

The tables below indicate the percentage of students who chose each option. The correct answer is indicated by shading.

Section A

Core

Question	% A	% B	% C	% D	% E	% No Answer
1	91	2	2	1	3	0
2	0	82	13	4	0	0
3	72	5	3	6	13	0
4	2	13	18	61	6	0
5	11	15	66	3	5	0
6	1	9	75	6	9	0
7	13	6	55	23	3	0
8	6	16	21	51	5	0
9	26	9	9	49	7	1
10	23	17	12	10	37	0
11	1	1	3	5	91	0
12	10	22	37	17	14	1
13	7	11	16	11	53	1

The Core section was generally well done, although Questions 10 and 12 were poorly answered.

Question 10 assessed the second dot point in the topic 'Displaying and describing relationships in bivariate data' (see page 121 of the *Mathematics VCE Study Design*). The fact that only 37 per cent of students correctly answered this relatively straightforward question suggests that this dot point may require further emphasis in the classroom.

Question 12 required students to determine the slope of a three median line fitted to a scatterplot. This is a standard technique, but one which students clearly find difficult. Only 37 per cent of students obtained the correct answer, 167 (option C). The key to successfully answering Question 13 was to correctly locate the coordinates of the median points of the bottom one third of data points and top one third of data points and then use these points to determine the slope.

- From the graph, for the bottom one third of data points, the median point is seen to lie midway between the data points for month 6 and month 7. It has the coordinates (6.5, 3000).
- From the graph, for the top one third of data points, the median point is seen to lie midway between the data points for month 30 and month 31. It has the coordinates (30.5, 7000).
- The slope of the three median line is then given by $\frac{7000-3000}{30.5-6.5} = 166.67$, correct to two decimal places, which is closest to 167 (option C).

A common mistake was to use month 6 and month 30 to determine the median, giving a slope closest to 146 (option B).

Section B
Module 1: Number patterns and applications

Question	% A	% B	% C	% D	% E	% No Answer
1	3	78	1	17	1	0
2	0	3	73	16	8	0
3	13	71	7	4	5	1
4	7	15	56	6	15	0
5	15	6	10	11	58	1
6	4	85	3	5	3	0
7	12	16	6	8	58	0
8	7	5	5	73	9	0
9	53	13	17	10	7	1

This module was well done, with no question causing particular difficulties.

Module 2: Geometry and trigonometry

Question	% A	% B	% C	% D	% E	% No Answer
1	75	5	14	2	4	0
2	6	79	6	8	1	0
3	13	66	9	8	3	1
4	66	4	12	15	2	1
5	4	18	14	58	7	0
6	12	25	27	28	7	1
7	17	29	46	4	3	1
8	11	23	8	46	12	0
9	18	16	11	17	37	1

This module was generally well done with the exception of Questions 5, 6 and 9.

Only 18 per cent of students correctly answered Question 5. In this question, students were given the actual area of a block of land. They were also given its area on a map. From this information a scale factor for area ($k^2 = 400$) can be determined. The majority of students (58 per cent) apparently obtained this area scale factor but then incorrectly applied it directly to scaling the given length rather than first converting it into the corresponding linear scale factor, $k = \sqrt{400} = 20$.

Question 6 involved finding the total surface area of a solid. While only 28 per cent of students successfully answered the question, most students had started correctly but either failed to include the base of the solid (the 27 per cent who chose option C) or focussed solely on the hemispherical surface (the 25 per cent who chose option B). To ensure that they include **all** of the surfaces involved, students might find it helpful to begin the solution to questions such as this by writing down a statement like: Total surface area = surface area of the hemispherical bowl + surface area of the side of the cylinder + surface area of the base of the cylinder.

In Question 9, students were given three alternative methods for calculating a length in a bearings problem. They were then asked to identify which of the three solutions were correct. The fact that one or more methods could be correct was signalled in the options offered. To answer this question, students were required to draw a diagram and then carefully assess all of the alternatives offered before arriving at a solution. As it turned out, all three alternatives given were valid methods (option E), but this option was chosen by only 37 per cent of students. The relatively even distribution of student responses over the other incorrect options suggests that most students either failed to systematically test all of the alternative methods, or failed to get started and just guessed.

Module 3: Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	4	3	4	71	18	0
2	4	4	86	2	4	0
3	1	3	1	0	94	0
4	8	75	7	7	2	1
5	10	13	12	60	5	1
6	2	43	15	6	34	0
7	8	18	26	21	27	1
8	8	59	15	9	9	1
9	53	15	12	10	8	1

This module was well done, with the exception of Questions 6 and 7.

In Question 6, the correct answer of $x \geq 2y$ (option E) was obtained by only 34 per cent of students. The most popular, but incorrect, choice made by 43 per cent of students was $y \leq 2x$ (option B). In terms of the problem at hand, this inequality directly translates to, 'the number of bottles of white wine is less than or equal to twice the number of bottles of red wine'. This is a true statement but is not the constraint specified.

Question 7 proved to be challenging for the vast majority of students. The key to answering this question was to recognise that, in the linear plot, the coordinates (3, 1) represented the values of x^3 and y respectively not of x and y. A possible approach is as follows.

- The equation of a linear graph of y plotted against x^3 as shown is $y = kx^3$.
- The coordinates on the graph tell us that when $x^3 = 3$, $y = 1$ so that $1 = k \times 3$ or $k = \frac{1}{3}$, so $y = \frac{1}{3}x^3$.
- When y is plotted against x (in the first quadrant), $y = \frac{1}{3}x^3$ is a cubic curve with the shape shown in all options A to E.
- The next step is to determine which of the coordinates shown lies on the curve $y = \frac{1}{3}x^3$. Inspection, or systematic testing, shows that when $x = 1$, $y = \frac{1}{3} \times 1^3 = \frac{1}{3}$. Thus the graph shown in option C is correct.

Module 4: Business-related mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	79	6	3	10	1	0
2	19	8	64	6	3	1
3	17	4	68	8	2	0
4	6	14	9	63	8	1
5	2	8	8	6	75	1
6	17	10	9	42	21	1
7	13	42	14	8	23	1
8	16	30	11	36	6	1
9	37	11	16	27	8	1

This module was well done, with the exception of Questions 7, 8 and 9.

Question 7 was very poorly done with only 23 per cent of students obtaining the correct solution of \$39 977 (option E). A possible solution approach is as follows.

- Let Joe's salary two years ago be x.
- After a year of 2% inflation followed by a year of 3% inflation, his salary is \$42 000.
- Therefore, $42000 = (x \times 1.02) \times 1.03$ **or** $x = \frac{42000}{1.02 \times 1.03}$

An incorrect solution strategy, which was used by the 42 per cent of students who chose option B, was to first depreciate his current salary of \$42 000 by 3% and then by 2% to give the answer \$39 925.20 ($42\,000 \times 0.97 \times 0.98 = 39\,925.20$). The error here is that, for example, a 3% depreciation of this year's salary is not equal to a 3% increase in the previous year's salary. This is because the percentages are applied to different bases.

Question 8 was based on a two-year hire purchase agreement. The correct answer was \$27.90 (option D). With a success rate of 36 per cent, this question was not well done. However, another 30 per cent of students started correctly, but only took into account one year's interest, not two, to obtain the answer \$25.20 (option B).

As expected, Question 9 was challenging and only 37 per cent of students arrived at the correct solution of \$215 000 (option A). This question required students to determine the total amount of interest paid when repaying a loan over 20 years. The key to answering this question was to realise that:

total interest paid = total amount repaid over 20 years – the amount borrowed
= number of monthly repayments × amount of each repayment – 250 000

A TVM solver can then be used to determine the monthly repayments, giving \$1938.2473. Therefore, the total interest paid = $240 \times 1938.2473 - 250\,000 = 215\,179.352$, or around \$215 000.

Module 5: Networks and decision mathematics

Question	% A	% B	% C	% D	% E	% No Answer	Comments
1	7	37	7	5	44	0	
2	5	7	76	7	5	0	
3	93	2	2	1	2	0	
4	1	66	28	2	4	0	
5	14	11	29	43	2	0	
6	21	7	10	42	19	0	An arrow was not shown on the dummy activity for Question 6 on the original paper as is the normal convention. Due to this, credit was given to students who chose either option D (using a down arrow) or option E (using an up arrow).
7	13	18	54	8	6	1	
8	22	65	5	5	2	0	
9	20	37	20	17	6	0	

As usual, this module was well done. However, Question 9, with a success rate of 37% per cent, proved to be difficult for the majority of students.

The context for Question 9 was a volleyball competition, with five teams who play each other once each. Students were asked to determine the number of extra games that needed to be scheduled if two more teams joined the competition. As well as trial and error, two systematic approaches could be used to obtain a solution. The key to both approaches is that the games played in the competition can be represented by a complete graph. In this graph, a game corresponds to an edge joining two vertices.

Approach 1
First add one new team (the sixth) to the competition. This is equivalent to adding one extra vertex to the graph. The graph now has six vertices but is incomplete; five new edges (games) must be added to the graph to make it complete. Now add an extra team (the seventh) to the competition. The graph now has seven vertices, but is incomplete; six new edges (games) must be added to this graph to make it complete. Thus, 11 extra games (5 + 6 = 11 – option B) must be scheduled to ensure that every one of the seven teams plays each other in the competition.

Approach 2

- A complete graph with n vertices has $\frac{n(n-1)}{2}$ edges.
- A complete graph with five vertices (a competition with five teams) has $\frac{5(5-1)}{2} = 10$ edges.
- A complete graph with seven vertices (a competition with seven teams) has $\frac{7(7-1)}{2} = 21$ edges.

- Therefore, 11 (21 – 10) extra games must be scheduled.

Module 6: Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	97	1	1	0	0	0
2	0	3	1	93	2	0
3	2	65	8	2	22	1
4	26	64	3	4	3	0
5	51	12	14	21	1	0
6	17	15	10	51	6	1
7	33	4	7	8	48	1
8	8	48	19	14	11	1
9	23	9	50	12	4	1

This module was well done with no questions proving to be particularly difficult. However, a diagrammatic approach to answering Question 8 is useful when analysing the properties of a transition matrix.

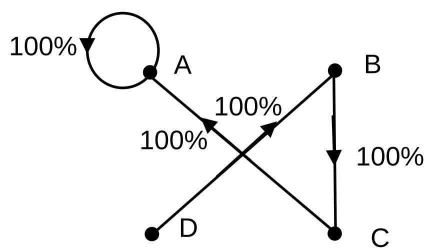
In Question 8, a student completes a six-question multiple-choice test. Each question has four options, *A*, *B*, *C* and *D*. The student chooses *D* for his answer to the first question and then uses the transition matrix shown to guide his choice of answers for the remaining five questions.

Transition matrix

This question

$$\text{Next question} \quad \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

Diagrammatical representation



From the diagram, and by following the arrows, we see that an answer of *D* to the first question leads to an answer of *B* for the second, *C* for the third, *A* for the fourth, and then *A* for all subsequent questions.

Thus his answers to the six questions will be *D B C A A A*. This corresponds to option B.