## GENERAL COMMENTS

There were 27677 students who sat the Further Mathematics examination 2 in 2010, slightly more than in 2009 . The selection of modules by the students in 2009 and 2010 is shown in the table below.

| MODULE | \% <br> $\mathbf{2 0 0 9}$ | \% <br> $\mathbf{2 0 1 0}$ |
| :--- | :---: | :---: |
| 1 - Number patterns | $37 \%$ | $33 \%$ |
| 2 - Geometry and trigonometry | $85 \%$ | $78 \%$ |
| 3 - Graphs and relations | $49 \%$ | $45 \%$ |
| 4 - Business-related mathematics | $46 \%$ | $42 \%$ |
| 5 - Networks and decision mathematics | $46 \%$ | $44 \%$ |
| 6 - Matrices | $48 \%$ | $59 \%$ |

Most students completed three modules. In general, students were able to answer the first question in each module very well but were challenged by the more complex questions they encountered later in the examination.

It was pleasing to see that many teachers and students had responded to issues raised in previous assessment reports. In 2010, only a small proportion of students ignored rounding instructions when dealing with a sum of money that had to be rounded off to the 'nearest cent'.

Most students used a straight edge to draw lines on graphs. Freehand lines were often insufficiently accurate to gain marks or to be useful for reading values. All students are encouraged to bring a ruler or other straight edge to use in the examination.

Most students read questions carefully. Some, however, did not attempt some questions or missed them altogether. This seemed to happen particularly where a graph was involved and where instructions immediately followed a graph. Students are advised to use the reading time wisely and to read each question carefully. Underlining or highlighting important information can also be a useful strategy.

Some students seemed to have difficulty with standard vocabulary; for example, misinterpreting 'proportion', 'predicting a value using the least squares regression equation' and 'perpetuity'. The development of a glossary throughout the year may be beneficial.

Many students struggled with relating explanations to the context of a problem. This occurred particularly in Question 1bi. in Number Patterns, Question 3a. in Graphs and Relations, and Question 1c. in Matrices. Formulaic answers were common, with some students referring to 'the first term of the series' rather than to the cost of the first section of tollway, or ' $x$ and $y$ cannot be more than 150 ' instead of referring to the number of pillows sold.

Students are expected to be familiar with the technology they use and its default settings. Some students obtained unreasonable answers in trigonometry, which may have been caused by calculating in radian rather than in degree mode. Further Mathematics students should regard a negative angle or an absurdly large (or small) calculated length as a signal to check their calculation.

Some students wrote answers without showing working and missed out on marks. Students are reminded that such instances do not allow access to consequential mark(s) that may be available or to rounding-error considerations.

There is a maximum of one rounding-error mark deducted per paper. For example, if a student made four clear rounding errors in the entire examination, they could be allowed marks for three of their incorrectly rounded answers. However, to qualify for this allowance, the source of each rounding error must be clear. This can be shown by writing the calculation and/or the unrounded answer before the rounded answer.

The following examples illustrate how this would be applied in Question 1d. of the Geometry and Trigonometry module.

## Assessment

## Report

- Length $=1558.8457 \approx 1558.9$ - This answer is incorrect; however, because the source of the rounding is apparent, this answer would qualify for a rounding-error allowance.
- Length $=1558.9$ - This answer is incorrect. Because the source of this answer is not apparent, this answer does not qualify for a rounding-error allowance.
- Length $=1558.85 \approx 1558.9$ - The answer and the source of the rounding are both incorrect and this answer does not qualify for a rounding-error.


## Areas of strength

Data Analysis

- calculating the median and range of a data set
- identifying the independent variable from a graph


## Number patterns

- working with arithmetic progressions


## Geometry and trigonometry

- substitution into the cosine rule
- applying the sine rule
- calculating an angle of elevation


## Graphs and relations

- drawing a line for an equation
- explaining an inequality in context

Business-related mathematics

- simple interest calculations

Networks and decision mathematics

- explaining adjacency matrices
- shortest path
- interpreting a table of data for activities in a project


## Matrices

- calculating matrix products using technology
- interpreting a diagrammatic version of a transition matrix


## Areas of weakness

## Data Analysis

- explaining when a mean and a median are both appropriate measures of centre for a given set of data
- interpreting the coefficients of a regression equation in context
- explaining the unreliability of extrapolated results when predicting with a least squares line
- plotting points on a scale with intervals of 500
- explaining a general trend


## Number patterns

- working with difference equations


## Geometry and trigonometry

- calculating the base dimensions of a square pyramid given the height and volume
- similar triangles


## Business-related mathematics

- understanding the definition of a perpetuity
- the significance of the first term when applying fixed simple and compound interest rates on the same investment
- applying a TVM solver function using technology


## Assessment

## Report

Networks and decision mathematics

- applying an understanding of an Euler path and two vertices of odd degree in context
- finding the critical path through a project described by a table


## Matrices

- generalising an explanation of state changes over an extended period of time


## SPECIFIC INFORMATION

## Core

Question 1a-b.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: |
| \% | 5 | 10 | 29 | 56 | $\mathbf{2 . 4}$ |

1a.
0.5

Common incorrect answers were Norway and 11.

1 b.
Median $=28$
Range $=56$
$\mathrm{IQR}=17$

## Question 1c-d.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | Average |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 2 | 59 | 39 | $\mathbf{1 . 4}$ |

1c.
$1 \mid 246$

1d.
The distribution is approximately symmetric.
This question was not well answered. Many students simply defined median and mean without reference to the given data. Some described the data in vague terms such as 'evenly distributed', while many referred to the absence of outliers. Neither of these terms is sufficient or specific enough to justify the assertion that both mean and median are appropriate measures of centre for this data set. Several explanations suggested that the data was 'symmetrically skewed'; a skewed distribution is not symmetrical.

Question 2a-c.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 7 | 14 | 36 | 29 | 14 | $\mathbf{2 . 3}$ |

2a.
Male income
2b.
\$350

Common incorrect answers included \$0.35, \$13 000 and \$13 000.35.

## 2ci.

\$18 250

## Assessment

## Report

2cii.
Making the required prediction involved going beyond the data used (extrapolation) to determine the regression equation.

There was no evidence that the relationship continued into income levels below those given in the dotplot. Therefore, the mathematical model established by the regression equation cannot be relied upon outside the range of the presented data set.

This question was poorly answered. Many students referred to their own view of the real world rather than the given data. The suggestions that 'Female income is not dependent on male income' or that 'it is a known fact that female income is always lower due to discrimination' were common. Incorrect or unacceptable answers included 'male income must be higher than the female income because that is what the graph shows' or 'any prediction is unreliable, whether or not it is derived from a formula.’

Question 3a-d.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 5 | 9 | 24 | 8 | 25 | 12 | 17 | $\mathbf{3 . 4}$ |

3a.


Many students had difficulty plotting the required values along the vertical scale with intermediate intervals of 500 .
3b.
An increasing trend
A common unacceptable answer described the general trend by explaining what occurred in each five-year period.
3c.
$G D P=20000+524 \times$ time
Students were expected to use the variables given in the question rather than just $x$ and $y$. While most students found the least squares regression equation using technology, some merely chose two points and found the equation of the straight line that joined them.

3d.
\$752
$20000+524 \times 27=34148$
Error in prediction $=34900-34148=752$
An answer of -752 was also accepted.
Many students calculated the predicted GDP of \$34 148 but then did not continue to find the corresponding error.
A number of students inappropriately substituted GDP = \$34900 into the regression equation, determined that the date could not have been 2007 and stated that 'The error is the time.'

## Module 1 - Number patterns

Question 1
1ai-1av.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 2 | 4 | 7 | 10 | 16 | 17 | 19 | 26 | $\mathbf{4 . 9}$ |

## 1ai.

$6.20-4.50=1.70$ and $7.90-6.20=1.70$
1aii.
\$11.30
$4.50+(5-1) \times 1.70=11.3$
1aiii.
8 sections
$4.50+(n-1) \times 1.70=16.40$
$\therefore n=8$
1aiv.
\$246
$\mathrm{S}_{15}=\frac{15}{2}(2 \times 4.50+(15-1) \times 1.70)=246$
Many students added 15 calculated terms rather than using the formula. This method was acceptable but it increased the possibility of errors.

1av.
$m=1, k=1.7$
A common incorrect value for $m$ was given as 4.5.
Question 1bi-1c.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 29 | 19 | 26 | 8 | 11 | 7 | $\mathbf{1 . 8}$ |

1 bi.
The cost of travel along the first section of road
This question was poorly answered, and many students did not make reference to the context. Answers such as 'the cost of a single trip', 'the entry fee', 'the first term' and 'the starting value' were not accepted.

1bii.
\$9.75
$B_{2}=0.9 \times 5+3=7.50$
$B_{3}=0.9 \times 7.50+3=9.75$
1biii.
\$30
Maximum charge when $B_{\mathrm{n}+1}=B_{\mathrm{n}}$
That is, when $x=0.9 x+3 \Rightarrow x=30$.

## Assessment

## Report

Many students were unable to apply the given difference equation to this question. Although the sequence was not geometric, some students incorrectly applied the infinite geometric series formula with $a=5$ and $r=0.9$.

## 1c.

Use Pass A for a single trip of up to 10 sections.
Use Pass B for a trip that involves more than 10 sections.
The simplest way to answer this question was to use technology and show the relevant section of the resulting table written down as the 'working out'.

| $\mathbf{n}$ | $\boldsymbol{A}_{\mathbf{n}}$ | $\boldsymbol{B}_{\mathbf{n}}$ |
| ---: | :---: | :---: |
| 8 | 16.40 | 18.04 |
| 9 | 18.10 | 19.24 |
| 10 | 19.80 | 20.31 |
| 11 | 21.50 | 21.28 |
| 12 | 23.20 | 22.15 |

This question was poorly answered. Some students made progress with the mathematics of this question, but said that Pass A was suitable for fewer than 10 sections and Pass B was suitable for more than 10 sections. This answer did not make comment on which pass to use for exactly 10 sections. A common unacceptable response was to suggest that Pass A should be 'used for short trips only'.

Question 2a-b.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 75 | 7 | 6 | 12 | $\mathbf{0 . 6}$ |

2a.
14.7 km
$S_{6}=100=\frac{a\left(1.05^{6}-1\right)}{1.05-1}$
$\therefore a \approx 14.70$
This question was poorly answered.
2b.
$L_{n+1}=1.05 L_{n}, L_{1}=14.7$
Of those students who attempted this question, a number omitted the specification of $L_{1}$. If the student's answer for Question 2a. was incorrect, the mark for this question was awarded if the specification for $L_{1}$ was the same as the answer found in Question 2a.

## Module 2 - Geometry and trigonometry

Question 1a-c.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 10 | 15 | 24 | 51 | $\mathbf{2 . 2}$ |

1a.
$120^{\circ}$
1b.
$240^{\circ}$
True (three-figure) bearings, such as $240^{\circ}$, are required material and in preference to quadrant bearings, such as $\mathrm{S} 60^{\circ}$ W . However, the correct quadrant bearing of $\mathrm{S} 60^{\circ} \mathrm{W}$ was accepted. A common incorrect answer was $\mathrm{N} 60^{\circ} \mathrm{E}$.

1c.
20 m

## Assessment

## Report

## Question 1di-1dii.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | Average |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 23 | 35 | 42 | $\mathbf{1} .2$ |

## 1di.

$1558.8 \mathrm{~m}^{2}$
$A=\frac{1}{2} \times 40 \times 90 \times \sin \left(120^{\circ}\right) \approx 1558.8457$
1dii.
$G W^{2}=40^{2}+90^{2}-2 \times 40 \times 90 \times \cos \left(120^{\circ}\right)$
$\therefore G W=\sqrt{40^{2}+90^{2}-2 \times 40 \times 90 \times \cos 120^{\circ}}$
$\therefore G W \approx 115.3256 \approx 115.3$
Question 1ei-ii.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | Average |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 23 | 20 | 57 | $\mathbf{1 . 4}$ |

## 1ei.

$160^{\circ}$

1eii.
45.7 m
$C K=\frac{45}{\cos \left(10^{\circ}\right)} \approx 45.694$
The simplest method was to apply the correct trigonometric ratio to the right-angled triangle found from half of the isosceles triangle. Some students correctly used the sine rule on the right triangle with $\sin \left(90^{\circ}\right)=1$. Others correctly used the sine rule on the isosceles triangle.

Question 2a-c.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 30 | 27 | 24 | 7 | 12 | $\mathbf{1 . 5}$ |

2a.
9 m

2b.
$\theta=\tan ^{-1}\left(\frac{10}{9}\right) \approx 48.01^{\circ}>45^{\circ}$
Students needed to show a correct calculation that resulted in $\theta \approx 48^{\circ}$, with a conclusion that this was greater than $45^{\circ}$.
Many students did not understand that a distance on a map is the horizontal distance between two points on the ground. Some students drew a right triangle with a hypotenuse of 10 and then used a sine ratio; however, this was incorrect.

## Assessment

## Report

2c.
2.5 cm
$\tan (\theta)=0.8=\frac{4}{x}$
$\therefore x=5 \mathrm{~m}$ on the ground
On the map, this will be 2.5 cm .
Many students found the distance on the ground to be 5 m , but then did not calculate the corresponding distance on the map.

## Question 3

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | Average |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 57 | 5 | 38 | $\mathbf{0 . 8}$ |

1.47 m
$1.8=\frac{1}{3} \times x^{2} \times 2.5$
$\therefore x \approx 1.4697$
This question was poorly answered. It was concerning that a number of students seemed confused by the volume being given as $1.8 \mathrm{~m}^{3}$ and wrote an equation involving $1.8^{3}$.

Question 4

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | Average |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 89 | 1 | 10 | $\mathbf{0 . 2}$ |

7.75 m


Similarity could be applied to two triangles found by first drawing a horizontal line through point $S$.
$\frac{P B}{8}=\frac{65}{160}$
$\therefore P B=\frac{8 \times 65}{160}=3.25$ metres
and $P H=P B+B H=3.25+4.5=7.75 \mathrm{~m}$
Another correct method considered $V S$ as a straight line on a pair of $X Y$ axes with the origin at point $T$, finding its equation and then substituting $x=95$.

A third correct method was to find the area of the trapezium VSKT. Equating this to the sum of the areas of two trapezia, $V P H T$ and $P S K H$, with a common unknown length, $P H$, resulted in an equation with one unknown to be solved.

The most common error was to inappropriately apply similarity to 'corresponding' lengths of two trapezia to give an incorrect answer of 5.08 m .

Many students were unable to complete this question correctly. Some were unable to gain any method marks as their working out could not be followed.

## Report

## Module 3 - Graphs and relations

Question 1a-d.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 4 | 5 | 17 | 15 | 59 | $\mathbf{3 . 2}$ |

1 a.
$R=65 x$
1 b.
\$1700

1c.


Some students did not attempt this question.
1d.
20
$65 x=500+40 x$
$\therefore 5 x=500 \Rightarrow x=20$
Question 2-3a.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | Average |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 9 | 22 | 69 | $\mathbf{1 . 6}$ |

2
$m=40$
$65 \times 35+50 \times m=4275$
$\therefore m=40$

3a.
The total number of pillows sold cannot exceed 150 .
Question 3b-d.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 12 | 6 | 14 | 15 | 14 | 39 | $\mathbf{3} 3$ |

3b.
$k=45$
3ci.


If this was the only line drawn on the graph, then labelling was not necessary. However, some students also drew another line as working for a later part of the question. In this case the line $x=30$ had to be clearly identified.

## Assessment

Report

3cii.


The instruction to shade the feasible region was included in the question; however, many students found the feasible region by shading the excluded region. Such answers could only be accepted if there is notation or legend that clearly identified the feasible region as blank or shaded in a different style.

3d.
\$9075
$R=65 x+50 y$
$(105,45) \Rightarrow \$ 9075$
$(30,120) \Rightarrow \$ 7950$
Question 3e-3g.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 61 | 10 | 13 | 4 | 12 | $\mathbf{1}$ |

3e.
Let $z=$ number of Snorestop pillows.
$\therefore x+y+z \leq 150$
and $z=2 x$
$\therefore x+y+2 x \leq 150$
$\therefore 3 x+y \leq 150$
Many students attempted only written explanations and were not successful in presenting the mathematics that resulted in the equation $3 x+y \leq 150$.

3f.
$R=175 x+50 y$
$R=65 x+50 y+55 z$
But $z=2 x$
$\therefore R=65 x+50 y+55 \times 2 x$
$\therefore R=175 x+50 y$
3g.
\$8375
New intersection at $(35,45)$

$$
R=175 \times 35+50 \times 45=8375
$$

A method mark was available for finding the new intersection point due to Inequality 4.

## Module 4 - Business-related mathematics

Question 1a-2b.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 5 | 6 | 14 | 16 | 25 | 23 | 11 | $\mathbf{3 . 6}$ |

1 ai.
\$2640

## Assessment

## Report

1aii.
\$640
$2640-2000=640$

1aiii.
8\%
$640=\frac{2000 \times r \times 4}{100}$

1 b.
\$2091
$(2000 \times 1.025) \times 1.02=2091$
2a.
\$1560
$(360000 \times 0.052) \times \frac{1}{12}=1560$

2b.
\$360 000
This question was poorly answered. Most students did not seem to understand that a perpetuity maintains the same principal amount while only regularly paying out $100 \%$ of all interest earned.

Many students gave $360000-72 \times 1560=\$ 247680$ as their answer; however, this was incorrect.
Question 3a-b.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | Average |
| :---: | :---: | :---: | :---: | :---: |
| $\%$ | 52 | 28 | 20 | $\mathbf{0 . 7}$ |

3 a .
Simple Saver annual interest rate is the highest as the first-year increase is larger.
This question was very poorly answered. The key to understanding the graph was to consider what happens after the first interest payment. Most students seemed to ignore the word 'rate', or they may have confused 'rate' with 'return'. Common incorrect responses were that 'the Growth Plus interest is eventually higher' and 'Growth Plus is compounding and so will eventually be higher'. Others suggested that 'Simple Saver is best for investments under ten years.'

3b.
\$920
$\frac{21800-8000}{15}=920$

Question 3ci-4c.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 41 | 8 | 11 | 7 | 12 | 6 | 10 | 5 | $\mathbf{2} .3$ |

3ci.
$24000=8000 \times\left(1+\frac{r}{100}\right)^{15}$

## Assessment

## Report

Some students gave their answer as $24000=8000 \times r^{15}$; however, this was incorrect.

3cii.
7.6\%

4ai.
\$1827.32
$\mathbf{N}=240$
$\mathbf{I}=6.25$
PV $=250000$
PMT $=-1827.3205 \ldots$
$\mathbf{F V}=0$
$\mathbf{P} / \mathbf{Y}=12$
$\mathbf{C} / \mathbf{Y}=12$
Some answers were calculated by using the annuities formula and these answers were usually incorrect. Such questions are easily answered with the accurate use of the technology TVM function.

## 4aii.

\$188.557
$1827.32 \times 240=438556.80$
$\therefore 438556.80-250000=188556.80$

4b.
\$213 118
$\mathbf{N}=60$
$\mathbf{I}=6.25$
$\mathbf{P V}=250000$
PMT $=\mathbf{- 1 8 2 7 . 3 2}$
$\mathbf{F V}=-213117.8071 \ldots$
$\mathbf{P} / \mathbf{Y}=12$
$\mathbf{C} / \mathbf{Y}=12$
4c.
212
$\mathbf{N}=103.75659 \ldots$
$\mathbf{I}=6.25$
PV $=100000$
$\mathbf{P M T}=-1250$
$\mathbf{F V}=0$
$\mathbf{P} / \mathbf{Y}=12$
$\mathbf{C} / \mathbf{Y}=12$
$\therefore 104+108=212$
Many students did not add the initial 108 weeks at the end.

## Module 5 - Networks

Question 1a-b.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | Average |
| :---: | :---: | :---: | :---: | :---: |
| \% | 3 | 37 | 60 | $\mathbf{1 . 6}$ |

1a.
Are not allowed to communicate with each other

## Assessment

## Report

Responses had to be relevant to the context of the question. A number of students seemed to quote material directly from their notes and gave unacceptable answers such as 'there is no connection or edge' or 'cannot get to one point from another' with no link to the context at hand.

1b.
$f=1$
$g=0$
Question 2a-bii.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 4 | 9 | 35 | 52 | $\mathbf{2 . 4}$ |

2a.
11

A common incorrect answer was 12.
$2 b i$.
Hamiltonian path
2bii.


Question 3a-c.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 9 | 26 | 29 | 17 | 19 | $\mathbf{2 . 1}$ |

3a.
D

3bi.
$B, D$ and $F$
3bii.
32 km
3c.
$B$ and D

## Question 4a-f.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 16 | 13 | 13 | 12 | 15 | 13 | 18 | $\mathbf{3 . 1}$ |

4 a .
2

4b.
9

## Assessment

## Report

4c.
A and C
4d.
4
4e.
16
4f.
A-B-D-H
The project time and the critical path could be found without a drawn network:
Activities that are not predecessors for any other activity will not have any successor before completion of the project. These were activities I, H, G, F, E in this problem.
The minimum time to complete the project is the maximum of earliest start time + activity time for all these activities. The critical path for the project will then end at the activity that has this maximum value.
That is, the maximum of

| E: | $5+7$ | $=12$ |
| :--- | ---: | :--- |
| F: | $9+6$ | $=15$ |
| G: | $9+1$ | $=10$ |
| H: | $13+3$ | $=\mathbf{1 6}$ |
| I: | $10+2$ | $=10$ |

Therefore the critical path ends with H where - H depends upon D; D depends upon B; B depends upon A.

## Module 6 - Matrices

Question 1a-2d.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 1 | 3 | 11 | 7 | 30 | 13 | 21 | 14 | $\mathbf{4 . 6}$ |

1a.
$\left[\begin{array}{lll}4 & 8 & 2\end{array}\right]$
1b.
[26]
The required answer needed to be a matrix.
1c.
The total of all the points scored by Oscar in this game
Explanations should have been applicable to the context of the question. Since there were different points available for different types of shots at goal, an answer of 'The total of all the goals shot by Oscar' was not accepted.

2a.
$20 \%$ of those doing heavy training one week will change to moderate training the next week.
Some students seemed confused about what the given percentage referred to. A common incorrect answer was ' $20 \%$ of players will change to moderate training the next week'. This would mean that $20 \%$ of a fixed number of 300 will change, whereas it is only $20 \%$ of a variable number (initially 90 ). Others suggested that the change would (only) occur in the first week rather than a regular transitional change.

2b.
66

## Assessment

## Report

$\left[\begin{array}{lll}0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.5 \\ 0.3 & 0.3 & 0.4\end{array}\right]\left[\begin{array}{c}90 \\ 150 \\ 60\end{array}\right]=\left[\begin{array}{c}66 \\ 138 \\ 96\end{array}\right]$
Some students relied on a formulaic approach and squared the transition matrix to find $S_{2}$. Such students incorrectly applied $n=2$ and used $S_{2}=T^{2} \times S_{0}$. However, the initial state matrix was given as $S_{1}$ rather than $S_{0}$ and so $S_{2}=T \times S_{1}$.

2c.
6
$S_{3}=\left[\begin{array}{lll}0.5 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.5 \\ 0.3 & 0.3 & 0.4\end{array}\right]^{2}\left[\begin{array}{c}90 \\ 150 \\ 60\end{array}\right]=\left[\begin{array}{c}56.4 \\ 144 \\ 99.6\end{array}\right]$
$\therefore 150-144=6$
A common incorrect answer was 3, found by using $T^{3}$ instead of $T^{2}$.
2d.
$\mathrm{S}_{7}=\mathrm{S}_{8}=\left[\begin{array}{c}50 \\ 150 \\ 100\end{array}\right]$

In order to show a steady state had occurred 'after 7 weeks', it was necessary to demonstrate that a steady state had been achieved for two consecutive weeks.

The question gave the state matrix in the first week as $\mathrm{S}_{1}$. Therefore the state matrix in the seventh week is $\mathrm{S}_{7}$.
It was considered that 'after 7 weeks' could have been interpreted either as $S_{7}$ or as $S_{8,}$ so full marks were awarded for answers that showed either $S_{7}=S_{8}=\left[\begin{array}{c}50 \\ 150 \\ 100\end{array}\right]$ or $S_{8}=S_{9}=\left[\begin{array}{c}50 \\ 150 \\ 100\end{array}\right]$

Further, since it can also be shown that $S_{6}=S_{7}=\left[\begin{array}{c}50 \\ 150 \\ 100\end{array}\right]$, it follows that a steady state had been found at $\mathrm{S}_{6}$ and that $\mathrm{S}_{6}=$ $\mathrm{S}_{7}=\mathrm{S}_{8}=\mathrm{S}_{9}$. Consequently, any consecutive pair of matrix calculations from $\mathrm{S}_{6}$ to $\mathrm{S}_{9}$ was accepted.

Many students found $S_{7}$ and then, for example $S_{100}$, and incorrectly concluded that since $S_{7}=S_{100}$ this fully justified the assertion that a steady state existed at $S_{7}$. However, such a pair of non-consecutive calculations did not eliminate the possibility that $S_{7}$ may not equal $S_{8}$.

Question 3a-4d.

| Marks | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{\%}$ | 12 | 8 | 11 | 13 | 12 | 10 | 15 | 7 | 12 | $\mathbf{4}$ |

3a.
$\left[\begin{array}{ccc}1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & 1\end{array}\right]$

## Assessment Report

Some students gave their answer as $\left[\begin{array}{c}1+1+1 \\ 2-1+3 \\ 1+2+1\end{array}\right]$, which represents the column matrix $\left[\begin{array}{l}3 \\ 4 \\ 4\end{array}\right]$.

3b.
$x=-5$
3c.
10
$\left[\begin{array}{ccc}7 & -1 & -4 \\ -1 & 0 & 1 \\ 5 & 1 & 3\end{array}\right]\left[\begin{array}{l}33 \\ 40 \\ 43\end{array}\right]=\left[\begin{array}{c}19 \\ 10 \\ 4\end{array}\right]$
4ai
$\left[\begin{array}{l}2100 \\ 1100\end{array}\right]$

4aii.
3200
4b.
$\left[\begin{array}{l}2613 \\ 1613\end{array}\right]$

The equation for each subsequent state matrix was given as the product of the matrix $G$ and the previous state matrix, $A_{\mathrm{n}+1}=G A_{\mathrm{n}}$.

It should be noted that the numbers in the columns of $G$ did not add up to one. This meant that, depending upon the initial state matrix, the total weekly number attending the two games could gradually increase or decrease. If $G$ had been a transition matrix with columns adding up to one, then the total numbers attending each week would remain constant regardless of the initial state matrix.

The attendance matrix for game 10 is $A_{10}$.
$A_{10}=G^{9} A_{1}$
$\therefore A_{10}=\left[\begin{array}{cc}1.2 & -0.3 \\ 0.2 & 0.7\end{array}\right]^{9}\left[\begin{array}{l}2000 \\ 1000\end{array}\right]=\left[\begin{array}{l}2612.5795 \\ 1612.5795\end{array}\right]$
A common error was to use the incorrect power for the matrix $G$.
4c.
Attendance at Dinosaurs' games is expected to gradually increase to 3000, at which level it will remain steady.
A description of 'the way in which the number of people attending the Dinosaur's games would change' was required. The correct answer had to acknowledge that a steady state of 3000 would occur after about 80 weeks.

Many students simply calculated the predicted attendance after 80 weeks and gave this number as their answer. Others merely said that 'the attendance was increasing.' However, neither of these responses was accepted.

4d.
Attendance at Dinosaurs' games is expected to gradually decrease to 600, at which level it will remain steady.
A description was required but many students gave the predicted number after 80 weeks or said that it 'was decreasing'.

