2009 Further Mathematics GA 2: Written examination 1

GENERAL COMMENTS

As in 2008, the majority of students appeared to be well prepared for examination 1 in 2009, with the average proportion of correct responses in the Core – Data Analysis and each of the modules exceeding 50 per cent. The number of students who sat for Further Mathematics Examination 1 in 2009 was 26 256, compared to 25 769 in 2008.

SPECIFIC INFORMATION

The tables below indicate the percentage of students who chose each option. The correct answer is indicated by shading.

Section A
Core: Data Analysis

Question	% A	% B	% C	% D	% E	% No Answer
1	3	1	1	94	1	0
2	4	85	6	3	1	0
3	85	10	2	1	1	0
4	8	6	3	4	79	0
5	1	1	4	90	4	0
6	9	43	14	21	13	0
7	2	10	14	69	4	0
8	3	4	3	2	88	0
9	49	22	10	13	6	0
10	70	6	6	9	9	0
11	4	11	9	28	47	0
12	8	15	59	10	7	0
13	13	17	19	40	10	1

All except four questions on the Core material were answered correctly by more than 50 per cent of students. Seven questions were answered correctly by 70 per cent or more of students.

Question 6 involved matching a boxplot with a given histogram and 43 per cent of students were able to answer correctly. To successfully complete this task, students needed to recognise that a boxplot is a graphical display of the five-number summary of a data set, namely, the minimum value, the first quartile (Q_1), the median (M), the third quartile (Q_3) and the maximum value. As all boxplots had whiskers extending to the same minimum and maximum values, a systematic approach to this question would have been to estimate the values of the median and the first and third quartiles, then look for a match (option B). Students who attempted to answer the question purely by inspection would have found it difficult to obtain the correct answer.

Question 9 had a success rate of 49 per cent. A further 22 per cent of students chose option B, making the common error of automatically assuming (incorrectly in this case) that the first variable appearing in the table was the independent variable.

In Question 11, students were given the body weights and brain weights of nine animals, and the equation to a least squares regression line determined from this data. The correct response was chosen by 47 per cent of students. Students were expected to first use the equation of the regression line to determine the predicted brain weight for the designated animal (the baboon) and then use this value, along with the animal's actual brain weight, to determine the residual value (the error of prediction). A significant number of students, 28 per cent, correctly completed the first part of the task – correctly calculating the predicted brain weight – but did not go any further and this led them to choose option D.

To answer Question 13, students were required to determine the slope of a three median line when fitted to a scatterplot. This is a standard technique, but one which students clearly find difficult. Only 40 per cent of students chose the correct answer, 0.45% (option D). The key to successfully answering Question 13 was to correctly locate the coordinates of the median points of the bottom one third of data points and top one third of data points. These two points are then used to determine the slope of the three median line.

From the graph, for the bottom one third of data points, it can be seen that the median point is the data point for 1990. It has the co-ordinates (1990, 0.6).

From the graph, for the top one third of data points, it can be seen that the median point is the data point for 1996. It has the co-ordinates (1996, 3.3).

The slope of the three median line is then given by $\frac{3.3-0.6}{1996-1990}=\frac{2.7}{6}$ = 0.45 (option D).

Section B
Module 1: Number patterns

Question	% A	% B	% C	% D	% E	% No Answer
1	2	4	91	2	0	0
2	91	3	1	4	0	0
3	7	80	5	6	2	0
4	68	13	5	5	8	0
5	6	11	6	52	23	1
6	28	22	8	9	33	1
7	12	19	62	4	3	1
8	6	24	34	28	7	1
9	12	25	25	26	11	1

Questions 5, 6, 8 and 9 were not well answered in 2009.

Only 23 per cent of students chose the correct response for Question 5 (option E). More than half of the students chose option D, indicating that they had failed to recognise that the solution required them to compute the **sum** of the first five terms of a geometric sequence, not just the fifth **term**.

In Question 6, students were given an expression for the *n*th term in a sequence and asked to identify a difference equation that would generate the same sequence. Only 33 per cent of students gave the correct response $t_{n+1} = t_n - 20$, $t_1 = 80$ (option E). A significant number of students chose either $t_{n+1} = 100 - 2t_n$, $t_1 = 80$ (option A) or $t_{n+1} = 100t_n - 20$, $t_1 = 1$(option B), suggesting that they did not reason their way through the solution. By generating a few terms, it is easy to show that the sequences generated by the difference equations given in options A and B do not generate a sequence for which $t_n = 100 - 20n$ is the *n*th term.

For Question 8, 34 per cent of students chose the correct response (option C). A systematic approach to answering this question could have been:
If, as given, D_n is the number of milligrams of the drug in the patient's body immediately after taking the drug at the start of the *n*th day, then

$D_1 = 15$	[the only drug in the patient's body is 15 mg taken at the start of the first day]
$D_2 = 0.15\,D_1 + 15$	[the amount of drug remaining from day 1 ($0.15\,D_1$) plus the new dose (15 mg)]
$D_3 = 0.15\,D_2 + 15$	[the amount of drug remaining from day 2 ($0.15\,D_2$) plus the new dose (15 mg)]

and so on, to obtain
$D_{n+1} = 0.15\,D_n + 15$

Question 9 was correctly answered by only 26 per cent of students. To correctly answer this question, students had to recognise that the distances between the checkpoints on the orienteering course followed an arithmetic sequence, and that the critical missing information was the first term in this sequence: the distance d_1 between checkpoints 1 and 2.

An approach to answering this question could have been:
The sum of all the distances was 4500, so, using the formula for the sum of an arithmetic sequence,

$$S_9 = \frac{9}{2}\left(2\times d_1 + \left(9-1\right)\times 50\right) = 4500$$

Solving for d_1 gives $d_1 = 300$, so that the distance between checkpoints 2 and 3 is $d_2 = d_1 + 50 = 350$ (option D).

Common errors included:

- not recognising that if there are ten checkpoints then there are nine distances between the checkpoints (25 per cent of students, by choosing option B, appeared to make this mistake)
- correctly calculating d_1, but not adding 50 to obtain the distance between checkpoints 2 and 3 (25 per cent of students, by choosing option C, appeared to make this mistake).

Module 2: Geometry and trigonometry

Question	% A	% B	% C	% D	% E	% No Answer
1	1	2	1	95	1	0
2	2	5	3	86	4	0
3	8	27	7	3	54	0
4	8	69	10	2	11	0
5	9	15	59	11	4	1
6	53	9	8	28	3	0
7	55	11	10	11	12	1
8	12	43	21	14	9	1
9	19	19	27	19	14	1

This module was generally well done, with only two questions, 8 and 9, having a success rate of less than 50 per cent.

In Question 8, students were asked to determine the volume of water contained in a length of hose with a circular cross section and a length of 85 metres. The outside diameter of the hose was 29 millimetres and the walls of the hose were two millimetres thick. The correct answer was option B and was chosen by 43 per cent of students. The most common error, made by 21 per cent of students, was to incorrectly calculate the inside diameter of the pipe as 27 mm (outside diameter – one wall thickness) rather than 25 mm (outside diameter – two wall thicknesses).

Question 9 was not well answered with only 27 per cent of students choosing the correct option (option C). The key to answering this question was to recognise that the maximum height occurs when the pole falls at right angles to the wall.

Module 3: Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	4	92	2	1	1	0
2	2	12	65	19	2	0
3	85	5	2	6	1	0
4	2	93	3	2	1	0
5	2	3	14	7	73	0
6	10	10	12	51	16	1
7	12	11	61	10	5	1
8	8	15	65	8	3	1
9	7	30	12	37	14	1

This module was generally well done with the exception of Question 9, for which only 37 per cent of students chose the correct option (option D).

A possible solution strategy could have been:
Profit = Revenue – Costs = $R - C$
Let S be the selling price of one card. Then, the revenue from selling 150 cards is $R = 150 \times S$
From the graph, the cost C of producing n cards is given by $C = 50 + 2n$, so the cost of producing 150 cards is
$C = 50 + 2 \times 150 = \$350$
The profit from selling 150 cards is \$175, so
$175 = 150 \times S - 350$ or $S = \$3.50$
Profit Revenue Costs

Module 4: Business-related mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	5	91	1	1	3	0
2	4	2	5	80	9	0
3	20	44	16	9	10	1
4	2	3	5	13	76	1
5	6	14	16	54	9	1
6	8	9	16	63	4	1
7	11	67	8	7	6	1
8	45	13	25	11	5	1
9	22	11	45	12	9	1

This module was generally well done, with only three questions (Questions 3, 8 and 9) having a success rate of less than 50 per cent.

Module 5: Networks and decision mathematics

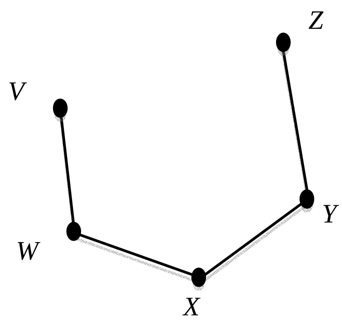
Question	% A	% B	% C	% D	% E	% No Answer
1	2	3	74	4	16	0
2	1	93	3	2	1	0
3	4	44	17	28	6	0
4	68	6	10	9	6	0
5	30	15	7	45	3	0
6	14	9	15	10	52	0
7	5	14	61	14	5	1
8	15	28	15	16	25	1
9	6	18	6	10	58	1

This module was generally well done with the exceptions of Questions 8 and 9.

Only 25 per cent of students chose the correct option (option E) for Question 8. A strategy to answer this question was to construct a simple graph that satisfied the given conditions, and to use this graph to test each of the options in a systematic manner.

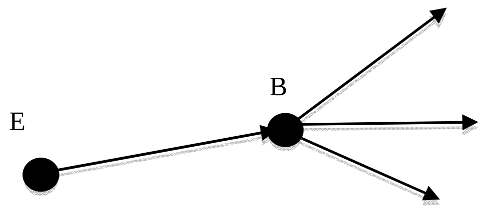
The graph shown is one such graph; it has five vertices, three of even degree (X, Y and W) and two of odd degree (V and Z). From this graph it can be seen that:

- adding an edge between V and Z gives a graph in which all vertices are even and all vertices are of equal degree, so both option A and option B are possible
- adding an edge between W and Y gives a graph with one vertex of even degree and four of odd degree, so option C is possible
- adding an edge between V and X gives a graph with three vertices of even degree and two of odd degree, so option D is possible
- there is no way that a single edge can be added to the graph so that it has four vertices of even degree and one of odd degree, so option E was the correct answer.

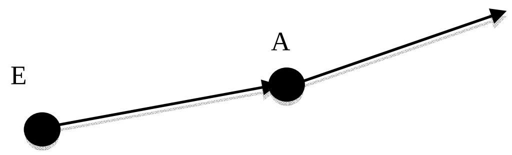


Question 9 was a challenging question. A key to answering Question 9 was to realise that if team A beats team B, then team B's one-step dominances becomes team A's two-step dominances. One solution strategy was to work through the possibilities systematically, starting with option A.

Option A – Elephants defeated Bears
The Elephants have only one one-step dominance plus one two-step dominance. The sub-graph below shows that the implication of the Elephants defeating the Bears is that they will have three two-step dominances, which is not the case. Therefore, the Elephants could not have defeated the Bears.



Option B – Elephants defeated Aardvarks
The sub-graph below shows that one of the implications of the Elephants defeating the Aardvarks is that the Elephants will have only a single one-step dominance plus a single two-step dominance, as required. Furthermore, in this tournament, the Elephants could not have defeated any other team and still have only a single two-step dominance. So option B was the correct answer.



A similar process could have been used to eliminate options C, D and E as possible correct answers.

Module 6: Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	93	2	1	4	0	0
2	2	4	6	2	85	0
3	16	75	1	1	7	0
4	4	6	5	82	3	0
5	5	5	10	4	75	1
6	6	3	87	1	2	0
7	3	8	3	84	1	0
8	12	76	2	8	1	0
9	6	8	65	6	15	1

This module was very well done, with no questions having a success rate of less than 50 per cent.