<u>**2006**</u>   <u>**Further Mathematics GA 3: Written examination 2**</u>

GENERAL COMMENTS

There were 23 993 students who sat the Further Mathematics examination 2 in 2006. The selection of modules by the students in 2005 and 2006 is shown in the table below. Matrices was a new module for 2006.

MODULE	% 2005	% 2006
1 – Number patterns	49	40
2 – Geometry and trigonometry	91	81
3 – Graphs and relations	53	49
4 – Business-related mathematics	64	54
5 – Networks and decision mathematics	41	42
6 – Matrices	n/a	14

Students performed well on the first questions for every module; however, a significant number of students either did not attempt the Core questions or did not complete them very well. As they progressed through each module, the questions became more complex. Overall, the Core section and the six modules were of comparable difficulty.

Students' ability to read and understand the question seemed to be a problem this year. Some students rounded off an answer in the middle of a calculation and then continued working with the rounded number, thus compounding a round-off error. This was especially evident in Question 2bi. of the Core and Question 4 of the Business-related mathematics module.

As in 2006, rounding errors were penalised once per paper. Answers written to fewer decimal places than required were not considered rounding errors and scored zero. Where students engaged with a question beyond the required answer, a penalty was applied if the extension was in error (for example, an incorrect simplification).

On many occasions, students provided answers that were related to questions but did not directly answer them. For example, many students provided only a decimal equivalent instead of the percentage that was specifically required by some questions.

In sections of the examination that required written explanations, some student responses seemed to be copied directly from their 'bound book of notes' and usually demonstrated little understanding of the answer or the question.

It appeared that some of the students who did not remove (as instructed on the front cover) the green formula sheet in the middle of the examination booklet may not have seen the last questions of the Graphs and relations module.

Poor use of calculators was evident again this year, especially when brackets should have been used to group a numerator of a fraction before dividing by a denominator.

Some students had crossed out a solution without replacing it. In several such cases, the crossed out work was partly correct. However, deleted work cannot be assessed at all.

Areas of strength
Core
- calculating the coefficient of determination from a given value of r
- using a boxplot to state a median value
- understanding the significance of the coefficient of a variable in a linear model

Number patterns
- showing, by at least two calculations, that a sequence has a common ratio of 0.8
- finding the fifth term of a geometric sequence

Geometry and trigonometry
- area of a triangle
- applying the cosine rule

Graphs and relations
- interpreting step graphs
- finding the equation of a straight line

Business-related mathematics
- using simple interest and compound interest formulas
- finding flat rate depreciation values

Networks and decision mathematics
- interpreting a bipartite graph
- interpreting a one-step dominance graph
- finding a critical path
- recognising activities that are not on the critical path

Matrices
- identifying the order of a matrix
- writing information in matrix form
- evaluating products

Areas of weakness
It is recommended that teachers address the following issues with students.
- It is essential that students read questions carefully. Many students lost marks because they did not answer questions as asked.
- Students often failed to employ brackets when using their calculators. Consequently, a calculation for $\frac{83.1-89.3}{4.5}$ often became one for $83.1-\frac{89.3}{4.5}$
- Many students still gave answers only, without showing any working. Where the answer was correct, full marks were awarded, but if working was absent or difficult to follow, method and consequential marks could not be awarded if the answer was wrong. Students who use a TVM function are advised to show a table of their inputs.
- Rounding off should only occur at the final answer stage in a question, not halfway through the calculation. However, in cases where the answer to a previous question must be used, the rounded version of the previous answer may be used without penalty. (Students could also use the unrounded version of the previous answer without penalty.)

Core
- converting a decimal to a percentage
- using a calculator to find a z-score from the formula
- calculating percentage below a standard z-score of 2
- plotting and reading boxplots accurately
- finding the points for a three median line

Number patterns
- finding a sum of terms between t_3 and t_{12} (also a problem in 2005)

Geometry and trigonometry
- recognising a non-right-angled triangle
- reading the requirements of a question
- interpreting a question and matching it to features on a diagram
- angle of depression

Graphs and relations
- writing an inequality from given information

- recognising when only discrete answers are possible within a feasible region

Business-related mathematics
- interpreting a rate of depreciation from a given formula
- determining when flat rate depreciation and reducing balance depreciation values are equal
- interpreting and calculating a compounding investment or loan situation involving payments
- using the annuities formula, despite this no longer being a requirement of the study design (students are now expected to apply a TVM Solver calculator function in these cases)

Networks and decision mathematics
- explaining two-step (or more) dominance
- recognising that crashing is relevant only for activities on a critical path

Matrices
- writing an explanation of why two matrices may not be conformable for multiplication
- showing two calculations that produce a steady state matrix

SPECIFIC INFORMATION
Question 1
1a–b.

Marks	0	1	2	Average
%	20	36	44	**1.3**

1a.
3.8

A common incorrect answer was 4.5.

1b.
–1.4

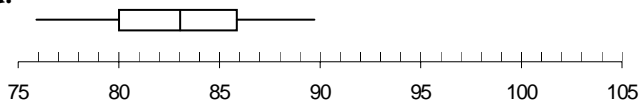Poor calculator use produced many incorrect answers.

1c–f.

Marks	0	1	2	3	4	5	Average
%	7	8	15	37	21	11	**3.0**

1c.
97.5%

This was a very poorly answered question. Incorrect answers included 95% and 2.5%.

1d.

Many students lost marks through inaccurate, careless placement of quartiles.

1e.
89.5

1f.
The median height increases with age.

This question was poorly answered. Some students referred to skewing (possibly after seeing the phrase 'positively related' in the question). Reference to a boxplot statistic was required, therefore, consideration of the **mean** height was not appropriate.

Question 2

Marks	0	1	2	3	Average
%	31	26	24	19	**1.3**

2a.
0.53

2bi.
56.9%

Many students did not convert the decimal into a percentage as required and gave an answer of 0.6. Some who did the conversion rounded the decimal first before converting to an incorrect answer of 60%.
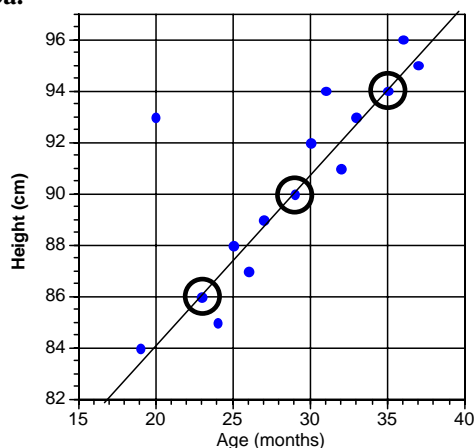
2bii.
56.9% of the variation in height is explained by the variation in age.

Common incorrect interpretations referred to **causation**. A common incorrect answer was '56.9% of the variation in height is **due** to the variation in age.'

Question 3

Marks	0	1	2	3	4	5	Average
%	31	17	19	15	11	7	**1.8**

3a.

Many students did not find the required three points. If they had drawn a suitable line through their two end points and then done a one-third shift toward their middle point, a method mark may have been gained.

3b.
Height = 70.7 + 0.7 *Age*

One mark was awarded for each of the coefficients found from any two of the correct median points, but the variables *Height* and *Age* were required rather than *x* and *y*. A number of students used estimated values for their line despite having quite precise values for these three (circled) points.

3c.
The three median line is not as influenced by extreme values such as (20, 93).

This question was quite poorly answered. Several students seemed to give quotations from textbooks but usually did not explain the relevance to the relationship in this question.

Module 1 – Number patterns
Question 1

Marks	0	1	2	3	Average
%	5	15	39	41	**2.2**

1a.
42 000

1b.
d = –3000

Many students omitted the negative sign.

1c.
16 days

Question 2

Marks	0	1	2	3	4	5	6	7	Average
%	18	4	6	11	16	16	15	15	**3.9**

2a.
$$\frac{500}{625} = \frac{400}{500} = 0.8$$

Two calculations were needed to establish the **common** ratio of 0.8.

2b.
256

2c.
$$H_n = 625 \times (0.8)^{n-1}$$

A difference equation was not appropriate here as a formula in terms of n was required.

2d.
41

2e.
10th month

2f.
1385

As in 2005, many students simply found the value of S_9 which, in most cases such as this, does not equal $S_{12} - S_3$.

Question 3

Marks	0	1	2	3	4	5	Average
%	26	26	14	11	15	8	**1.9**

3a.
r = 0.9, d = 2000

An incorrect value of $r = 0.1$ was common.

3b.
38 225

3c.
Day 10

3d.
20 000

Many students did not recognise this as a sum to infinity. An incorrect value of 2000 was a common response.

Module 2 – Geometry and trigonometry
Question 1
1a–d.

Marks	0	1	2	3	4	5	Average
%	10	15	19	16	11	30	**3.0**

1a.
45°

1b.
77.8 m

1c.
033°

Again, bearings proved to be a problem for many students.

1d.
102.9 m

This question was often poorly done. Some students incorrectly assumed that angle *AXC* was 90° or that triangle *YXC* was isosceles.

1e–g.

Marks	0	1	2	3	4	5	Average
%	23	17	22	23	5	10	**2.0**

1e.
12 601 m^2

Students who used Heron's formula to get an answer of 12 602 were awarded full marks.

1fi.
$$BC^2 = x^2 = 251^2 + 142^2 - 2(251)(142)\cos 45°$$
$$BC = x = \sqrt{251^2 + 142^2 - 2(251)(142)\cos 45°} \approx 181$$

Students were required to substitute into the formula. Listing the inputs for a cosine rule calculator application was not acceptable.

1fii.
33.7°

1g.
80 m

This question was very poorly answered. Most students began with $\frac{1}{2}bc = 3200$ or $bc = 6400$ but then failed to

recognise the equality of *b* and *c* and did not take the square root. Some students seemed to confuse the line *MN* with the line *XY* on the previous page, and some tried to use an area ratio but were often unsuccessful.

Question 2

Marks	0	1	2	Average
%	34	32	34	**1.0**

2a.
45.5 m

2b.
24.9°

Finding the size of the incorrect angle was a common error.

Question 3

Marks	0	1	2	3	Average
%	34	22	3	41	**1.5**

3a.
1.5 m

An incorrect value of 1.75 was a common response.

3b.
13 m^3

Many students did not find the correct internal height (1.9 m) of the tank, which was necessary for a method mark.
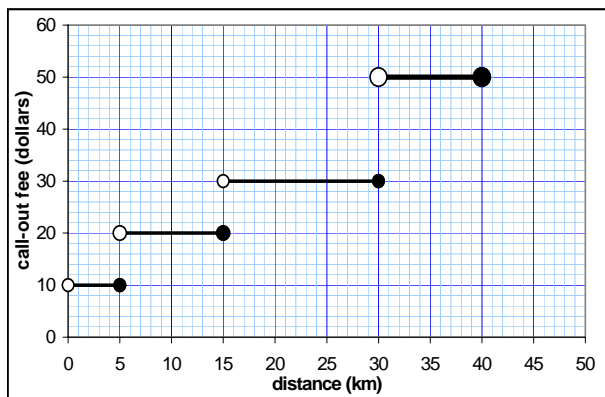
Module 3 – Graphs and relations

Question 1

Marks	0	1	2	3	Average
%	2	6	24	68	**2.6**

1ai.
$30

1aii.
5 km.

1b.

Question 2

Marks	0	1	2	3	4	Average
%	18	17	21	20	24	**2.2**

2a.

$$f = -\frac{1}{8}d + 50$$

The negative sign on the gradient was missed by a number of students.

2b.
240 km

An incorrect answer of 400 km was common, due to incomplete reading of the question.
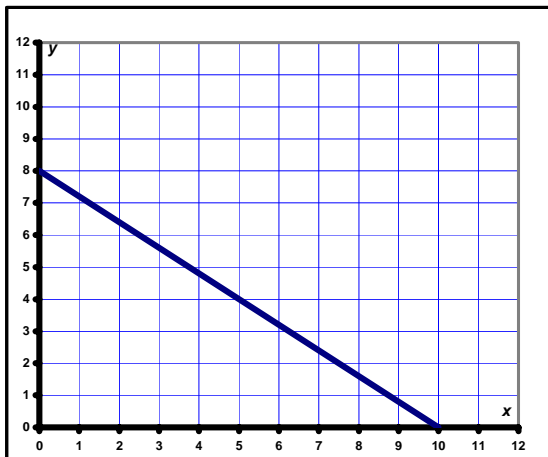
2c.
75 litres

An incorrect answer of 63 litres was common, due to incomplete reading of the question.

Question 3
3a–b.

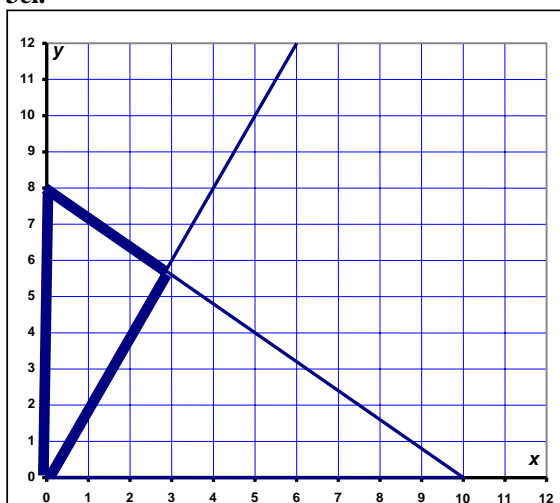| Marks	0	1	2	Average
%	33	38	29	**1.0**

3a.

3b.
$y \geq 2x$

The equation $2y \geq x$ was a common incorrect response.

3c–eii.

Marks	0	1	2	3	4	5	6	Average
%	29	27	16	12	10	1	4	**1.7**

3ci.

A method mark was available for correctly plotting an incorrect equation from Question 3b

Although the question asked for the **boundaries** of the region to be highlighted, most students shaded the feasible region, which was accepted.

3cii.
2 dogs

A consequential mark was available here but many students were unable to interpret the *x* value from their graph when
$y = 5$.

3d.
$P = 40x + 30y$

3ei.
2 washes and 6 clips

The maximum value was not at a vertex of the feasible region. Incorrect answers often included decimal values.

3eii.
$260

Module 4 – Business-related mathematics

Question 1

Marks	0	1	2	3	4	5	6	7	8	Average
%	5	6	7	18	13	18	13	7	12	**4.5**

1ai.
$6000

1aii.
$42 000

1aiii.
8

1bi.
15%

An incorrect answer of 85% was common.

1bii.
$36 847.50

Many students got this answer despite having part i. incorrect.

1biii.
10th year

1c.
7th year

2–3c.

Marks	0	1	2	3	4	5	Average
%	17	13	22	19	16	13	**2.5**

Question 2
$70 300

Question 3
3a.
$10 500

Many students earned only one mark by showing the calculation but only finding the interest earned – $3500.

3b.
$16 103

3c.
$25 935

Question 4

Marks	0	1	2	Average
%	76	7	17	**0.4**

$2150

This question was often poorly done, if attempted at all. Student first had to find the total interest paid over 24 months ($922.90 per month) using a TVM Solver function on their calculator.

Total paid = 24 × 922.90 = $22 149.60
∴ Interest = 22 150 − 20 000 = $2150

Many students rounded off the $922.90 to $923 before progressing with their calculation and therefore arrived at an inaccurate answer. Rounding off should only be done at the last step in a calculation. Others misread the question and simply found the future value (FV) without allowing for any instalments.

Module 5 – Networks
Question 1
1a–b.

Marks	0	1	2	3	Average
%	4	5	7	84	**2.7**

1a.
George

1b.

Person	Position
Harriet	drums
Ian	saxophone
Keith	keyboard

One mark was awarded if any two of these instruments were attributed to the correct person.
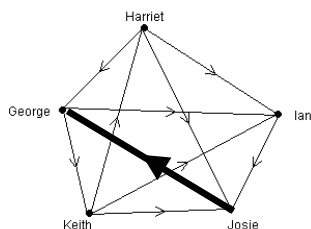
Question 2
2a–b.

Marks	0	1	2	Average
%	7	17	76	**1.7**

2a.
No musician competes against him/her self.

Answers that referred to 'the absence of loops in the directed graph' were not accepted as they did not **explain** why these values are zero.

2b.

2c–e.

Marks	0	1	2	3	4	Average
%	18	21	19	22	20	**2.1**

2c.

Two-step dominance is via dominance over a person (Keith) who beats Ian.

A common incorrect response simply suggested that George had won more games than Ian.

2d.

$x = 2$

This figure could be found from following the edges on the network diagram.
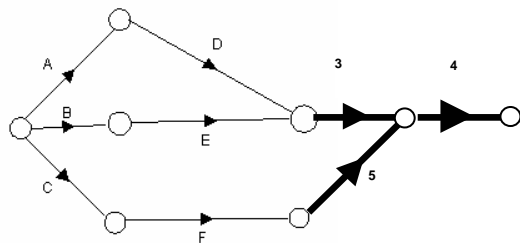
2e.

- First: Keith
- Last: Ian

The total of Keith's one-step plus two-step dominances is the highest, while Ian's is the lowest

Question 3

3a–b.

Marks	0	1	2	3	Average
%	12	14	44	30	**1.9**

3a.

Arrows were required for full marks.

3b.

5

There are only three complete paths from start to finish. Without even considering the duration of any activities, any one of these three paths leaves five activities unaccounted for in the other two paths.

3c–e.

Marks	0	1	2	3	Average
%	38	27	24	11	**1.1**

3c.

B–E–G–I

Every activity on a critical path has **zero** slack time (float). From the given information, this means that *A* and *C* cannot be on a critical path, which leaves only one complete path as an option.

3d.

7 hours

3e.

8 hours

Many students incorrectly answered 9 hours.

Activity C has a duration of 3 hours and a float of 1 hour. Since activity I must commence after 12 hours, this leaves a maximum of 8 hours for activities F and H combined.

Module 6 – Matrices
Question 1

Marks	0	1	2	3	4	Average
%	8	11	17	30	34	**2.7**

1a.

2×3

1bi.

$$M = \begin{bmatrix} 145978.00 \\ 171848.50 \end{bmatrix}$$

1bii.

Total revenue from selling products A, B and C at Eastown and Noxland

1c.

number of columns in $P \neq$ number of rows in Q

Many answers illustrated a poor understanding of the need for the number of columns in matrix P to be the same as the number of rows in matrix Q. Some students simply said that the 'second number' in matrix P had to equal the 'first number' in matrix Q. Although this was accepted in 2006, it did not indicate whether the student knew which numbers referred to rows and which to columns.

Question 2
2a–b.

Marks	0	1	2	3	Average
%	7	18	10	65	**2.4**

2a.

$$T = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}$$

Some students wrote percentages despite being asked for proportions. Some simply copied the figures in order, as printed into the three columns. The percentage, 8%, was sometimes written as a proportion of 0.8.

2b.

$$K_0 = \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix}$$

2c–d.

Marks	0	1	2	3	4	Average
%	27	12	30	10	20	**1.8**

2c.

$$T K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix} \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 268800 \\ 136200 \\ 195000 \end{bmatrix}$$

The expression and the result were both required for full marks.

2d.

Any two products $T^n K_0$ where $n \geq 38$. For example:

$$K_{38} = T^{38}.K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}^{38} \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$

and

$$K_{39} = T^{39}.K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}^{39} \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix} \text{ and this is the same as } K_{38}$$

Question 3

Marks	0	1	2	3	4	Average
%	45	14	12	18	11	**1.4**

3a.

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \\ 6 \end{bmatrix}$$

Many students did not deal well with the missing x term in the third equation. A common error was to put the zero in the wrong position on the third line or to simply leave a gap. Neither of these two options was awarded a mark.

3b.

There is a unique solution since $\det \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix} = 1 \neq 0$

Incorrect answers included those where students justified their response by quoting the actual solutions to the equations.

3c.

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & -1 \\ 2 & -4 & -3 \end{bmatrix}$$

A common incorrect answer was $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix}^{-1}$

3d.
- 3 bookshops
- 4 sports shoe shops
- 2 music stores