2007 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

There were 25 644 students who sat the Further Mathematics examination 2 in 2007, compared with 23 993 students in 2006. The selection of modules by the students in 2006 and 2007 is shown in the table below.

MODULE	% 2006	% 2007
1 – Number patterns	40	42
2 – Geometry and trigonometry	81	84
3 – Graphs and relations	49	52
4 – Business-related mathematics	54	54
5 – Networks and decision mathematics	42	41
6 – Matrices	14	30

Students performed well on the first questions for every module; however, a significant number of students either did not attempt the Core questions or did not complete them very well. As they progressed through each module, the questions became more complex. Overall, the Core section and the six modules were of comparable difficulty.

Students are advised to ensure that they use their reading time effectively, to carefully read all questions they will attempt and think about what each question is asking. Careful reading can also help prevent students from overlooking questions or parts of questions.

Some assessors reported that legibility of written responses seemed to be more of an issue this year. Figures that had been written over were sometimes indecipherable, and poorly constructed 2s and 7s, 4s and 9s and 0s and 6s could easily have been misread. Where an answer was not clear, marks could not be awarded.

Each year, several students write all of their answers within explanatory sentences. This not only takes up time unnecessarily but often takes up space that has been allowed for a possible calculation or other relevant working to be shown. Except where an explanation is explicitly required by the question (for example, when the question states ‘show that’), a clearly identified answer is usually sufficient.

All students are encouraged to show their working out in all questions that require some calculation. This may allow a method mark to be awarded if a question is worth more than one mark. Also, if a question requires a previous answer for its calculation and that previous answer was incorrect, a consequential mark is only be available if the working shows the correct application of that previous incorrect answer. However, many students continue to give answers only, without showing any relevant working. Where the answer was correct, full marks were awarded, but if working was absent or difficult to follow, method and consequential marks could not be awarded where the answer was incorrect. Assessors only mark the student’s presented work. They cannot assume that ‘a student must have done this to get that answer’. Where a formula is used, students could write the formula and then rewrite it with the correct terms substituted in the correct positions. If a TVM function is used, a table of their inputs, including relevant negative signs, is acceptable.

Students should be encouraged to read assessment reports for previous years as these can assist them to minimise preventable errors. Again this year, some students rounded off an answer in the middle of a calculation and then continued working with the rounded number, thus compounding a round-off error. This was most evident in the Geometry and trigonometry module.

As in 2006, rounding errors were penalised once per paper. Answers written to fewer decimal places than required were not considered rounding errors and scored zero. Where students engaged with a question beyond the required answer, a penalty was applied if the extension was in error (for example, an incorrect simplification).

Several questions required students to ‘show that’ a dimension had a given value. These were often poorly answered as many students were unable to clearly **communicate** their mathematical reasoning.

Simple statements such as $\sqrt{12^2 + 32^2}$ were frequently disconnected from the question as they were not labelled but merely presented as a statement that may have had something to do with the question. Such questions need sufficient practice; they require a written explanation of a result as if it was being explained to a person who could understand the mathematics but may not be capable of answering the question themselves.

Of growing concern is evidence that some teachers appear to be presenting formulaic or 'black-box' solutions for students. This was apparent where answers were provided using an incorrect, but related, formula that the student clearly did not understand conceptually. This occurred in particular in the Geometry and trigonometry module where the surface area of a triangular prism was required. While it may be expedient to provide some students with formulaic solutions for certain questions, this does not enable them to tackle variations where one or more of the required conditions do not apply. Such 'black-box' solutions to examination questions are often attempted in inappropriate circumstances.

With the increasing capability of graphics and CAS calculators to store and execute various programs, students must expect that their mathematical knowledge will continue to be examined in ways that require sensible use of such technology. This includes interpreting and communicating results, along with demonstrating an understanding of underlying mathematical ideas and the ability to appropriately formulate and analyse key components of the Further Mathematics study, as described in Outcomes 1–3.

All students must be able to interpret a calculated answer. In practical situations a final answer such as 4.8E-34 is likely to indicate a zero value. Similarly, a final answer of, for example, 14.9999993, should be taken as 15 in most situations, especially where integer values are expected.

Some students had crossed out a solution that was, in fact, partially correct. Deleted work cannot be assessed.

Areas of strength

Core

- describing the shape of a histogram that has a positive skew
- determining the coefficients of the equation for a least squares regression line
- describing the strength, direction and form of a relation with a high value of r

Number patterns

- extrapolating a pattern that follows an arithmetic sequence
- understanding that a common ratio of 0.95 corresponds to a percentage reduction of 5%
- demonstrating that a sequence is neither arithmetic nor geometric

Geometry and trigonometry

- calculating an angle with an appropriate trigonometric ratio
- applying Pythagoras' theorem in two and three dimensions
- applying Heron's formula

Graphs and relations

- calculating average speed from data given on a graph
- interpreting a distance–time graph
- drawing a graph with an equation $G = 15 - 0.06\ s$
- explaining an inequality in terms the quantities specified

Business-related mathematics

- calculating a discounted price, given the rate of discount
- calculating the monthly repayment of a loan, given the principle, term and compounding interest rate
- writing an amount of money to the nearest cent and not just to the nearest five cents

Networks and decision mathematics

- determining the sum of the degrees in a weighted network diagram
- finding the minimum time to complete a project
- determining the slack time for an activity

Matrices

- multiplying two matrices on the calculator
- identifying the order of a matrix
- interpreting the elements of a matrix when applied to real data

Areas of weakness

Students' ability to read and understand the questions was again a problem this year, and many students lost marks because they did not answer the questions as asked. For example, for Question 2a. in the Core section a number of students quoted the minimum actual temperature, when the question asked for the year in which the lowest temperature was recorded. Many students seemed to miss Question 3bii. in the Core section entirely. Others left the last page of modules blank, perhaps because they missed reading these questions.

Rounding off should only occur at the final answer stage in a question, not part way through the calculation. Teachers should emphasise that the continued use of a rounded answer will often compound the rounding error and should provide relevant examples in a range of contexts. In cases where the answer to a previous question must be used, the rounded version of the previous answer can be used without penalty. (Despite this, students who use the unrounded version of the previous answer are not penalised as long as their mathematics is correct.)

It was again evident this year that some students did not give consideration to the appropriateness of their answers. Many examples of unreasonable answers were seen; for example, that monthly repayments of $5 280 938.76 would be needed to repay a loan of $30 000 over five years at an interest rate of 9% pa; or that the mean surface temperature of Australia in 2010 would be -24 819.48°C in 2010. It is strongly recommended that estimation and consideration of the reasonableness (or otherwise) of results become an integral part of every mathematics class so that students explicitly consider the validity of their answers.

Correct expression formulation continues to be a concern where technology is used for calculation. Divisions, such as required by $\frac{37+37+24}{2}$, often seemed to be done on a calculator without brackets or any other approach to sum the numerator before dividing by 2.

Similarly, some students appeared to have difficulty in dealing with the fraction on their calculator when using the formula $V = \frac{1}{3} \times \text{area of the base} \times \text{height}$ when finding the volume of a pyramid.

Core

- interpreting a histogram where each column represents a range of values
- calculating a residual value (despite the fact that the relevant formula was given on the formula sheet)
- accurately drawing a least squares regression line on a graph by using its formula. Students should determine and connect two points that are widely spaced. Of those who attempted Question 3bii., many drew an inaccurate line by choosing two points that were apparently too close together
- providing a clear explanation of why the given residual plot supported the assumption of linearity of a relationship

Number patterns

- using given data points to develop the equation for a quantity in terms of the term number for an arithmetic sequence and for a geometric sequence
- showing a logical calculation that finds the 'total kilojoule intake from day 8 to day 14'. Unless the method was clear, marks could not be earned if the answer here was incorrect
- recognising that 'from day 8 to day 14' meant that $S_{14}–S_7$ is required rather than $S_{14}–S_8$
- showing a clear method for a calculation (or table) where a term in an arithmetic sequence first exceeds the corresponding term of a geometric sequence
- working with difference equations

Geometry and trigonometry

- showing a clear explanation of the quantities, mathematics and results that apply in 'show that' questions
- calculating the total surface area of a triangular prism (quite a number of students used the same, incorrect, formula)
- similar figures

- assuming incorrectly that removing the top $\frac{3}{4}$ of the *height* of a pyramid leaves $\frac{1}{4}$ of the *volume* remaining

Graphs and relations
- when fuel usage for gas must be less than fuel usage for petrol, this does not occur **at** the speed indicated by the intersection of two lines; rather, it applies at a speed **greater** than that indicated by the intersection
- shading the unacceptable region instead of the feasible region as instructed; such answers can be accepted only if a legend clearly indicates the correct feasible region
- justifying whether a certain condition satisfies **all** given constraints

Business-related mathematics
- showing working out clearly for a two mark question
- deducting a deposit before using the remainder as the principal amount for an interest calculation
- understanding a context that describes an ordinary perpetuity
- correctly applying a TVM calculator function and interpreting the result
- reverse GST (or similar) calculations where a given percentage change has been applied to a quantity and the final result is given and the initial quantity must be found
- reducing balance depreciation
- equating a reducing balance rate required to achieve a given projected book value

Networks and decision mathematics
- understanding the definition of a ‘connected graph’
- determining the capacity of a cut that includes an edge where the flow is in the opposite direction
- determining the maximum flow through a network by correctly finding the minimum cut
- recognising that crashing is relevant only for activities on a critical path
- recognising that crashing activities on a critical path may create a new critical path and that this new one may also be reduced by crashing an activity on it

Matrices
- interpreting the outcome of the product of two matrices in a practical situation
- testing whether a steady state has been obtained through multiplying the previous result by the transition matrix and noting no significant change
- interpreting a calculator result of something such as 4.8E-34 as zero in context
- giving only the entire matrix as the answer (Question 2dii.) rather than explicitly answering the question that required identification of an element of that matrix

SPECIFIC INFORMATION

Core

Question 1

Marks	**0**	**1**	**2**	**3**	**Average**
%	6	22	30	42	**2.1**

1a.
Positively skewed

1bi.
26

Many students seemed unable to interpret the ranges that applied to the columns of the histogram. A common incorrect answer was 13.

1bii.
$\frac{20}{103} = 19.4\%$

A common incorrect answer was $\frac{23}{103} = 22.3\%$

Question 2

Marks	0	1	2	3	4	Average
%	4	19	25	27	25	**2.6**

2a.
1964

A number of students misread the question and gave the lowest temperature of 13.01°C as their answer.

2bi
13.77

2bii.
–0.09

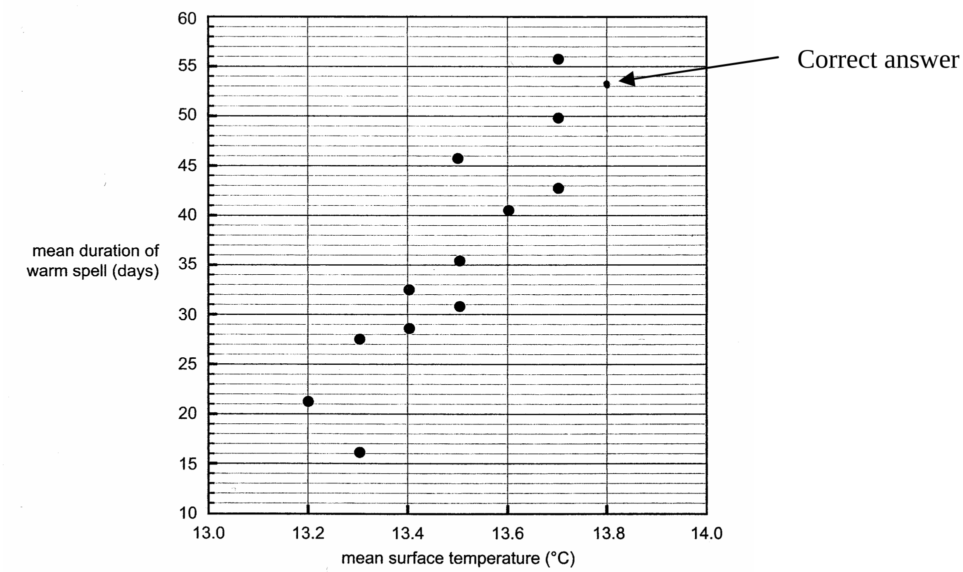
The negative sign was required, as determined by the correct use of the residual formula on the formula sheet.

2biii.
0.013

The gradient form of the given equation was expected here. Sometimes an incorrect answer was calculated from the data.

Question 3a.

Marks	0	1	Average
%	14	86	**0.9**



Scatterplot 1

This point had to be on the 13.8 coordinate line, with the value for the duration of the warm spell between 53 and 54.

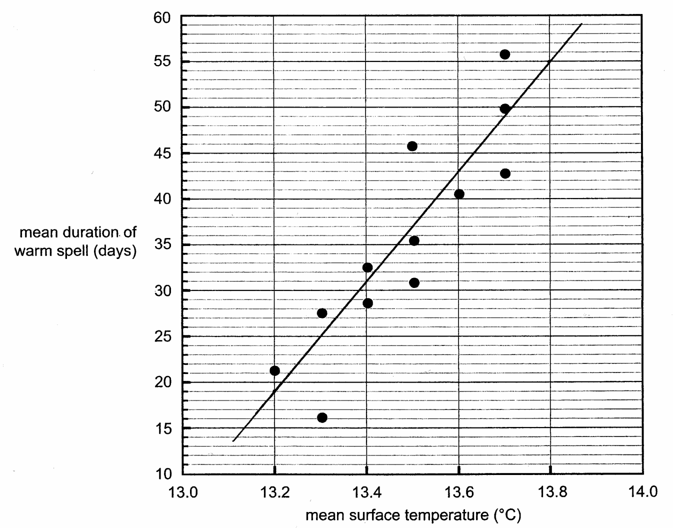
Questions 3bi–3e.

Marks	0	1	2	3	4	5	6	7	Average
%	26	12	10	9	8	9	15	11	**3.1**

3bi.
Mean duration = –776.9 + 60.3 × *temperature*

The appropriate variables were required and $y = -776.9 + 60.3x$ received only one mark. Some students got the variables back to front.

3bii.



Scatterplot 1

A straight line that had to pass through the points (13.2, 19) and (13.8, 55)

A significant number of students missed this question entirely. This highlights the need for students to use their reading time effectively.

To draw a straight line, students are expected to bring a straight edge (for example, a ruler) into the examination.

The line was very often badly plotted with two close points apparently chosen to locate the line. It is expected that the points be a reasonable distance apart. Values could have been calculated for mean surface temperatures of 13.2°C and 13.8°C.

Some students incorrectly used the point plotted in Question 3a.

3c.
There is no clear pattern to the random scatter of points.

3d.
83%

3e.
Strong, positive and linear

A common error was to analyse the **residual** plot for strength, direction and form, incorrectly concluding that there was no relationship between the variables.

Module 1 – Number patterns

Question 1

Marks	0	1	2	3	4	Average
%	1	15	25	32	28	**2.7**

1a.
8700

1b.
7950

A common incorrect answer was 7945.

1c.
8850

A common incorrect answer was 8700.

1d.
Day 14

A common incorrect answer was Day 13.

Question 2

Marks	0	1	2	3	4	5	6	Average
%	26	10	9	10	14	15	15	**2.9**

2a.
5%

A common incorrect answer was 95% or 0.95.

2b.
$12\,000 \times 0.95^{(3-1)} = 10\,830$

A frequent error was to use 1200 instead of 12 000.

2c.
$R_n = 12\,000 \times 0.95^{n-1}$

A difference equation was not appropriate here as a formula in terms of *n* was required. A frequent error was to use 1200 instead of 12 000.

2d.

$$\begin{aligned} t_9 - t_{10} &= 12\,000 \times 0.95^{(9-1)} - 12\,000 \times 0.95^{(10-1)} \\ &= 7961.045 - 7562.993 \\ &= 398.052 \\ &= 398 \text{ kJ} \end{aligned}$$

2e.

$$\begin{aligned} S_{14} - S_7 &= \frac{12\,000}{-0.05}\left[\left(0.95^{14} - 1\right) - \left(0.95^{7} - 1\right)\right] \\ &= 50\,559 \text{ kJ} \end{aligned}$$

Many students calculated and listed S_8, S_9, S_{10} etc. and then added the result rather than subtracting the results of two geometric series. Others tried $S_{14} - S_8$ rather than $S_{14} - S_7$.

Questions 3–4

Marks	0	1	2	3	4	5	Average
%	38	8	16	12	10	16	**2.0**

A number of students left this page entirely blank. This is either because they missed reading it or did not understand the questions.

3a.
25.25

3b.

$\frac{23}{20} \approx 1.15 \quad \neq \quad \frac{25.25}{23} \approx 1.10$ and $23 - 20 = 3 \quad \neq \quad 25.25 - 23 = 2.25$

For one mark, students had to show two subtractions not having the same result and two quotients not having the same result. Some only showed the calculations and did not state that the results within each pair were not equal. Others merely showed the results of the calculations without indicating where these results came from, or their significance

3c.
16 minutes

Many students found a reverse application of a difference equation, that is formulating and solving a linear equation, demanding.

The equation $M_2 = 0.75 \times M_1 + 8$ had to be solved for M_1 where $M_2 = 20$

Question 4
Day 12

A method mark was available for setting up an equation to find where the two distances were equal. Many students reached the wrong answer with working out that could not be clearly followed and thus earned no marks.

A suitable method could have been:
Solve $100 + 50(n-1) > 500 \times 1.02^{n-1}$
$n > 11.25$

Module 2 – Geometry and trigonometry

Question 1a.

Marks	0	1	Average
%	5	95	**1.0**

12 cm

A number of students inappropriately tried to apply trigonometric ratios or Pythagoras' theorem to this length that is half of *AB*, since *W* was a midpoint.

Questions 1b–2b.

Marks	0	1	2	3	4	5	6	Average
%	15	12	12	15	17	13	17	**3.1**

1b.

$$\angle WAQ = \tan^{-1}\left(\frac{12}{32}\right) = 20.556°$$
$$= 20.6°$$

1c.
$\angle AWB = 2 \times \angle WAQ = 2 \times 20.556 = 41.112 \approx 41.1°$
or, by using a rounded figure for $\angle WAQ = 20.6$, $\angle AWB = 2 \times \angle WAQ = 2 \times 20.6 = 41.2$

Either 41.1° or 41.2° was accepted.

1d.
$\frac{1}{2}$ (50% and 0.5 were also accepted.)

2a.

$$AW = \sqrt{32^2 + 12^2}$$
$$= \sqrt{1168} = 34.18 \approx 34$$

This calculation was often accompanied by a suitable diagram.

2b.

SA = area of base + area of two sides + area of two ends

$$= 24 \times 28 + 2 \times (28 \times 34) + 2 \times \left(\frac{1}{2} \times 24 \times 32\right)$$
$$= 672 + 1904 + 768$$
$$= 3344 \text{ cm}^2$$

The solid in this question is a triangular prism where the triangle does **not** have a right angle.

A significant number of students used an incorrect formula that had obviously been copied from a text and failed to score any marks. The formula $SA = bh + bl + hl + l\sqrt{b^2 + h^2}$ applies only to a right triangular prism. Further, the question directed that a length AW = 34 cm had to be used. If this length was not used at all, then no marks were available.

Question 3

Marks	0	1	2	3	4	5	Average
%	19	14	13	17	11	27	**2.7**

3a.

$$\text{Volume} = \frac{1}{3} \times 24 \times 28 \times 32$$
$$= 7168 \text{ cm}^3$$

3b.

$$AC = \sqrt{24^2 + 28^2} \approx 36.878$$
$$AY = \sqrt{32^2 + \left(\frac{AC}{2}\right)^2} = \sqrt{32^2 + \left(\frac{36.878}{2}\right)^2}$$
$$= \sqrt{32^2 + (18.439)^2} \approx 36.93 \approx 37$$

3c.

$$s = \frac{37 + 37 + 24}{2} = 49$$
$$A = \sqrt{49(49-24)(49-37)(49-37)}$$
$$= \sqrt{49 \times 25 \times 12 \times 12}$$
$$= \sqrt{176400}$$
$$= 420$$

This question required the correct use of Heron's formula. One mark was allowed for the correct and identified value of *s*.

Ineffective calculator usage again seems to have led to incorrect values for *s*, often due to not bracketing the numerator of the fraction before dividing by 2. Another quite common error involved using 32 cm for one of the sides of triangle *YAB* rather than recognising that it was an isosceles triangle.

Question 4

Marks	0	1	2	3	Average
%	58	37	2	4	**0.5**

4a.

$\frac{24}{32} = \frac{3}{4}$

4b.

Volume removed $= \left(\frac{3}{4}\right)^3 = \left(\frac{27}{64}\right)$ of the original volume

Hence, volume remaining $= 1 - \frac{27}{64}$

$= \frac{37}{64}$

Module 3 – Graphs and relations

Questions 1a–d.

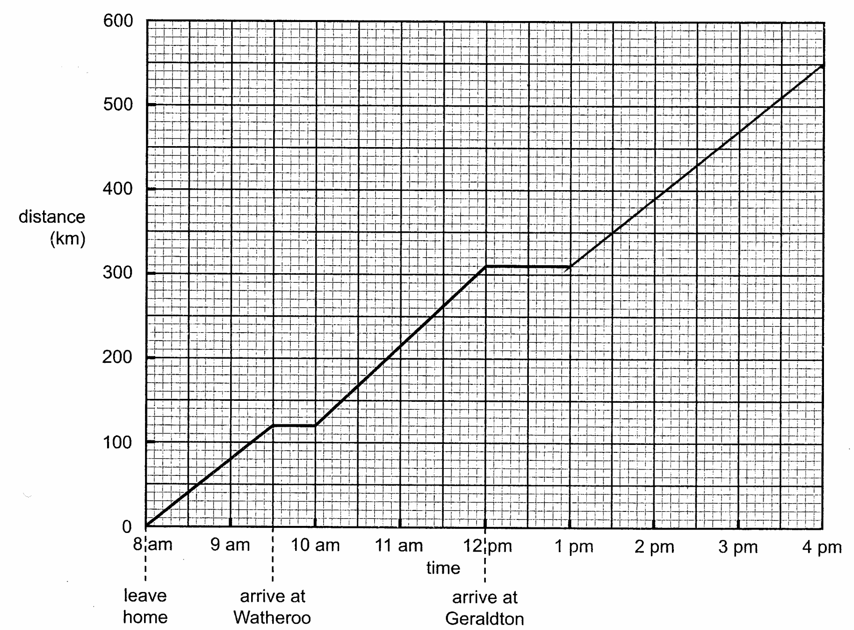
Marks	0	1	2	3	4	Average
%	2	11	19	27	41	**3.0**

1a.
30 minutes

1b.
190 km

1c.
95 km/h

1d.



A line segment that began at 1 pm and a distance of 310 km and ended at 4 pm and a distance of 550 km.

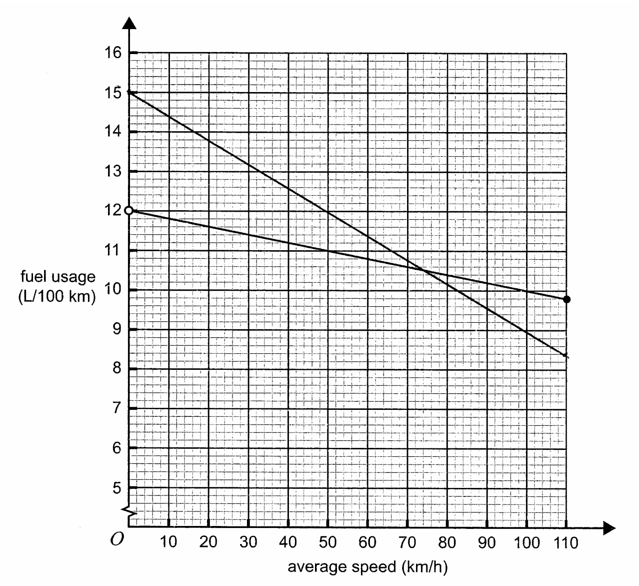
Some students stopped their line segment before 4 pm. A positive gradient was required as the vertical axis represented the distance traveled on their holiday.

Questions 2a–c.

Marks	0	1	2	3	Average
%	19	27	34	20	**1.6**

2a.
10.8

2b.



A line joining (0, 15) and (110, 8.4) was required.

A common incorrect line was drawn from (0, 15) to (110, 9.8).

2c.
Any speed **greater than** 75 km/h and up to 110 km/h was accepted.

The question referred to fuel usage being **less** for gas than for petrol. This could not include a speed that was **equal** to 75 km/h. The given equation was stated as being valid for average speeds up to 110 km/h.

Questions 2d–3a.

Marks	0	1	2	3	Average
%	29	29	15	27	**1.4**

2d.
$7.92

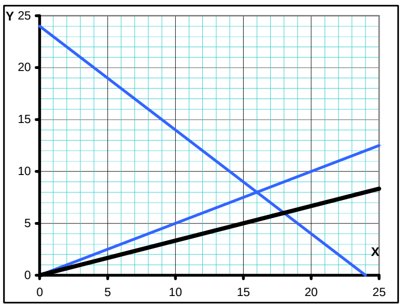
Looking at the reasonableness of the answer is an important aspect of mathematical calculations. Some answers that were given for this question, such as $165 for gas to travel 100 km, were totally unrealistic.

3a.
The total number of hours of driving with petrol plus the hours driving with gas must be no more than 24.

Questions 3bi–d.

Marks	0	1	2	3	4	5	Average
%	35	17	17	14	11	6	**1.7**

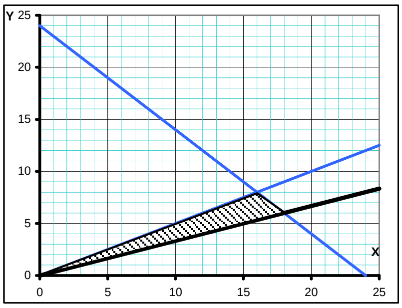
3bi.



It was easy to see if this line had been drawn accurately or not since the correct line $y = \frac{1}{3}x$ went through the point (15, 5). Some students tried to plot the point for $x = 25$ rather than one that can be more accurately located, such as x equals any multiple of three.

Accuracy of graph drawing is very important. A straight edge is expected for lines drawn in this module.

3bii.



It was difficult for the correct feasible region to be shaded if the line $y = \frac{1}{3}x$ had been drawn in the wrong location. If it had been drawn **near** the correct location and the feasible region had been shaded in the relevant area, the mark was awarded.

The instruction was to shade the feasible region. Some texts lead students to shade the **excluded** region. This was accepted only if a legend or arrow highlighted their correct (**included** and **unshaded**) feasible region.

3c.

Yes. Ten hours on gas and five hours on petrol lies on the line $y = \frac{1}{2}x$ and is within the feasible region.

A reference had to be made to the need for five hours using petrol.

3d.
maximum = 18
minimum = 17

Very few students obtained these two answers. Many thought the minimum time was 16 hours but this, with seven hours driving on petrol, would only account for 23 hours, whereas the question stated that 'the Goldsmiths plan to drive for 24 hours'. Of course, this same reasoning applies to those who felt that zero hours on gas would be the minimum.

Module 4 – Business-related mathematics

Although Australian currency has a five cent coin as its lowest denomination, financial calculations are expected to be correct to a stated accuracy. Unless stated otherwise, answers correct to the nearest cent are expected, and these should not be rounded to the nearest five cents. Domestic bills are often paid by electronic means and are usually calculated and debited correct to the nearest cent.

Sums of money correct to fractions of a cent are common in financial transactions. The price of petrol is given correct to the nearest tenth of a cent. The exchange rate for the Australian dollar is often quoted correct to the nearest hundredth of a cent. Students must carry out calculations to the nearest cent or an appropriate accuracy as specified in the question.

Question 1

Marks	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**Average**
%	4	6	8	17	33	18	14	**3.8**

1ai.
$1750

1aii.
$218.75

1bi.
$8420

1bii.
7.3%

This question was quite poorly answered and the deposit was often not allowed for. One mark was available for clearly identifying the interest of $1420 that was then to be used in the simple interest formula.

1c.
$5950

Question 2a.

Marks	**0**	**1**	**2**	**3**	**Average**
%	43	23	18	16	**1.1**

2a.

$$\text{Monthly interest} = \frac{0.09}{12} \times 30000$$
$$= \$225$$

Many students were puzzled by not being given a period of time for this loan. A loan where the interest only is paid each month is not a reducing balance question as the balance does not decrease or increase. It becomes a simple interest calculation for one period (one month in this case). This situation is like a **perpetuity** in reverse, where Khan is paying monthly instalments **in perpetuity**.

2b.
$16 801

N = 60
I = 9
PV = 30 000
PMT = –400
***FV* = 16 800.77604**
P/Y = 12

A number of students used N = 5.

2c.
$622.75

N = 60

$I = 9$
$PV = 30\ 000$
$\boldsymbol{PMT = -622.75065}$
$FV = 0$
$P/Y = 12$

This was a straightforward TVM question. Substitution into the annuities formula was still seen from a few students, despite it no longer being required by the study design. Students should be familiar with the use of the TVM function on their calculator.

Questions 3a–4b.

Marks	0	1	2	3	4	5	6	Average
%	24	14	16	13	9	10	13	**2.5**

3a.

$$\frac{900}{1.1} = 818.182$$
$$= \$818.18$$

Reverse GST questions such as this continue to be a problem for many students. Answers of $818.20 were not accepted as accuracy correct to the nearest cent was required. Some students simply reduced $900 by 10% of $90 to get $810.

3bi.

$$\frac{(900-300)}{5} = \$120$$

3bii.

$\$900 - 0.46 \times 250 \times 5 = \$325$

Some students did not include the deposit in the 'total amount paid'.

4a.

$$10000 \times 0.88^5 = 5277.319$$
$$= \$5277$$

4b.

16.7%

$N = 5$
$\boldsymbol{I = 16.744679}$
$PV = 10\ 000$
$PMT = 0$
$FV = -4000$
$P/Y = 1$

Students were also able to solve $10000 \times \left(1 - \frac{y}{100}\right)^5 = 4000$

Module 5 – Networks

Question 1

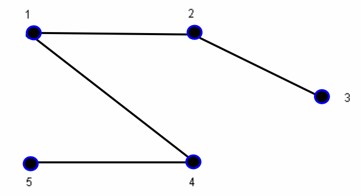
Marks	0	1	2	Average
%	6	37	57	**1.5**

1a.

4

Many students did not seem to understand what a **connected graph** was.

1b.



This is only one of the possible answers. Any graph with four edges that connected all five points, whether directly or indirectly, with each other was acceptable.

Question 2

Marks	0	1	2	3	4	Average
%	18	29	24	20	10	**1.8**

2a.
24

2bi.
C or G

An Euler path was required. This had to start at one of the two odd nodes and finish at the other.

2bii.
2800 m

Many students misread this question. The question was about the length of the Euler path, but some seemed to find the **shortest Hamiltonian path** and gave an answer of 1050.

2bi.
F–G–A–B–C–D–E–F or *F–E–D–C–B–A–G–F*

This question required the **shortest** Hamiltonian **circuit** commencing at *F*. Therefore, it had to finish at *F*. A common incorrect answer was *F–A–B–C–D–E–G–F*.

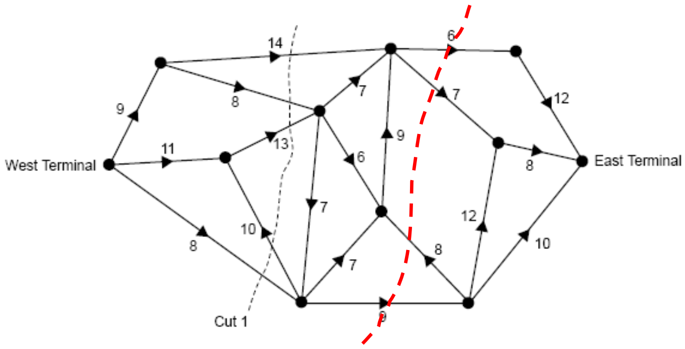
Question 3

Marks	0	1	2	3	Average
%	35	37	23	5	**1.0**

3a.
43

This question was poorly answered, with many students giving an answer of 53. The edge with the 10 should not have been counted as its flow was in the reverse direction.

3b.
22



Minimum cut ⇒ maximum flow. Despite this, some answers given here were greater than the answer for Q3a.

The minimum cut is shown here. The edge marked 8 is not counted as its flow is in the reverse direction.

3c.
7

A common incorrect answer was 8.

Question 4

Marks	0	1	2	3	4	5	6	Average
%	31	10	11	14	15	12	7	**2.4**

4a.
19 weeks

4b.
5 weeks

EST of D = 4, G = 5
LST of D = 9, G = 10
EFT of G = 8, H = 13
Therefore 9 – 4 = 5 or 10 – 5 = 5 or 13 – 8 = 5

4c.
A, E and G

While these three activities are not on a critical path, **crashing** any of these will not affect the completion time of the project.

4d.
15 weeks

Reducing C and F by two weeks each reduces the length of the path B–C–F–H–I from 19 to 15 weeks. However, this creates a new critical path B–E–H–I which is 16 weeks long. This means we should reduce activity E by one week so that both paths B–C–F–H–I and B–E–H–I are new critical paths and both are 15 weeks long.

4e.
$25 000

Reduce C by two weeks, F by two weeks and E by one week = 5 weeks at $5000 per week.

Module 6 – Matrices

Questions 1a–b.

Marks	0	1	2	3	4	Average
%	3	9	15	39	34	**3.0**

1a.

$$\begin{bmatrix} 1.2 & 20.1 & 4.2 \\ 6.7 & 0.4 & 0.6 \end{bmatrix}$$

Matrix brackets were required.

1bi.
[1890]

1bii.
4×4

1biii.
The total energy content of the servings of these four foods in one sandwich.

This question referred to the ingredients to make one single sandwich. Many students incorrectly referred to the 'total energy in these four foods' and did not specify it was for the particular quantity of 'these four foods' needed to make one sandwich.

Questions 1c–2b.

Marks	0	1	2	3	4	Average
%	7	15	22	11	46	**2.8**

1c.
$b = 4$, $m = 4$, $p = 2$, $h = 1$

This question required accurate data entry into the calculator and then correct multiplication of the inverse of the square matrix by the column matrix. Errors in typing the matrix elements into the calculator seemed common.

There was little room for method marks here, with two marks awarded for all four correct answers. If an error in data entry had been made, none of these answers would have appeared and thus the student scored zero.

2a.
700

2b.
0.5

50% and $\frac{1}{2}$ were also accepted.

Question 2c.

Marks	0	1	2	3	4	5	Average
%	23	6	16	20	24	11	**2.5**

2ci.

$$\begin{bmatrix} 160 \\ 280 \\ 180 \\ 80 \end{bmatrix}$$

$$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 160 \\ 280 \\ 180 \\ 80 \end{bmatrix}$$

2cii.
280

2ciii.
56

$$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}^4 \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 10.24 \\ 56.32 \\ 312.96 \\ 320.48 \end{bmatrix}$$

2civ.
7 weeks

2cv.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 700 \end{bmatrix}$$

$$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix}^{80} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 0 \\ 0 \\ 700 \end{bmatrix}$$

Some students apparently did not raise their transition matrix to a high enough power to find this steady state matrix. Others did not interpret a calculator answer such as 5.8460065E¯30 as representing zero or 699.999828 as representing 700 in this practical context. Such unrounded answers were not accepted.

Question 2d.

Marks	0	1	2	Average
%	50	39	11	**0.6**

Many otherwise strong students left the two questions on this page blank. The 'Turn over' instruction is provided at the bottom of the page to encourage students to read the last page of the examination paper.

2di.

$$\begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix}$$

$$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 400 \\ 200 \\ 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix}$$

2dii.
130

$$\begin{bmatrix} 0.4 & 0 & 0 & 0 \\ 0.5 & 0.4 & 0 & 0 \\ 0 & 0.5 & 0.8 & 0 \\ 0.1 & 0.1 & 0.2 & 1 \end{bmatrix} \begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 190 \\ 280 \\ 180 \\ 80 \end{bmatrix} = \begin{bmatrix} 130 \\ 207 \\ 284 \\ 163 \end{bmatrix}$$

Simply stating the resulting 4×1 matrix did not answer this question.