VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
Year

GENERAL MATHEMATICS

Written examination 1

Day Date

Reading time: *. to *.** (15 minutes)**
Writing time: *. to *.** (1 hour 30 minutes)**

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
40	40	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question book of 21 pages
- Formula sheet
- Answer sheet for multiple-choice questions
- Working space is provided throughout the book.

Instructions
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book and the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

March 2023

Instructions

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1; an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Use the following information to answer Questions 1 and 2.

The *blood pressure* (low, normal, high) and the *age* (under 50 years, 50 years or over) of 110 adults were recorded. The results are displayed in the two-way frequency table below.

Blood pressure	**Age**	
	Under 50 years	**50 years or over**
low	15	5
normal	32	24
high	11	23
Total	58	52

Question 1

The variables *blood pressure* (low, normal, high) and *age* (under 50 years, 50 years or over) are

A. both categorical variables.
B. both numerical variables.
C. a numerical variable and a categorical variable respectively.
D. a categorical variable and a numerical variable respectively.
E. a discrete variable and a continuous variable respectively.

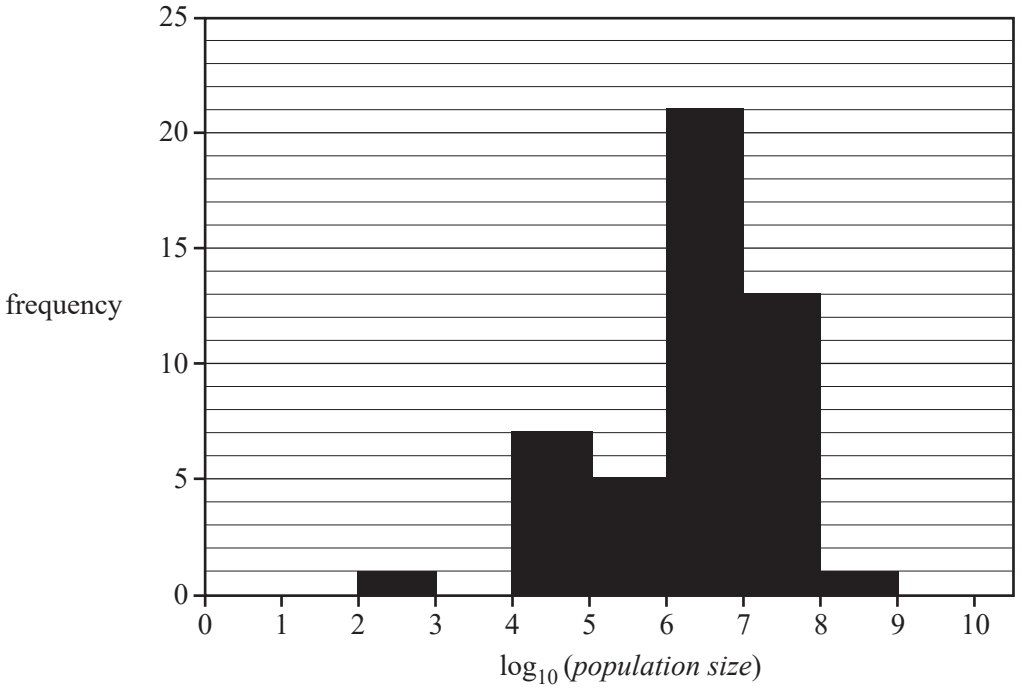
Question 2

The percentage of adults under 50 years of age who have high blood pressure is closest to

A. 11%
B. 19%
C. 26%
D. 44%
E. 58%

Question 3

The histogram below shows the *population size* for 48 countries plotted on a logarithmic (base 10) scale.



Data: Worldometers, <www.worldometers.info/>

Based on this histogram, the number of countries with a *population size* that is less than 100 000 people is

A. 1

B. 7

C. 8

D. 13

E. 48

Use the following information to answer Questions 4–6.

The pulse rates of a population of Year 12 students are approximately normally distributed with a mean of 69 beats per minute and a standard deviation of 4 beats per minute.

Question 4

A student selected at random from this population has a standardised pulse rate of $z = -2.5$

This student's actual pulse rate is

A. 59 beats per minute.

B. 63 beats per minute.

C. 65 beats per minute.

D. 73 beats per minute.

E. 79 beats per minute.

Question 5

Another student selected at random from this population has a standardised pulse rate of $z = -1$.

The percentage of students in this population with a pulse rate greater than this student is closest to

A. 2.5%

B. 5%

C. 16%

D. 68%

E. 84%

Question 6

A sample of 200 students was selected at random from this population.

The number of these students with a pulse rate of less than 61 beats per minute or greater than 73 beats per minute is closest to

A. 19

B. 37

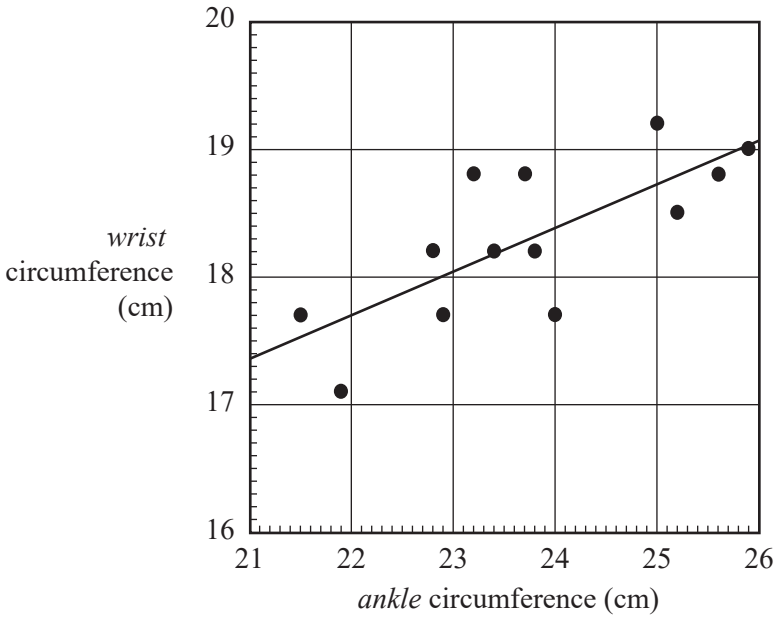
C. 64

D. 95

E. 190

Use the following information to answer Questions 7–9.

The scatterplot below shows the *wrist* circumference and *ankle* circumference, both in centimetres, of 13 people. A least squares line has been fitted to the scatterplot with *ankle* circumference as the explanatory variable.



Question 7

The equation of the least squares line is closest to

A. *ankle* = 10.2 + 0.342 × *wrist*

B. *wrist* = 10.2 + 0.342 × *ankle*

C. *ankle* = 17.4 + 0.342 × *wrist*

D. *wrist* = 17.4 + 0.342 × *ankle*

E. *wrist* = 17.4 + 0.731 × *ankle*

Question 8

When the least squares line on the scatterplot is used to predict the wrist circumference of the person with an ankle circumference of 24 cm, the residual will be closest to

A. –0.7

B. –0.4

C. –0.1

D. 0.4

E. 0.7

Question 9

The residuals for the least squares line have a mean of 0.02 cm and a standard deviation of 0.4 cm.

The value of the residual for one of the data points is found to be –0.3 cm.

The standardised value of this residual is

A. –0.8

B. –0.7

C. –0.3

D. 0.7

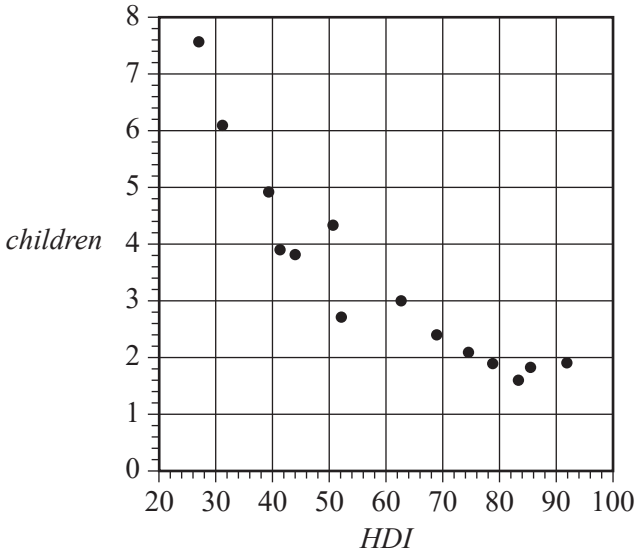
E. 0.8

Question 10

The table below gives the Human Development Index (*HDI*) and the mean number of children per woman (*children*) for 14 countries in 2007.

A scatterplot of the data is also shown.

HDI	***Children***
27.3	7.6
31.3	6.1
39.5	4.9
41.6	3.9
44.0	3.8
50.8	4.3
52.3	2.7
62.5	3.0
69.1	2.4
74.6	2.1
78.9	1.9
85.6	1.8
92.0	1.9
83.4	1.6



Data: Gapminder

The scatterplot is non-linear.

A log transformation applied to the variable *children* can be used to linearise the scatterplot.

With *HDI* as the explanatory variable, the equation of the least squares line fitted to the linearised data is closest to

A. $\log_{10}(children) = 1.1 - 0.0095 \times HDI$

B. $children = 1.1 - 0.0095 \times \log_{10}(HDI)$

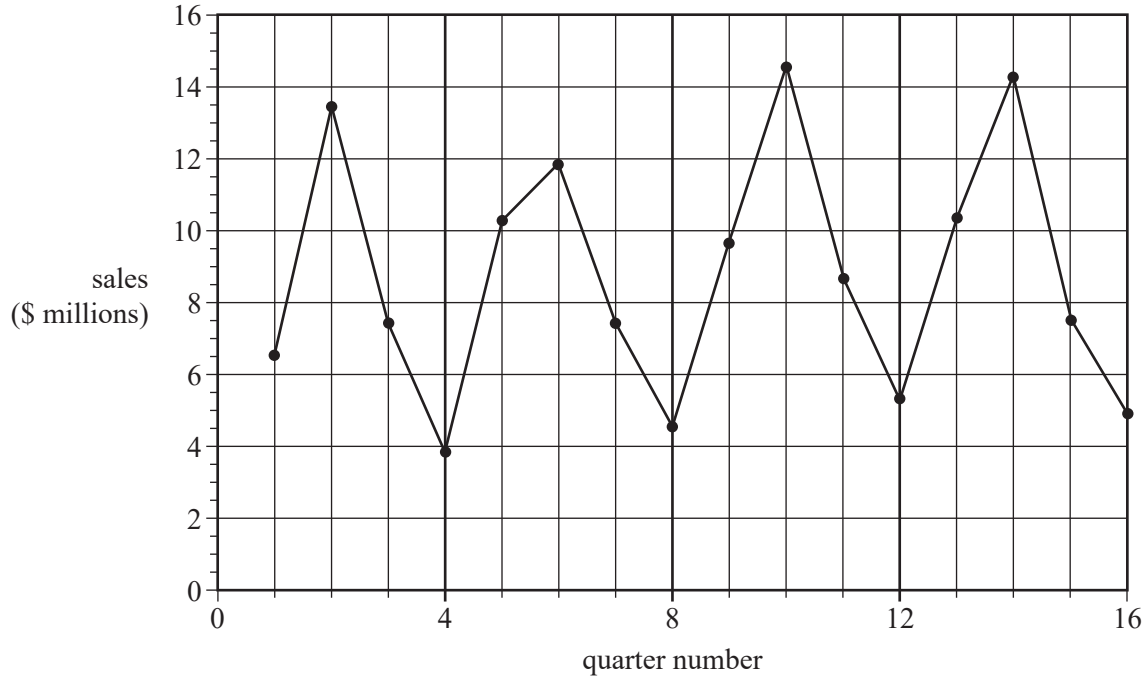
C. $\log_{10}(children) = 8.0 - 0.77 \times HDI$

D. $children = 8.0 - 0.77 \times \log_{10}(HDI)$

E. $\log_{10}(children) = 21 - 10 \times HDI$

Use the following information to answer Questions 11 and 12.

The time series plot below charts the quarterly sales figures, in millions of dollars, of a small manufacturing business over a period of four years.



Question 11

The time series plot is best described as having

A. seasonality only.

B. irregular fluctuations only.

C. seasonality with irregular fluctuations.

D. a decreasing trend with seasonality and irregular fluctuations.

E. an increasing trend with seasonality and irregular fluctuations.

Question 12

The sales figures used to generate this time series plot are displayed in the table below.

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2018	6.5	13.4	7.4	3.8
2019	10.2	11.8	7.4	4.5
2020	9.6	14.5	8.6	5.3
2021	10.3	14.2	7.5	4.9

The four-mean smoothed sales with centring for Quarter 3 in 2020, in millions of dollars, was closest to

A. 8.6

B. 9.3

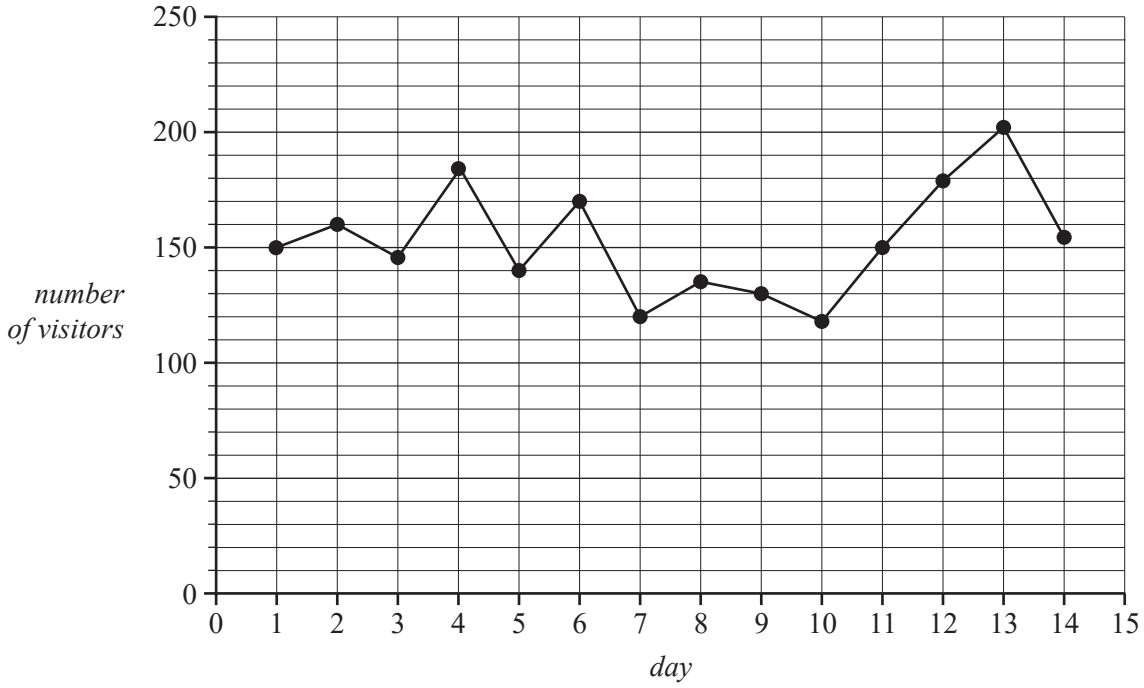
C. 9.5

D. 9.6

E. 9.7

Question 13

The time series plot below shows the daily number of visitors to a historical site over a two-week period.



This time series plot is to be smoothed using seven-median smoothing.

The seven-median smoothed number of visitors on day 4 is closest to

A. 120

B. 140

C. 145

D. 150

E. 160

Use the following information to answer Questions 14–16.

The table below shows the long-term average of the number of meals served each day at a restaurant. Also shown is the daily seasonal index for Monday through to Friday.

	Day of the week						
	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**	**Saturday**	**Sunday**
Long-term average	89	93	110	132	145	190	160
Seasonal index	0.68	0.71	0.84	1.01	1.10		

Question 14

The seasonal index for Wednesday is 0.84

This tells us that, on average, the number of meals served on a Wednesday is

A. 16% less than the daily average.

B. 84% less than the daily average.

C. the same as the daily average.

D. 16% more than the daily average.

E. 84% more than the daily average.

Question 15

Last Tuesday, 108 meals were served in the restaurant.

The deseasonalised number of meals served last Tuesday was closest to

A. 77

B. 93

C. 110

D. 131

E. 152

Question 16

The seasonal index for Saturday is closest to

A. 1.22

B. 1.31

C. 1.38

D. 1.45

E. 1.49

Recursion and financial modelling

Question 17

Consider the recurrence relation shown below.

$$A_0 = 3, \qquad A_{n+1} = 2A_n + 4$$

The value of A_3 in the sequence generated by this recurrence relation is given by

A. $2 \times 3 + 4$

B. $2 \times 4 + 4$

C. $2 \times 10 + 4$

D. $2 \times 24 + 4$

E. $2 \times 52 + 4$

Question 18

Geoff purchased a computer for $4500. He will depreciate the value of his computer by a flat rate of 10% of the purchase price per annum.

A recurrence relation that Geoff can use to determine the value of the computer after n years, V_n, is

A. $V_0 = 4500, \quad V_{n+1} = V_n - 450$

B. $V_0 = 4500, \quad V_{n+1} = V_n + 450$

C. $V_0 = 4500, \quad V_{n+1} = 0.9\ V_n$

D. $V_0 = 4500, \quad V_{n+1} = 1.1\ V_n$

E. $V_0 = 4500, \quad V_{n+1} = 0.1\ (V_n - 450)$

Question 19

Manu invests $3000 in an account that pays interest compounding monthly.

The balance of his investment after n months, B_n, can be determined using the recurrence relation

$$B_0 = 3000, \qquad B_{n+1} = 1.0048 \times B_n$$

The total interest earned by Manu's investment after the first five months is closest to

A. $57.60

B. $58.02

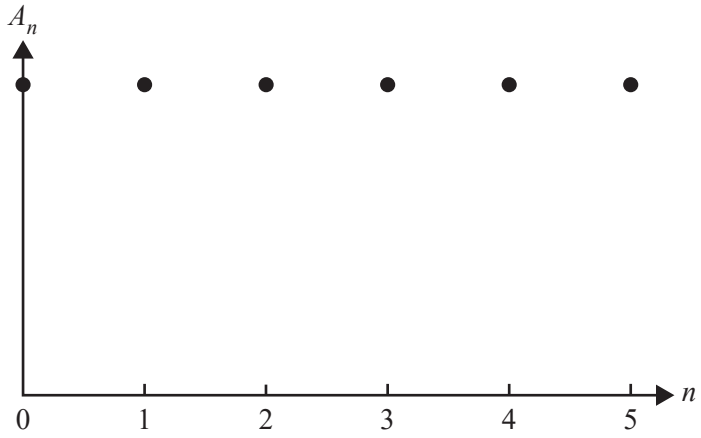
C. $72.00

D. $72.69

E. $87.44

Question 20

The graph below represents the value A_n, in dollars, of an annuity investment for five time periods.



A recurrence relation that could match this graphical representation is

A. $A_0 = 200\,000, \quad A_{n+1} = 1.015A_n - 2500$

B. $A_0 = 200\,000, \quad A_{n+1} = 1.025A_n - 5000$

C. $A_0 = 200\,000, \quad A_{n+1} = 1.03A_n - 5500$

D. $A_0 = 200\,000, \quad A_{n+1} = 1.04A_n - 6000$

E. $A_0 = 200\,000, \quad A_{n+1} = 1.05A_n - 8000$

Question 21

Ray deposited \$7000 in an investment account earning interest at the rate of 3% per annum, compounding quarterly.

A rule for the balance, R_n, in dollars, after n years is given by

A. $R_n = 7000 \times 0.03^n$

B. $R_n = 7000 \times 1.03^n$

C. $R_n = 7000 \times 0.03^{4n}$

D. $R_n = 7000 \times 1.0075^n$

E. $R_n = 7000 \times 1.0075^{4n}$

Question 22

The value of a van purchased for \$45 000 is depreciated by k% per annum using the reducing balance method.

After three years of this depreciation, it is then depreciated in the fourth year under the unit cost method at the rate of 15 cents per kilometre.

The value of the van after it travels 30 000 km in this fourth year is \$26 166.24

The value of k is

A. 9

B. 12

C. 14

D. 16

E. 18

Question 23

Barb took out a personal loan, borrowing $5000.

Interest for this loan compounds quarterly.

Barb makes no repayments in the first year and after one year she owes $5325.14

The effective annual rate of interest for the first year of Barb's loan is closest to

A. 6.34%

B. 6.35%

C. 6.50%

D. 6.54%

E. 6.56%

Question 24

Twenty years ago, Hector invested a sum of money in an account earning interest at the rate of 3.2% per annum, compounding monthly.

After 10 years, he made a one-off extra payment of $10 000 to the account.

For the next 10 years, the account earned interest at the rate of 2.8% per annum, compounding monthly.

The balance of his account today is $686 904.09

The sum of money Hector originally invested is closest to

A. $355 000

B. $370 000

C. $377 000

D. $384 000

E. $385 000

Matrices

Question 25

The matrix below shows how five people, Anita (A), Beverly (B), Christy (C), Dion (D) and Eddie (E), can communicate with each other.

$$\begin{array}{rc} & \begin{array}{ccccc} \multicolumn{5}{c}{\textit{receiver}} \\ A & B & C & D & E \end{array} \\ \begin{array}{r} \\ A \\ B \\ \textit{sender}\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

A '1' in the matrix shows that the person named in that row can send a message directly to the person named in that column.

For example, the '1' in row 3 and column 4 shows that Christy can send a message directly to Dion.

Eddie wants to send a message to Beverly.

Which one of the following shows the order of people through which the message is sent?

A. Eddie – Beverly

B. Eddie – Christy – Beverly

C. Eddie – Christy – Anita – Beverly

D. Eddie – Christy – Dion – Beverly

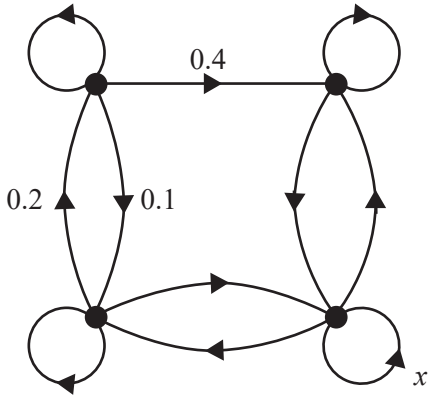
E. Eddie – Christy – Dion – Anita – Beverly

Question 26

A transition matrix, V, is shown below.

$$V = \begin{array}{c} \\ L \\ T \\ F \\ M \end{array} \begin{array}{c} \text{this month} \\ \begin{array}{cccc} L & T & F & M \end{array} \\ \begin{bmatrix} 0.6 & 0.6 & 0.2 & 0.0 \\ 0.1 & 0.2 & 0.0 & 0.1 \\ 0.3 & 0.0 & 0.8 & 0.4 \\ 0.0 & 0.2 & 0.0 & 0.5 \end{bmatrix} \end{array} \begin{array}{l} \\ L \\ T \\ F \\ M \end{array} \quad \text{next month}$$

The transition diagram below has been constructed from the transition matrix V.
The labelling in the transition diagram is not yet complete.



The proportion for one of the transitions is labelled x.
The value of x is

A. 0.1
B. 0.2
C. 0.5
D. 0.6
E. 0.8

Question 27

Four teams, A, B, C and D, competed in a round-robin competition in which each team played each of the other teams once. There were no draws.

The results are shown in the matrix below.

$$\begin{array}{cc} & \text{loser} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \text{winner} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 0 & 0 & f & 1 \\ 1 & 0 & 0 & 0 \\ 1 & g & 0 & 1 \\ 0 & 1 & 0 & h \end{bmatrix} \end{array}$$

A '1' in the matrix shows that the team named in that row defeated the team named in that column.

For example, the '1' in row 2 shows that team B defeated team A.

In this matrix, the values of f, g and h are

A. $f = 0, \quad g = 1, \quad h = 0$

B. $f = 0, \quad g = 1, \quad h = 1$

C. $f = 1, \quad g = 0, \quad h = 0$

D. $f = 1, \quad g = 1, \quad h = 0$

E. $f = 1, \quad g = 1, \quad h = 1$

Question 28

Consider the matrix P, where $P = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$.

The element in row i and column j of matrix P is p_{ij}.

The elements in matrix P are determined by the rule

A. $p_{ij} = 4 - j$

B. $p_{ij} = 2i + 1$

C. $p_{ij} = i + j + 1$

D. $p_{ij} = i + 2j$

E. $p_{ij} = 2i - j + 2$

Question 29

The following Leslie matrix, L, can be used to model how a population of female animals of three age groups changes over time.

$$L = \begin{bmatrix} 0 & 0 & k \\ 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}$$

where k is the average number of female offspring from a female animal in the third age group during one time period.

Let S_n represent the state matrix showing the population of each of the three age groups after n time periods. The matrix recurrence rule $S_{n+1} = LS_n$ is used to model this situation.

Given $S_0 = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$, how many of the following statements are true?

- When $k = 1$, the population will remain at 100 over time.
- When $k = 2$, the population will decrease in the long term.
- When $k = 3$, the population will increase in the long term.
- When $k = 4$, the population will return to the same population age distribution every three years.

A. 0
B. 1
C. 2
D. 3
E. 4

Question 30

The table below shows information about three matrices, A, B and C.

Matrix	Order
A	2×4
B	2×3
C	3×4

The transpose of matrix A, for example, is written as A^T.
What is the order of the product $C^T \times (A^T \times B)^T$?

A. 2×3
B. 3×4
C. 4×2
D. 4×3
E. 4×4

Question 31

Matrix P has inverse matrix P^{-1}.

Matrix P is multiplied by the scalar $w\,(w \neq 0)$ to form matrix Q.

Matrix Q^{-1} is equal to

A. $\frac{1}{w}P^{-1}$

B. $\frac{1}{w^2}P^{-1}$

C. wP^{-1}

D. w^2P^{-1}

E. P^{-1}

Question 32

Consider the matrix recurrence relation below.

$$S_0 = \begin{bmatrix} 30 \\ 20 \\ 40 \end{bmatrix}, \qquad S_{n+1} = T\,S_n \qquad \text{where } T = \begin{bmatrix} j & 0.3 & l \\ 0.2 & m & 0.3 \\ 0.4 & 0.2 & n \end{bmatrix}$$

Matrix T is a regular transition matrix.

Given the information above and that $S_1 = \begin{bmatrix} 42 \\ 28 \\ 20 \end{bmatrix}$, which one of the following is true?

A. $m > l$

B. $j + l = 0.7$

C. $j = n$

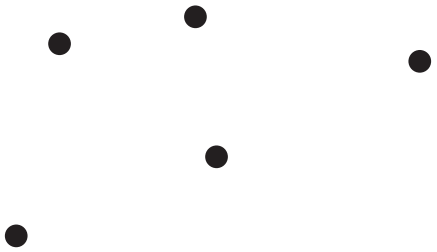
D. $j > m$

E. $l = m + n$

Networks and decision mathematics

Question 33

Consider the graph with five isolated vertices shown below.

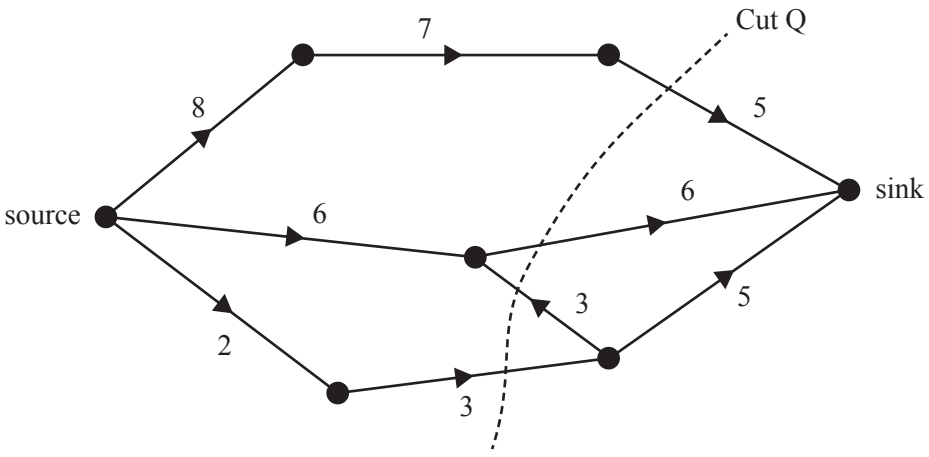


To form a tree, the minimum number of edges that must be added to the graph is

A. 1
B. 4
C. 5
D. 6
E. 10

Question 34

The flow of water through a series of pipes is shown in the network below.
The numbers on the edges show the maximum flow through each pipe in litres per minute.



The capacity of Cut Q, in litres per minute, is

A. 11
B. 13
C. 14
D. 16
E. 17

Question 35

A planar graph has five faces.

This graph could have

A. eight vertices and eight edges.

B. six vertices and eight edges.

C. eight vertices and five edges.

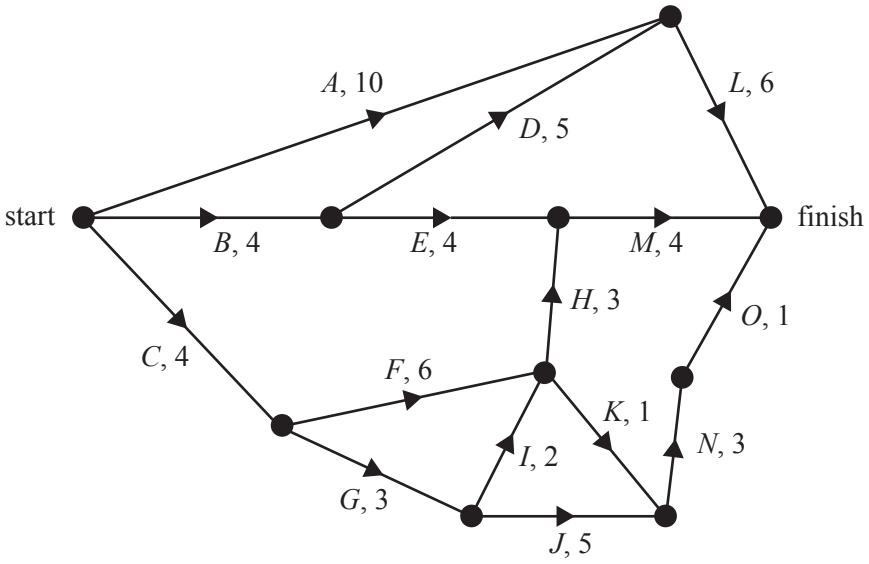
D. eight vertices and six edges.

E. five vertices and eight edges.

Use the following information to answer Questions 36 and 37.

The directed graph below shows the sequence of activities required to complete a project.

The time to complete each activity, in hours, is also shown.



Question 36

The earliest starting time, in hours, for activity *N* is

A. 3

B. 10

C. 11

D. 12

E. 13

Question 37

To complete the project in minimum time, some activities cannot be delayed.

The number of activities that cannot be delayed is

A. 2

B. 3

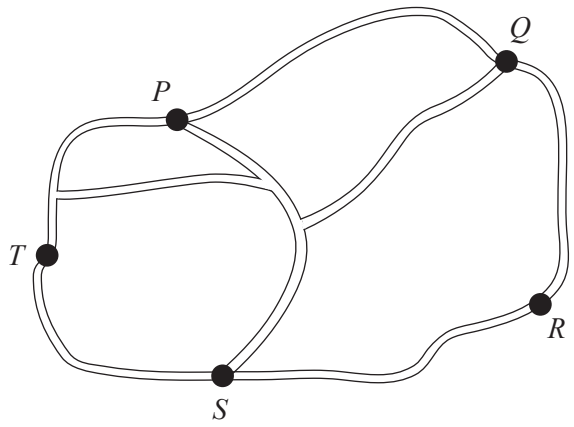
C. 4

D. 5

E. 6

Question 38

The map below shows all the road connections between five towns, P, Q, R, S and T.



The road connections could be represented by the adjacency matrix

A.

	P	Q	R	S	T
P	1	3	0	2	2
Q	3	0	1	1	1
R	0	1	0	1	0
S	2	1	1	0	2
T	2	1	0	2	0

B.

	P	Q	R	S	T
P	1	2	0	2	2
Q	2	0	1	1	1
R	0	1	0	1	0
S	2	1	1	0	2
T	2	1	0	2	0

C.

	P	Q	R	S	T
P	0	3	0	2	2
Q	3	0	1	1	1
R	0	1	0	1	0
S	2	1	1	0	2
T	2	1	0	2	0

D.

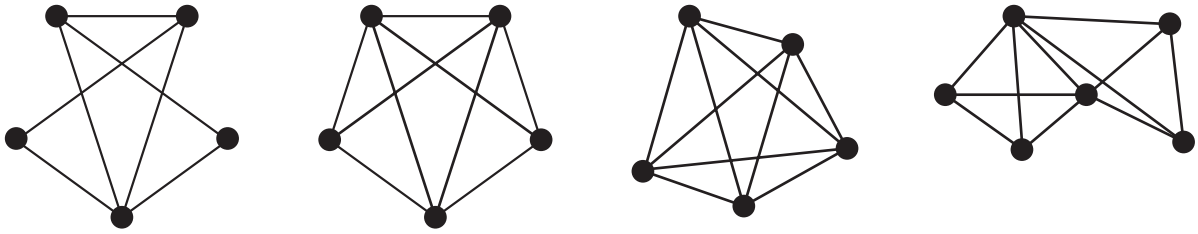
	P	Q	R	S	T
P	0	2	0	2	2
Q	2	0	1	1	1
R	0	1	0	1	0
S	2	1	1	1	2
T	2	1	0	2	0

E.

	P	Q	R	S	T
P	1	2	0	2	2
Q	2	0	1	1	1
R	0	1	0	1	0
S	2	1	1	1	1
T	2	1	0	1	0

Question 39

Consider the following four graphs.



How many of these four graphs are planar?

A. 0

B. 1

C. 2

D. 3

E. 4

Question 40

Five children, Alan, Brianna, Chamath, Deidre and Ewen, are each to be assigned a different job by their teacher. The table below shows the time, in minutes, that each child would take to complete each of the five jobs.

	Alan	**Brianna**	**Chamath**	**Deidre**	**Ewen**
Job 1	5	8	5	8	7
Job 2	5	7	6	7	4
Job 3	9	5	7	5	9
Job 4	7	7	9	8	5
Job 5	4	4	4	4	3

The teacher wants to allocate the jobs so as to minimise the total time taken to complete the five jobs.

In doing so, she finds that two allocations are possible.

If each child starts their allocated job at the same time, then the first child to finish could be either

A. Alan or Brianna.

B. Brianna or Deidre.

C. Chamath or Deidre.

D. Chamath or Ewen.

E. Deidre or Ewen.

END OF MULTIPLE-CHOICE QUESTION BOOK

Answers to multiple-choice questions

Question	Answer
1	A
2	B
3	C
4	A
5	E
6	B
7	B
8	A
9	A
10	A
11	C
12	D
13	D
14	A
15	E
16	D
17	D
18	A
19	D
20	B

Question	Answer
21	E
22	B
23	C
24	B
25	E
26	D
27	A
28	E
29	C
30	E
31	A
32	E
33	B
34	C
35	E
36	D
37	C
38	A
39	D
40	B