

**Updated – version 1.1**

Amendments to study design history

|  |  |  |  |
| --- | --- | --- | --- |
| Version | Status | Release Date | Comments |
| 1.1 | Current | November 2022 | Update to key knowledge for Matrices in Units 3 and 4 General Mathematics (page 91). |
| 1.0 | Superseded | February 2022 | Original study design. |

Authorised and published by the Victorian Curriculum and Assessment Authority
Level 7, 2 Lonsdale Street
Melbourne VIC 3000

ISBN: 978-1-925264-18-0

© Victorian Curriculum and Assessment Authority 2022

No part of this publication may be reproduced except as specified under the *Copyright Act 1968* or by permission from the VCAA. Excepting third-party elements, schools may use this resource in accordance with the [VCAA educational allowance](https://www.vcaa.vic.edu.au/Footer/Pages/Copyright.aspx). For more information read the [VCAA copyright policy](http://www.vcaa.vic.edu.au/Footer/Pages/Copyright.aspx).

The VCAA provides the only official, up-to-date versions of VCAA publications. Details of updates can be found on the [VCAA website](http://www.vcaa.vic.edu.au).

This publication may contain copyright material belonging to a third party. Every effort has been made to contact all copyright owners. If you believe that material in this publication is an infringement of your copyright, please email the Copyright Officer.

Copyright in materials appearing at any sites linked to this document rests with the copyright owner(s) of those materials, subject to the Copyright Act. The VCAA recommends you refer to copyright statements at linked sites before using such materials.

The VCAA logo is a registered trademark of the Victorian Curriculum and Assessment Authority.

|  |
| --- |
| Contact us if you need this information in an accessible format: for example, large print or audio.Telephone (03) 9032 1635 or email vcaa.media.publications@education.vic.gov.au |

Contents

[Important information 6](#_Toc71028293)

[**Introduction** **7**](#_Toc71028296)

[Scope of study 7](#_Toc71028297)

[Rationale 7](#_Toc71028298)

[Aims 7](#_Toc71028299)

[Structure 7](#_Toc71028300)

[Entry 9](#_Toc71028301)

[Duration 9](#_Toc71028302)

[Changes to the study design 9](#_Toc71028303)

[Monitoring for quality 9](#_Toc71028304)

[Safety and wellbeing 9](#_Toc71028305)

[Employability skills 9](#_Toc71028306)

[Legislative compliance 9](#_Toc71028307)

[Child Safe Standards 10](#_Toc71028307)

[**Assessment and reporting** **11**](#_Toc71028308)

[Satisfactory completion 11](#_Toc71028309)

[Levels of achievement 11](#_Toc71028310)

[Authentication 12](#_Toc71028311)

[**Unit 1: Foundation Mathematics** **13**](#_Toc71028312)

[Area of Study 1 13](#_Toc71028313)

[Area of Study 2 13](#_Toc71028314)

[Area of Study 3 14](#_Toc71028315)

[Area of Study 4 14](#_Toc71028316)

[Mathematical investigation 14](#_Toc71028317)

[Outcomes 16](#_Toc71028318)

[Assessment 19](#_Toc71028319)

[**Unit 2: Foundation Mathematics** **20**](#_Toc71028320)

[Area of Study 1 20](#_Toc71028321)

[Area of Study 2 20](#_Toc71028322)

[Area of Study 3 21](#_Toc71028323)

[Area of Study 4 21](#_Toc71028324)

[Mathematical investigation 21](#_Toc71028325)

[Outcomes 23](#_Toc71028326)

[Assessment 26](#_Toc71028327)

[**Unit 1: General Mathematics** **27**](#_Toc71028328)

[Area of Study 1 27](#_Toc71028329)

[Area of Study 2 27](#_Toc71028330)

[Area of Study 3 28](#_Toc71028331)

[Area of Study 4 29](#_Toc71028332)

[Mathematical investigation 29](#_Toc71028333)

[Outcomes 30](#_Toc71028334)

[Assessment 34](#_Toc71028339)

[**Unit 2: General Mathematics** **35**](#_Toc71028340)

[Area of Study 1 35](#_Toc71028341)

[Area of Study 2 35](#_Toc71028342)

[Area of Study 3 36](#_Toc71028343)

[Area of Study 4 36](#_Toc71028344)

[Mathematical investigation 37](#_Toc71028345)

[Outcomes 38](#_Toc71028346)

[Assessment 42](#_Toc71028351)

[**Unit 1: Mathematical Methods** **43**](#_Toc71028352)

[Area of Study 1 43](#_Toc71028353)

[Area of Study 2 44](#_Toc71028354)

[Area of Study 3 44](#_Toc71028355)

[Area of Study 4 45](#_Toc71028356)

[Mathematical investigation 45](#_Toc71028357)

[Outcomes 46](#_Toc71028358)

[Assessment 49](#_Toc71028359)

[**Unit 2: Mathematical Methods** **51**](#_Toc71028360)

[Area of Study 1 51](#_Toc71028361)

[Area of Study 2 52](#_Toc71028362)

[Area of Study 3 52](#_Toc71028363)

[Area of Study 4 53](#_Toc71028364)

[Mathematical investigation 53](#_Toc71028365)

[Outcomes 54](#_Toc71028366)

[Assessment 57](#_Toc71028367)

[**Unit 1: Specialist Mathematics** **58**](#_Toc71028368)

[Area of Study 1 58](#_Toc71028369)

[Area of Study 2 59](#_Toc71028370)

[Mathematical investigation 60](#_Toc71028371)

[Outcomes 61](#_Toc71028372)

[Assessment 64](#_Toc71028373)

[**Unit 2: Specialist Mathematics** **65**](#_Toc71028374)

[Area of Study 1 65](#_Toc71028375)

[Area of Study 2 66](#_Toc71028376)

[Area of Study 3 67](#_Toc71028377)

[Area of Study 4 67](#_Toc71028378)

[Mathematical investigation 68](#_Toc71028379)

[Outcomes 69](#_Toc71028380)

[Assessment 72](#_Toc71028381)

[**Units 3 and 4: Foundation Mathematics** **73**](#_Toc71028382)

[Area of Study 1 73](#_Toc71028383)

[Area of Study 2 74](#_Toc71028384)

[Area of Study 3 74](#_Toc71028385)

[Area of Study 4 75](#_Toc71028386)

[Outcomes 75](#_Toc71028387)

[School-based assessment 79](#_Toc71028388)

[External assessment 82](#_Toc71028389)

[**Units 3 and 4: General Mathematics** **84**](#_Toc71028390)

[Area of Study 1 84](#_Toc71028391)

[Area of Study 2 86](#_Toc71028392)

[Outcomes 89](#_Toc71028393)

[School-based assessment 94](#_Toc71028394)

[External assessment 97](#_Toc71028395)

[**Units 3 and 4: Mathematical Methods** **98**](#_Toc71028396)

[Area of Study 1 98](#_Toc71028397)

[Area of Study 2 99](#_Toc71028398)

[Area of Study 3 99](#_Toc71028399)

[Area of Study 4 100](#_Toc71028400)

[Outcomes 101](#_Toc71028401)

[School-based assessment 105](#_Toc71028402)

[External assessment 107](#_Toc71028403)

[**Units 3 and 4: Specialist Mathematics** **109**](#_Toc71028404)

[Area of Study 1 109](#_Toc71028405)

[Area of Study 2 110](#_Toc71028406)

[Area of Study 3 110](#_Toc71028407)

[Area of Study 4 110](#_Toc71028408)

[Area of Study 5 112](#_Toc71028409)

[Area of Study 6 113](#_Toc71028410)

[Outcomes 114](#_Toc71028411)

[School-based assessment 117](#_Toc71028412)

[External assessment 119](#_Toc71028413)

Important information

Accreditation period

Units 1–4: 1 January 2023 – 31 December 2027

Implementation of this study commences in January 2023.

Other sources of information

The [*VCAA Bulletin*](https://www.vcaa.vic.edu.au/news-and-events/bulletins-and-updates/bulletin/Pages/index.aspx) is the only official source of changes to regulations and accredited studies. The Bulletin also regularly includes advice on VCE studies. It is the responsibility of each VCE teacher to refer to each issue of the Bulletin. The Bulletin is available as an e-newsletter via [free subscription](https://www.vcaa.vic.edu.au/Footer/Pages/Subscribe.aspx) on the VCAA website.

To assist teachers in developing courses, the VCAA publishes online [Support materials](https://www.vcaa.vic.edu.au/curriculum/vce/vce-study-designs/Pages/vce-study-designs.aspx) (incorporating the previously known *Advice for teachers*).

The current [*VCE and VCAL Administrative Handbook*](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx) contains essential information on assessment processes and other procedures.

VCE providers

Throughout this study design the term ‘school’ is intended to include both schools and other VCE providers.

Copyright

Schools may use this VCE study design in accordance with the VCAA educational allowance, including making copies for students and use within the school’s secure online teaching and learning environment.
For details, access the full [VCAA copyright policy](https://www.vcaa.vic.edu.au/Footer/Pages/Copyright.aspx) on the VCAA website.

Introduction

Scope of study

Mathematics is the study of function and pattern in number, logic, space and structure, and of randomness, chance, variability, and uncertainty in data and events. It is both a framework for thinking and a means of symbolic communication that is powerful, logical, concise and precise. Mathematics also provides a means by which people can understand and manage human and natural aspects of the world and interrelationships between these. Essential mathematical activities include conjecturing, hypothesising and problem-posing; estimating, calculating, computing and constructing; abstracting, proving, refuting and inferring; applying, investigating, modelling and problem-solving.

Rationale

This study is designed to provide access to worthwhile and challenging mathematical learning in a way
which takes into account the interests, needs, dispositions and aspirations of a wide range of students, and introduces them to key aspects of the discipline and its applications. It is also designed to promote students’ awareness of the importance of mathematics in everyday life in a technological society and globalised world, and to develop confidence and the disposition to make effective use of mathematical concepts, processes and skills in practical and theoretical contexts.

Aims

This study enables students to:

* develop mathematical concepts, key knowledge and key skills
* apply mathematics to analyse, investigate and model a variety of contexts and solve practical and theoretical problems in situations that range from well-defined and familiar to open-ended and unfamiliar
* apply computational thinking and algorithms, and use technology effectively as a tool for working mathematically.

Structure

The study is made up of the following units:

* Foundation Mathematics Units 1–4
* General Mathematics Units 1–4
* Mathematical Methods Units 1–4
* Specialist Mathematics Units 1–4

Each unit covers specific content contained in areas of study and is designed to enable students to achieve a set of outcomes for that unit. Each outcome is described in terms of key knowledge and key skills.
A glossary defining terms used across Units 1 to 4 in the *VCE Mathematics Study Design* is included in the Support materials*.*

The areas of study from which content is drawn as applicable to each unit are: Algebra, number and structure; Calculus; Data analysis, probability and statistics; Discrete Mathematics; Functions, relations and graphs; and Space and measurement.

Units 1–4 have been developed as a sequence, with Units 1 and 2 covering assumed key knowledge and key skills as preparation for Units 3 and 4.

**Foundation Mathematics Units 1–4** provide for the continuing mathematical development of students with respect to problems encountered in practical contexts in everyday life at home, in the community, at work and in study.

**General Mathematics Units 1–4** provide for the study of non-calculus and discrete mathematics topics. They are designed to be widely accessible and provide preparation for general employment, business or further study, in particular where data analysis, recursion and financial modelling, networks and matrices are important. Students who have done only Mathematical Methods Units 1 and 2 will have had access to assumed key knowledge and key skills for General Mathematics Units 3 and 4 but may also need to undertake some supplementary study.

**Mathematical Methods Units 1–4** provide for the study of simple elementary functions, transformations and combinations of these functions, algebra, calculus, probability and statistics, and their applications in a variety of practical and theoretical contexts. They also provide background for further study in, for example, science, technology, engineering and mathematics (STEM), humanities, economics and medicine.

**Specialist Mathematics Units 1–4** provide for the study of various mathematical structures, reasoning and proof. The areas of study in Units 3 and 4 extend content from Mathematical Methods Units 3 and 4 to include rational and other quotient functions as well as other advanced mathematics topics such as logic and proof, complex numbers, vectors, differential equations, kinematics, and statistical inference. They also provide background for advanced studies in mathematics and other STEM fields. Study of Specialist Mathematics Units 3 and 4 assumes concurrent study or previous completion of Mathematical Methods
Units 3 and 4.

**Combinations of mathematics units**

|  |  |
| --- | --- |
| Units 1 and 2 | Units 3 and 4 |
| **Foundation Mathematics** | Foundation Mathematics |
| **General Mathematics** | General Mathematics or Foundation Mathematics |
| **Mathematical Methods** | Mathematical Methods orGeneral Mathematics |
| **General Mathematics and Mathematical Methods** | General Mathematics and Mathematical Methods  |
| **Mathematical Methods** | Mathematical Methods andSpecialist Mathematics\* |
| **Mathematical Methods and Specialist Mathematics** | Mathematical Methods and Specialist Mathematics |
| **Mathematical Methods and Specialist Mathematics** | General Mathematics, Mathematical Methods and Specialist Mathematics |

\*For this combination of units, students will need to undertake some supplementary study with respect to assumed knowledge and skills for Specialist Mathematics Units 3 and 4.

Entry

There are no prerequisites for entry to Units 1, 2 and 3; however, students undertaking Mathematical Methods Units 1 and 2 or Specialist Mathematics Units 1 and 2 are assumed to have a sound background in number, algebra, function, geometry, probability and statistics. Students must undertake Unit 3 and Unit 4 as a sequence. Units 1 to 4 are designed to a standard equivalent to the final two years of secondary education. All VCE studies are benchmarked against comparable national and international curriculum.

Enrolment in Specialist Mathematics Units 3 and 4 assumes a current enrolment in, or previous completion of, Mathematical Methods Units 3 and 4.

Duration

Each unit involves at least 50 hours of scheduled classroom instruction.

Changes to the study design

During its period of accreditation minor changes to the study will be announced in the [*VCAA Bulletin*](https://www.vcaa.vic.edu.au/news-and-events/bulletins-and-updates/bulletin/Pages/index.aspx). The Bulletin is the only source of changes to regulations and accredited studies. It is the responsibility of each VCE teacher to monitor changes or advice about VCE studies published in the Bulletin.

Monitoring for quality

As part of ongoing monitoring and quality assurance, the VCAA will periodically undertake an audit of
VCE Mathematics to ensure the study is being taught and assessed as accredited. The details of the audit procedures and requirements are published annually in the [*VCE and VCAL Administrative Handbook*](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx). Schools will be notified if they are required to submit material to be audited.

Safety and wellbeing

It is the responsibility of the school to ensure that duty of care is exercised in relation to the health and safety of all students undertaking the study.

Employability skills

This study offers a number of opportunities for students to develop employability skills. The *Advice for teacher’s* companion document provides specific examples of how students can develop employability skills during learning activities and assessment tasks.

Legislative compliance

When collecting and using information, the provisions of privacy and copyright legislation, such as the Victorian *Privacy and Data Protection Act 2014* and *Health Records Act 2001*, and the federal *Privacy Act 1988* and *Copyright Act 1968*, must be met.

Child Safe Standards

Schools and education and training providers are required to comply with the Child Safe Standards made under the Victorian *Child Wellbeing and Safety Act 2005*. Registered schools are required to comply with *Ministerial Order No. 1359 Implementing the Child Safe Standards – Managing the Risk of Child Abuse in Schools and School Boarding Premises*. For further information, consult the websites of the [Victorian Registration and Qualifications Authority](https://www.vrqa.vic.gov.au/childsafe/Pages/Home.aspx), the [Commission for Children and Young People](https://ccyp.vic.gov.au/) and the [Department of Education and Training](https://www2.education.vic.gov.au/pal/child-safe-standards/policy).

Assessment and reporting

Satisfactory completion

The award of satisfactory completion for a unit is based on the teacher’s decision that the student has demonstrated achievement of the set of outcomes specified for the unit. Demonstration of achievement of outcomes and satisfactory completion of a unit are determined by evidence gained through the assessment of a range of learning activities and tasks.

Teachers must develop courses that provide appropriate opportunities for students to demonstrate achievement of the set of outcomes.

The decision about satisfactory completion of a unit is distinct from the assessment of levels of achievement. Schools will report a student’s result for each unit to the VCAA as S (satisfactory) or N (not Satisfactory).

Levels of achievement

Units 1 and 2

Procedures for the assessment of levels of achievement in Units 1 and 2 are a matter for school decision. Assessment of levels of achievement for these units will not be reported to the VCAA. Schools may choose to report levels of achievement using grades, descriptive statements or other indicators.

Units 3 and 4

The VCAA specifies the assessment procedures for students undertaking scored assessment in Units 3
and 4. Designated assessment tasks are provided in the details for each unit in VCE study designs.

The student’s level of achievement in Units 3 and 4 will be determined by School-assessed Coursework (SAC) as specified in the VCE study design, and external assessment.

The VCAA will report the student’s level of achievement on each assessment component as a grade from A+ to E or UG (ungraded). To receive a study score the student must achieve two or more graded assessments in the study and receive an S for both Units 3 and 4. The study score is reported on a scale of 0–50; it is a measure of how well the student performed in relation to all others who took the study. Teachers should refer to the current [*VCE and VCAL Administrative Handbook*](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx) for details on graded assessment and calculation of the study score. Percentage contributions to the study score in VCE Mathematics are as follows:

**Foundation Mathematics**

Unit 3 School-assessed Coursework: 40 per cent

Unit 4 School-assessed Coursework: 20 per cent

Units 3 and 4 Examination: 40 per cent.

**General Mathematics**

Unit 3 School-assessed Coursework: 24 per cent

Unit 4 School-assessed Coursework: 16 per cent

Units 3 and 4 Examination 1: 30 per cent

Units 3 and 4 Examination 2: 30 per cent.

**Mathematical Methods**

Unit 3 School-assessed Coursework: 20 per cent

Unit 4 School-assessed Coursework: 20 per cent

Units 3 and 4 Examination 1: 20 per cent

Units 3 and 4 Examination 2: 40 per cent.

**Specialist Mathematics**

Unit 3 School-assessed Coursework: 20 per cent

Unit 4 School-assessed Coursework: 20 per cent

Units 3 and 4 Examination 1: 20 per cent

Units 3 and 4 Examination 2: 40 per cent.

Examination 1 for Mathematical Methods and Examination 1 for Specialist Mathematics are technology-free examinations. All other VCE mathematics examinations assume student access to VCAA approved technology.

Details of the assessment program are described in the sections on Units 3 and 4 in this study design.

Authentication

Work related to the outcomes of each unit will be accepted only if the teacher can attest that, to the best of their knowledge, all unacknowledged work is the student’s own. Teachers need to refer to the current [*VCE and VCAL Administrative Handbook*](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx) for authentication rules and strategies.

Unit 1: Foundation Mathematics

Foundation Mathematics Units 1 and 2 focus on providing students with the mathematical knowledge, skills, understanding and dispositions to solve problems in real contexts for a range of workplace, personal, further learning, and community settings relevant to contemporary society. They are also designed as preparation for Foundation Mathematics Units 3 and 4 and contain assumed knowledge and skills for these units.

In Unit 1 students consolidate mathematical foundations, further develop their knowledge and capability to plan and conduct activities independently and collaboratively, communicate their mathematical ideas, and acquire mathematical knowledge skills to make informed decisions in their lives. The areas of study for Foundation Mathematics Unit 1 are ‘Algebra, number and structure’, ‘Data analysis, probability and statistics’, ‘Discrete mathematics’, and ‘Space and measurement’. The content should be developed using contexts present in students’ other studies, work and personal or other familiar situations.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving integer, rational and real arithmetic, sets, lists and tables, contemporary data displays, diagrams, plans, geometric objects and constructions, algorithms, measures, equations and graphs, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic, statistical and financial functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Algebra, number and structure

In this area of study students cover estimation, and the use and application of different forms of number and related calculations in practical, everyday and routine work contexts.

This area of study includes:

* application of integers, fractions and decimals, their properties and related operations
* use of ratios, proportions, percentages and rates to solve problems
* estimation, approximation and reasonableness of calculations and results.

Area of Study 2

Data analysis, probability and statistics

In this area of study students cover collection, presentation and analysis of gathered and provided data from community, work, recreation and media contexts, including consideration of suitable forms of representation.

This area of study includes:

* collection and representation of data in diagrammatic, tabular and graphical forms and the features, conventions and terminology used in these processes
* construction of charts, tables and graphs to represent data
* interpretation of data to summarise and communicate findings and possible conclusions.

Area of Study 3

Discrete mathematics

Financial and consumer mathematics

In this area of study students cover the use and interpretation of different forms of numbers and calculations, and their application in relation to the understanding and management of personal, local and national financial matters.

This area of study includes:

* personal financial services and information such as borrowing, bills and banking
* income calculations including rates of pay and payslips
* personal taxation and superannuation
* taxation as a community and contribution to government
* fees and interest
* cost structures and related information associated with financial transactions.

Area of Study 4

Space and measurement

In this area of study students cover time, and the use and application of the metric system and related measurements in a variety of domestic, societal, industrial and commercial contexts.

This area of study includes:

* standard metric units and measures, including common derived metric measures
* reading and interpretation of scales on digital and analogue instruments
* estimation and approximation strategies
* time and duration including time and date specifications, conventions, schedules and timetables.

Mathematical investigation

This comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes.

Investigation is to be incorporated in the development of concepts, skills and processes for the unit, and can be used to assess the outcomes.

There are three components to mathematical investigation:

Formulation

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest.

Exploration

Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Communication

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

Possible contexts or scenarios

Some possible contexts or scenarios for the mathematical investigation in Unit 1 include:

* Planning and costing for a day trip, including route planning and costing using public transport.

Determining the location and modes of transport, addresses formulating the problem. For the exploration aspects of the problem, devising and costing a plan for the day, including key times, travel time and costings. Alternative itineraries could be incorporated in case of unforeseen delays. Results can be communicated in the form of a detailed itinerary incorporating a map and the costings.

* Collecting, reporting and analysing sports performance data or analysing and comparing population densities of business areas, suburbs and regions in a city or state.

Formulating the problem includes selecting and finding the relevant sports performance data or the areas to analyse in relation to population densities. Undertaking a statistical analysis of the data would be covered in the exploration phase, where suitable methods of data analysis and representation are selected and used. A report summarising the outcomes of analysis, and any interpretation and reflections on the results addresses the communication phases of the investigation.

* Calculating and comparing the amount of tax to be paid on different incomes, comparing PAYG rates and company rates.

Investigating different taxation rates requires research based on the different taxation tables available on the ATO website and selection of a range of different income brackets. The exploration phase requires the use of different taxation brackets to calculate the taxation amounts to be paid and possible comparison with business taxation tables. The final component of the investigation would be the communication of the results via a report or presentation.

* Workplace task requirements such as mathematical measurements and use of formulas required for a specific job or task.

This could consider the context of painting a room at home or in the school. Formulating the problem requires selecting the room or rooms to be painted, deciding on which walls or ceilings would be painted, and selecting the paints to be used. The exploration stage involves measuring the walls and ceilings and determining the requirements for a tradesperson. Results can be communicated in the form of a detailed quotation for painting works to be carried out.

Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to use and apply a range of mathematical concepts, skills and procedures from selected areas of study to solve practical problems based on a range of everyday and real-life contexts.

Algebra, number and structure

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 1.

Key knowledge

* integers, fractions, decimals, ratios, proportions, percentages and rates
* place value, truncation and rounding, leading digit approximation and order of magnitude as powers of 10
* numerals, symbols, number facts and operations and strategies for calculation

Key skills

* make estimates and carry out relevant calculations using mental and by-hand methods
* use technology effectively for accurate, reliable and efficient calculations
* solve practical problems which require the use and application of a range of numerical computations involving integers, decimals, fractions, proportions, percentages, rates, powers and roots
* check for accuracy and reasonableness of calculations and results

Data analysis, probability and statistics

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 2.

Key knowledge

* categorical and numerical data
* methods of data collection and organisation
* key features and conventions of diagrams, charts, tables and graphs
* purposes for using different forms of data representation and types of data scales (categorical and numerical)
* common measures of central tendency (mean, median, mode) and of spread (range, quartiles)
* characteristics and properties of data sets and the shape of their distributions
* terminology and language for description, comparison and analysis of data sets, graphs and summary statistics

Key skills

* collect, organise, collate and represent categorical and numerical data
* accurately read and interpret diagrams, charts, tables and graphs
* summarise statistical data and determine commonly used measures of central tendency and of spread
* describe, compare and analyse data sets and report on any trends, implications or limitations

Discrete mathematics

Financial and consumer mathematics

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 3.

Key knowledge

The numerical, data and algebraic knowledge that underpin the following financial and consumer topics and issues:

* cost structures and fees and interest for financial services such as savings, loans, debit and credit cards, purchase and payment services (including online payment systems and buy now-pay later systems)
* financial systems and related calculations such as personal taxation, GST and superannuation
* income payments and calculations such as casual versus permanent employment, rates of pay, pay scales and payslips

Key skills

* read, interpret and perform calculations related to financial services such as with banking, utility bills and GST
* demonstrate understanding and perform calculations related to personal financial services such as bank statements
* demonstrate understandings and perform calculations of fees and interest charged by banks and other financial institutions in relation to amount borrowed, interest rate, time and risk
* describe, calculate and interpret different income related taxes, payments and deductions and their impact on income, such as pay scales, personal tax, withholding tax, PAYE and PAYG

Space and measurement

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 4.

Key knowledge

* metric units for length, area, volume, capacity, time, mass, temperature and common derived units
* meaning and conventions of different metric units, relative scale and conversions, including International System of Units (SI)
* standard digital and analogue tools and instruments and scales
* standard formulas for calculating length, area, surface area, volume and capacity

Key skills

* calculate and interpret length, area, surface area, volume, capacity and duration for a range of personal, societal or workplace measurement problems with use of estimation, rounding and approximation strategies
* identify and use common metric and other relevant measurements
* convert between a range of standard metric units
* estimate and measure different quantities using appropriate measurement tools

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine practical contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* common uses and applications of mathematics in aspects of everyday life
* relevant and appropriate mathematics in areas relating to study, work, social or personal contexts
* common methods of presenting and communicating mathematics in everyday life, for example charts, graphs, maps, plans, tables, symbolic expressions and diagrams

Key skills

* identify and recognise how mathematics is used in everyday situations and contexts, making connections between mathematics and the real world
* extract the mathematics embedded in everyday situations and contexts and formulate what mathematics can be used to solve practical problems in both familiar and new contexts
* represent the mathematical information in a form that is personally useful as an aid to problem-solving, such as a table, summary, chart, numeric or algebraic representation, physical model or sketch
* undertake a range of mathematical tasks, applications and processes to solve practical problems, such as drawing, measuring, counting, estimating, calculating, generalising and modelling
* use estimation and other assessment skills to check the outcomes and decide on the appropriate accuracy for the outcome
* interpret results and outcomes of the application of mathematics in a context, including how appropriately and accurately they fit the situation, and to reflect on and evaluate the mathematics used and the outcomes obtained relative to personal, contextual and real-world implications
* represent, communicate and discuss the results and outcomes of the application of mathematics in a range of contexts

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in practical situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* the conventions for representations of mathematical information using technology
* contemporary technology and online and digital media, including software and applications based on computers, tablets, calculators and hand-held devices
* relevance and appropriateness of the use and application of technology
* the numerical, graphical, symbolic, geometric, statistical and financial functionalities of a range of technologies
* the conditions and settings for effective application of a given technology and its functionality

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* use technology to carry out computations and analysis, produce diagrams, tables, charts and graphs which model situations and solve practical problems
* interpret, evaluate and discuss the outputs of technology, including reflecting on and evaluating the technology used and the outcomes obtained relative to personal, contextual and real-world implications
* use technology to communicate the results of working mathematically

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass the areas of study in the unit.

Suitable tasks for assessment in this unit may be selected from the following.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

* portfolio
* assignments
* tests
* solutions to sets of worked questions
* summary notes or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

* portfolio
* modelling tasks
* problem-solving tasks
* mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for computational thinking and the effective and appropriate use of technology.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Unit 2: Foundation Mathematics

The focus of Unit 2 is on extending breadth and depth in the application of mathematics to solving practical problems from contexts present in students’ other studies, work and personal or other familiar situations. The areas of study for Foundation Mathematics Unit 2 are ‘Algebra, number and structure’, ‘Data analysis, probability and statistics’, ‘Discrete mathematics’, and ‘Space and measurement’.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving integer, rational and real arithmetic, sets, lists and tables, contemporary data displays, diagrams, plans, geometric objects and constructions, algorithms, measures, equations and graphs, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic, statistical and financial functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Algebra, number and structure

In this area of study students cover estimation and the use and application of the representation of generalisations and patterns in number, including formulas and other symbolic expressions, in everyday
and routine work contexts.

This area of study includes:

* construction, use and interpretation of formulas and symbolic expressions to describe relationships between variables and to model and represent generalisations and patterns
* manipulation of symbolic expressions and solution of equations
* estimation, approximation and reasonableness of calculations and results.

Area of Study 2

Data analysis, probability and statistics

In this area of study students cover the analysis of gathered and provided data from community, work, recreation and media contexts, including consideration of suitable forms of data summaries.

This area of study includes:

* creation of a range of charts, tables and graphs to represent and compare data
* measures of central tendency and simple measures of spread (such as range and interquartile range) to summarise and interpret data and compare sets of related data
* interpretation, summary and comparison of related data sets to report findings and draw possible conclusions.

Area of Study 3

Discrete mathematics

Financial and consumer mathematics

In this area of study students cover the use and interpretation of different forms of numbers and calculations and their application in relation to the understanding and management of personal, local and national financial matters.

This area of study includes:

* products and services such as comparison of health products, informed spending choices, decision making according to criteria
* managing money: earning and spending, life-stage financial planning, servicing of current and future commitments such as HECS-HELP debt, child-care support and other benefits
* local, community and national financial and economic data and trends over time (national/community/ local) such as CPI, interest rates, wages and house prices.

Area of Study 4

Space and measurement

In this area of study students cover shape and location concepts, and their use and application in a variety
of domestic, societal, industrial and commercial contexts.

This area of study includes:

* description, representation and properties of simple and composite shapes and objects
* two-dimensional plans, models and diagrams of objects
* location, maps, directories and digital maps including birds-eye and street views
* routes and itineraries, including location and direction, speeds, distances and estimated travel times,
for example daily work route and diversions, and itinerary for travel.

Mathematical investigation

This comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes.

Investigation is to be incorporated in the development of concepts, skills and processes for the unit, and can be used to assess the outcomes.

There are three components to mathematical investigation:

Formulation

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues
or problems of interest.

Exploration

Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Communication

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

Possible contexts or scenarios

Some possible contexts or scenarios for the mathematical investigation in Unit 2 include:

* Planning and comparing car purchase options, or house rental options.

For an investigation into house rental options, formulation involves finding and selecting several rental house options from the internet, and deciding how to best compare the properties using, for example, the weekly and monthly payments. This could include, for example, deciding to use spreadsheets to compare options. The exploration phase includes entering relevant data and information into the spreadsheets and undertaking any calculations in order to compare the properties. This could also incorporate doing a budget to see what rentals could be afforded. Results could be communicated as a client report.

* Calculating and comparing the costs and charges for different utilities (e.g. phone plans, gas or electricity plans).

Based around sets of anonymised utility bills and plans available on the internet from different providers, formulate how to analyse and compare the costs and charges for different utilities. Once a decision has been made on how to undertake the research and against what criteria, the exploration component would see an analysis of the bills to understand and compare the breakdown of charges and rates, potentially using spreadsheets to document and undertake the analysis. The analysis and the results could then be communicated in the third investigation component through a report that incorporated a comparative table.

* Comparing HECS-HELP and VET student loan schemes or analyse and compare interest charged under different borrowing schemes, in terms of amount and length of financial commitment, and servicing requirements.

An example could be to consider how to purchase a car funded through different borrowing schemes. The formulation stage would involve decisions about available budgets, which car and from which suppliers, and establishing what financial parameters needed to be considered. In the exploration stage a cost analysis of different products would be undertaken, including comparing different interest rates over different time frames. Results could be presented as recommendations for different options, detailing their mathematical reasons.

* Creating plans for the construction of an object such as a child’s cubby house, school garden, or the design, costing and construction of new packaging for a product.

In the example of designing a child’s cubby house, formulation of the problem would include the initial investigations about design, size, dimensions, and cost. The second stage would explore the problem, creating designs and models using scale drawing techniques. This stage might also cost the project. The design could be communicated as a set of useable plans with associated specifications and costings.

Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to use and apply a range of mathematical concepts, skills and procedures from selected areas of study to solve practical problems based on a range of everyday and real-life contexts.

Algebra, number and structure

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 1.

Key knowledge

* numerals, symbols, number facts and operations and strategies for calculation
* basic conventions of formal mathematical terminology, notations and processes in relevant symbolic expressions and formulas
* concepts of constant, pro-numeral, variable and formula

Key skills

* form estimates and carry out relevant calculations using mental and by-hand methods
* develop simple formulas describing generalisations, patterns and relationships between real-life variables
* solve practical problems using constants, symbols, variables, common formulas, expressions and equations
* check for accuracy and reasonableness of calculations and results

Data analysis, probability and statistics

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 2.

Key knowledge

* categorical and numerical data
* common measures of central tendency (mean, median, mode) and simple measures of spread (range, quantiles, interquartile range)
* characteristics and properties of data sets and their summary data and the shape of their distributions
* terminology and language for description, comparison and analysis of data sets, graphs and summary statistics

Key skills

* accurately read and interpret diagrams, charts, tables and graphs for categorical and numerical data
* summarise statistical data and calculate commonly used measures of central tendency and spread
* describe, compare and analyse data sets and summary data and report on any trends, implications or limitations

Discrete mathematics

Financial and consumer mathematics

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 3.

Key knowledge

The numerical, data and algebraic knowledge that underpin the following financial and consumer topics and issues:

* cost structures and fees for products and services
* factors that impact on personal income and informed spending choices such as age, advertising, scams and gambling
* charges for services and utilities such as rent, gas and electricity
* interest, repayments and relationship to amount borrowed, interest rates, time, CPI and risk
* financial systems and related calculations such as student loan schemes (HECS-HELP and VET student loans)

Key skills

* read, interpret and perform calculations related to financial services such as products and services
* compare products and services from an optimal spending perspective for services such as health care, insurance and internet provision
* identify credit options to best manage finances to meet short, medium and long terms goals, such as credit cards, personal loans
* describe, calculate and interpret different income related taxes, payments and deductions and their impact on income, such as student loan repayments, allowances, benefits and payments
* describe the effect of life-stage factors on personal income sources and the different perspectives on informed spending choices

Space and measurement

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 4.

Key knowledge

* names and properties of common geometric shapes and objects
* language, symbols and conventions for geometric and spatial representation, including point, vertex, line, ray, angle, diagonal, edge, curve, perimeter, boundary, face, surface and solid
* language, symbols and labelling and drawing conventions for diagrams, plans and models, including keys and scale
* language, symbols, labelling and conventions for maps and location related diagrams and directories, including keys, scale, direction, distance, coordinates and grid references

Key skills

* interpret and describe common shapes and objects using accurate and appropriate geometric and spatial language and conventions
* create and modify simple diagrams, plans, maps or designs using drawing equipment and digital drawing packages
* interpret diagrams, plans, maps and models and evaluate their accuracy
* interpret information on maps to plan and describe travel routes, including use of navigational software and tools

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine practical contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* common uses and applications of mathematics in aspects of everyday life
* relevant and appropriate mathematics in areas relating to study, work, social or personal contexts
* common methods of presenting and communicating mathematics in everyday life, for example charts, graphs, maps, plans, tables, symbolic expressions and diagrams

Key skills

* identify and recognise how mathematics is used in everyday situations and contexts, making connections between mathematics and the real world
* extract the mathematics embedded in everyday situations and contexts and formulate what mathematics can be used to solve practical problems in both familiar and new contexts
* represent the mathematical information in a form that is personally useful as an aid to problem-solving, such as a table, summary, chart, numeric or algebraic representation, physical model or sketch
* undertake a range of mathematical tasks, applications and processes to solve practical problems, such as drawing, measuring, counting, estimating, calculating, generalising and modelling
* use estimation and other assessment skills to check the outcomes and decide on the appropriate accuracy for the outcome
* interpret results and outcomes of the application of mathematics in a context, including how appropriately and accurately they fit the situation and to reflect on and evaluate the mathematics used and the outcomes obtained relative to personal, contextual and real-world implications
* represent, communicate and discuss the results and outcomes of the application of mathematics in
a range of contexts

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in practical situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* the conventions for representations of mathematical information using technology
* contemporary technology and online and digital media, including software and applications based on computers, tablets, calculators and hand-held devices
* relevance, appropriateness and validity of the use and application of technology
* the numerical, graphical, symbolic, geometric statistical and financial functionalities of a range of technologies
* the conditions and settings for effective application of a given technology and its functionality

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* use technology to carry out computations and analysis, and produce diagrams, tables, charts and graphs which model situations and solve practical problems
* interpret, evaluate and discuss the outputs of technology including reflecting on and evaluating the technology used and the outcomes obtained relative to personal, contextual and real-world implications
* use technology to communicate the results of working mathematically

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass the areas of study in the unit.

Suitable tasks for assessment in this unit may be selected from the following.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

* portfolio
* assignments
* tests
* solutions to sets of worked questions
* summary notes or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

* portfolio
* modelling tasks
* problem-solving tasks
* mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for computational thinking and the effective and appropriate use of technology.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Unit 1: General Mathematics

General Mathematics Units 1 and 2 cater for a range of student interests, provide preparation for the study of VCE General Mathematics at the Units 3 and 4 level and contain assumed knowledge and skills for these units. The areas of study for Unit 1 of General Mathematics are ‘Data analysis, probability and statistics’, ‘Algebra, number and structure’, ‘Functions, relations and graphs’ and ‘Discrete mathematics’.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists, tables and matrices, diagrams and geometric constructions, algorithms, algebraic manipulation, recurrence relations, equations and graphs, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic, financial and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Data analysis, probability and statistics

In this area of study students cover types of data, display and description of the distribution of data, summary statistics for centre and spread, and the comparison of sets of data.

Investigating and comparing data distributions

This topic includes:

* types of data, including categorical (nominal or ordinal) or numerical (discrete or continuous, interval, ratio)
* display and description of categorical data distributions of one or more groups using frequency tables and bar charts, and the mode and its interpretation
* display and description of numerical data distributions using histograms, stem plots and dot plots and choosing between plots according to context and purpose
* summarising numerical data distributions, including use of and calculation of the sample summary statistics, median, range, and interquartile range (IQR) or mean and standard deviation
* the five-number summary and the boxplot as its graphical representation and display, including the use of the lower fence (Q1 – 1.5 × IQR) and upper fence (Q3 + 1.5 × IQR) to identify possible outliers
* consideration of a range of distributions (symmetrical, asymmetrical), their summary statistics and the percentage of data lying within several standard deviations of the mean
* use of back-to-back stem plots or parallel boxplots, as appropriate, to compare the distributions of a single numerical variable across two or more groups in terms of centre (median) and spread (IQR and range), and the interpretation of any differences observed in the context of the data.

Area of Study 2

Algebra, number and structure

In this area of study students cover the concept of a sequence and its representation by rule, table and
graph, arithmetic or geometric sequences as examples of sequences generated by first-order linear recurrence relations, and simple financial and other applications of these sequences.

Arithmetic and geometric sequences, first-order linear recurrence relations and financial mathematics

This topic includes:

* the concept of an arithmetic or geometric sequence as a function with the set of non-negative integers as its domain
* tabular and graphical display of sequences, investigation of their behaviour (increasing, decreasing, constant, oscillating, limiting values)
* use of a first-order linear recurrence relation of the form  where and are constants, to generate the values of an arithmetic sequence
* use of a first-order linear recurrence relation of the form  where and are constants, to model and analyse practical situations involving discrete linear growth or decay such as a simple interest loan or investment, the depreciating value of an asset using the unit cost or flat-rate method
* use of a first-order linear recurrence relation of the form where and are constants, to generate the values of a geometric sequence
* use of a first-order linear recurrence relation of the form where and are constants,
to model growth and decay and analyse practical situations involving geometric sequences such as the reducing height of a bouncing ball, reducing balance depreciation, compound interest loans or investments
* generation of the explicit rule,, of an arithmetic or geometric sequence, its use and evaluation, including various practical and financial contexts
* percentage increase and decrease, mark-ups and discounts, and calculating GST in various financial contexts
* determining the impact of inflation on costs and the spending power of money over time
* the unitary method and its use in making comparisons and solving practical problems involving percentages and finance
* comparison of purchase options including cash, credit and debit cards, personal loans, buy now and pay later schemes.

Area of Study 3

Functions, relations and graphs

In this area of study students cover linear function and relations, their graphs, modelling with linear functions, solving linear equations and simultaneous linear equations, line segment and step graphs and their applications.

Linear functions, graphs, equations and models

This topic includes:

* the linear function , its graph, and interpretation of the parameters, and in terms of initial value and constant rate of change respectively
* graphing linear relations and equivalent forms
* formulation and analysis of linear models from worded descriptions or relevant data (including simultaneous linear equations in two variables) and their application to solve practical problems including domain of interpretation
* piecewise linear (line segment, step) graphs and their application to modelling practical situations, including tax scales and charges and payment.

Area of Study 4

Discrete mathematics

In this area of study students cover the concept of matrices and matrix operations to model and solve a range of practical problems, including population growth and decay.

Matrices

This topic includes:

* use of matrices to store and display information that can be presented in a rectangular array of rows and columns such as databases and links in social networks and road networks
* types of matrices (row, column, square, zero and identity) and the order of a matrix
* matrix addition, subtraction, multiplication by a scalar, and matrix multiplication including determining the power of a square matrix using technology as applicable
* use of matrices, including matrix products and powers of matrices, to model and solve problems, for example costing or pricing problems, and squaring a matrix to determine the number of ways pairs of people in a network can communicate with each other via a third person
* inverse matrices and their applications including solving a system of simultaneous linear equations
* introduction to transition matrices (assuming the next state only relies on the current state), working with iterations of simple models linked to, for example, population growth or decay, including informal consideration of long run trends and steady state.

Mathematical investigation

This comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes.

Investigation is to be incorporated in the development of concepts, skills and processes for the unit, and can be used to assess the outcomes.

There are three components to mathematical investigation:

Formulation

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest.

Exploration

Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures
or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Communication

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

Possible contexts or scenarios

Some possible contexts or scenarios for the mathematical investigation in Unit 1 include:

* Wind direction and speed at different geographic locations around the state during the year, for example, identifying and comparing potential sites for wind powered electricity generation.
* Use of matrices to represent product inventory, sales, discounts and profits in a sales context.
* Modelling tax scales and taxation changes, and how have these have varied in Australia over the past few decades.
* Superannuation, for example, consider projected future superannuation contribution rates from employers to employees that increase to at least 12.0% of gross income. What minimum base salary could use this projected rate and build an account balance that would ensure a sustainable and comfortable lifestyle in retirement? If salary increases occur annually, every second year, etc. and in line with inflation, what changes would this make to the overall balance? If employee voluntary contributions are made regularly, what changes would this make to the overall balance? If an extra amount is added to the contribution, what new balance is created?

Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Area of Study 1

Data analysis, probability and statistics

Investigating and comparing data distributions

Key knowledge

* types of data, including categorical (nominal or ordinal) or numerical (discrete and continuous)
* the concept of a data distribution and its display using a statistical plot
* the five-number summary and possible outliers
* mean and sample standard deviation

Key skills

* construct and interpret graphical displays of data, and describe the distributions of the variables involved and interpret in the context of the data
* calculate the values of appropriate summary statistics to represent the centre and spread of the distribution of a numerical variable and interpret in the context of the data
* construct and use parallel boxplots or back-to-back stem plots (as appropriate) to compare the distribution of a numerical variable across two or more groups in terms of centre (median), spread (range and IQR) and outliers, interpreting any observed differences in the context of the data

Area of Study 2

Algebra, number and structure

Arithmetic and geometric sequences, first-order linear recurrence relations and financial mathematics

Key knowledge

* the concept of an arithmetic or geometric sequence as a function and its recursive specification
* the use of a first-order linear recurrence relation to generate the values of an arithmetic or geometric sequence
* the explicit rule, , of an arithmetic or geometric sequence and its evaluation
* the use of a first-order linear recurrence relations of the form where and are constants to model linear growth and decay, including the rule for evaluating the value after periods of linear growth or decay in financial contexts, simple interest investments and loans, flat rate and unit cost depreciation of an asset over time, including the rule for the future value of the asset after depreciation periods
* the use of first-order linear recurrence relations of the form where and are constants to model compound interest investments and loans, reducing balance depreciation of an asset over time, including the rule for the future value of the asset after depreciation periods, compound interest investments and debts
* concepts of ratio, proportion, percentage, percentage change and rate, and unitary method

Key skills

* use a given recurrence relation to generate a sequence, deduce the explicit rule,  from the recursion relation, tabulate, graph and evaluate the sequence
* use a recurrence relation, table or graph to model and analyse practical situations involving discrete linear growth or decay
* demonstrate the use of a recurrence relation to determine the linear depreciating value of an asset after time periods for the initial sequence
* use a rule for the future value of a linear depreciating asset to solve practical problems

Area of Study 3

Functions, relations and graphs

Linear functions, graphs, equations and models

Key knowledge

* the properties of linear functions and their graphs
* the concept of a linear model and its properties, and simultaneous linear equations and their solutions
* the forms, rules, graphical images and tables for linear relations and equations
* situations that can be modelled by piecewise linear graphs

Key skills

* develop a linear model to represent and analyse a practical situation and specify its domain of application
* interpret the slope and the intercept of a straight-line graph in terms of its context and use the equation to make predictions
* construct graphs and/or tables of values for given linear models and formula and vice versa
* solve linear equations constructed from word problems, including simultaneous linear equations
* use piecewise linear graphs to model and analyse practical situations

Area of Study 4

Discrete Mathematics

Matrices

Key knowledge

* the concept of a matrix and its use to store, display and manipulate information
* types of matrices (row, column, square, zero, identity) and the order of a matrix
* matrix arithmetic: the definition of addition, subtraction, multiplication by a scalar, multiplication, the power of a square matrix, and the conditions for their use
* determinant and inverse of a matrix
* simple communication matrices and their application
* regular transition matrices and their identification

Key skills

* use matrices to store and display information that can be presented as a rectangular array
* identify row, column, square, zero and identity matrices and determine their order
* add and subtract matrices, multiply a matrix by a scalar or another matrix, and raise a matrix to a power
* use matrix sums, difference, products and powers and inverses to model and solve practical problems
* construct a transition matrix to model the transitions in a population with an equilibrium state

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the facts, concepts and techniques associated with the topics studied
* the standard mathematical models used in the topics studied
* the facts, concepts, techniques and/or models suitable to solve extended application problems or to conduct a structured investigation in context
* assumptions and conditions underlying the facts, concepts, techniques and models used when solving a problem or conducting a mathematical investigation

Key skills

* identify, recall and select the mathematical facts, concepts and techniques needed to solve a problem or conduct an investigation in a variety of contexts
* recall, select and use standard mathematical models to represent practical situations
* use specific models to comment on particular situations being analysed and to make predictions
* interpret and report the results of applying these models in terms of the context of the problem being solved, including discussing the assumptions underlying the application of such models

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques
or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* the difference between exact numerical and approximate numerical answers when using technology to perform computation, and rounding to a given number of decimal places or significant figures
* domain and range requirements for specification of graphs, and the role of parameters in specifying general forms of models, relations and equations
* the relation between numerical, graphical and symbolic forms of information about functions, relations and equations, and the corresponding features of those functions, relations and equations
* the similarities and differences between formal mathematical expressions and their representation by technology

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
* use technology to carry out numerical, graphical and symbolic computation as applicable
* produce results, using a technology, which identify examples or counter-examples for propositions
* produce tables of values, families of graphs and collections of other results using technology, which support general analysis in investigative, modelling and problem-solving contexts
* use appropriate domain and range specifications to illustrate key features of graphs of models and relations
* identify the connection between numerical, graphical and symbolic forms of information about functions, relations and equations, and the corresponding features of those functions, relations and equations
* specify the similarities and differences between formal mathematical expressions and their representation by technology
* select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
* apply suitable constraints and conditions, as applicable, to carry out required computations
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass the areas of study in the unit.

Suitable tasks for assessment in this unit may be selected from the following.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

* assignments
* tests
* solutions to sets of worked questions
* summary notes or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

* modelling tasks
* problem-solving tasks
* mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for computational thinking and the effective and appropriate use of technology.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Unit 2: General Mathematics

General Mathematics Units 1 and 2 cater for a range of student interests, provide preparation for the study of VCE General Mathematics at the Units 3 and 4 level and contain assumed knowledge and skills for these units. The areas of study for Unit 2 of General Mathematics are ‘Data analysis, probability and statistics’, ‘Discrete mathematics’, ‘Functions, relations and graphs’ and ‘Space and measurement’.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams, networks and geometric constructions, algorithms, algebraic manipulation, equations and graphs, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic, financial and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Data analysis, probability and statistics

In this area of study students cover association between two numerical variables, scatterplots, and lines of good fit by eye and their interpretation.

Investigating relationships between two numerical variables

This topic includes:

* response and explanatory variables
* scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction, form and strength
* informal interpretation of association and causation
* use of a line of good fit by eye to make predictions, including the issues of interpolation and extrapolation
* interpretation of a line of good fit, its intercept and slope in the context of the data.

Area of Study 2

Discrete mathematics

In this area of study students cover the use of graphs and networks to model and solve a range of practical problems, including connectedness, shortest path and minimum spanning trees.

Graphs and networks

This topic includes:

* introduction to the notations, conventions and representations of types and properties of graphs, including edge, loop, vertex, the degree of a vertex, isomorphic and connected graphs and the adjacency matrix
* description of graphs in terms of faces (regions), vertices and edges and the application of Euler’s formula for planar graphs
* connected graphs: walks, trails, paths, cycles and circuits with practical applications
* weighted graphs and networks, and an introduction to the shortest path problem (solution by inspection only) and its practical application
* trees and minimum spanning trees, greedy algorithms and their use to solve practical problems.

Area of Study 3

Functions, relations and graphs

In this area of study students cover direct and inverse variation, transformations to linearity and modelling of some non-linear data.

Variation

This topic includes:

* numerical, graphical and algebraic approaches to direct and inverse variation
* transformation of data to linearity to establish relationships between variables, for example and ,
 and , and and
* modelling of given non-linear data using the relationships , where and

, where .

Area of Study 4

Space and measurement

In this area of study students cover units of measurement, accuracy, computations with formulas for different measures, similarity and scale in two and three dimensions, and their practical applications involving simple and composite shapes and objects, trigonometry, problems involving navigation and Pythagoras’ theorem and their applications in the plane.

Space, measurement and applications of trigonometry

This topic includes:

* units of measurement of length, angle, area, volume and capacity
* exact and approximate answers, scientific notation, significant figures and rounding
* similar shapes including the conditions for similarity
* perimeter and areas of triangles, quadrilaterals, circles including arcs and sectors and composite shapes, and practical applications
* volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and composite objects) and practical applications, including simple applications of Pythagoras’ theorem in three dimensions
* the use of trigonometric ratios and Pythagoras’ theorem to solve practical problems involving a right-angled triangle in two dimensions, including the use of angles of elevation and depression
* the use of the sine rule, including the ambiguous case, the cosine rule, as a generalisation of Pythagoras’ theorem, and their application to solving practical problems involving non-right-angled triangles, including three-figure (true) bearings in navigation
* similar objects and the application of linear scale factor to scale lengths, surface areas and volumes with practical applications.

Mathematical investigation

This comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes.

Investigation is to be incorporated in the development of concepts, skills and processes for the unit, and can be used to assess the outcomes.

There are three components to mathematical investigation:

Formulation

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest.

Exploration

Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Communication

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

Possible contexts or scenarios

Some possible contexts or scenarios for the mathematical investigation in Unit 2 include:

* Trends in weather and climate: worldwide dialogue suggests future weather patterns will be significantly different to current patterns. Investigate a specific weather condition (wind, precipitation, air pressure, temperature) in an Australian state or region and discuss findings using data from the Bureau of Meteorology. Possible questions to be explored could include: Has there been a noticeable change in the chosen weather condition over the last 5, 10 or 15 years? Is there a trend in precipitation levels suggesting more droughts or floods?
* Cycling path networks in urban planning and renewal projects. Consider a large urban renewal project. Possible questions to be explored could include: What could a cycle path structure look like in this community? Could a series of one-way streets be integrated to help move vehicles safely alongside bicycles? Could sections of the community be restricted to bicycle travel only? What would the movement of vehicles look like around this restricted section?
* Touring on a holiday: consider a holiday tour planned around Australia with all capital cities being visited and the journey remaining as close to the coastline as possible. It is intended that a maximum of 20,000 km will be travelled. Possible questions to be explored: Can a journey, initially remaining on national highways, be created as close as possible to the coastline and with the total kilometres travelled being less than 20,000? Where could inland travel occur to ensure no more than 20,000 kilometres is travelled? If travel between capital cities was as close as possible to linear, what would be the total distance? What is the difference in the area covered between the coastline and direct route journeys?

Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Area of Study 1

Data analysis, probability and statistics

Investigating relationships between two numerical variables

Key knowledge

* response and explanatory variables and their role in modelling associations between two numerical variables
* scatterplots and their use in identifying and describing the association between two numerical variables
* the concepts of association and causation
* the equation of a line of good fit

Key skills

* use a scatterplot to describe an observed association between two numerical variables in terms of strength, direction and form
* identify the explanatory variable and use the equation of a line of good fit by eye to the data to model an observed linear association
* calculate the intercept and slope, and interpret the slope and intercept of the model in the context of data
* use a linear model to make predictions, including the issues of interpolation and extrapolation

Area of Study 2

Discrete mathematics

Graphs and networks

Key knowledge

* the language, properties and types of graphs, including edge, face, loop, vertex, the degree of a vertex, isomorphic and connected graphs, and the adjacency matrix, Euler’s formula for planar graphs, and walks, trails, paths, circuits, bridges and cycles in the context of traversing a graph
* weighted graphs and networks, and the shortest path problem
* trees, minimum spanning trees and the concept of a greedy algorithm

Key skills

* describe a planar graph in terms of the number of faces (regions), vertices and edges and apply Euler’s formula to solve associated problems
* apply the concepts of connected graphs: trails, paths, circuits, bridges and cycles to model and solve practical problems related to traversing a graph
* find the shortest path in a weighted graph (solution by inspection only)
* apply the concepts of trees and minimum spanning trees to solve practical problems using a variety of greedy algorithms
* construct graphs, digraphs and networks and their matrix equivalents to model and analyse practical situations

Area of Study 3

Functions, relations and graphs

Variation

Key knowledge

* the concepts of direct and inverse variation
* the methods of transforming data
* orders of magnitude, units of measure that range over multiple orders of magnitude, and the concept of a logarithmic (base 10) scale

Key skills

* solve problems which involve the use of direct or inverse variation
* model non-linear data by using suitable transformations
* use a logarithmic (base 10) scale to represent quantities that range over several orders of magnitude and to solve variation problems

Area of Study 4

Space and measurement

Space, measurement and applications of trigonometry

Key knowledge

* the measures of length, area, volume and capacity and their units of measurement
* Pythagoras’ theorem and the trigonometric ratios (sine, cosine and tangent) and their application including angles of elevation and depression and three figure bearings
* the definition of sine and cosine for angles up to 180°
* the sine rule (including the ambiguous case) and the cosine rule
* the perimeter and areas of triangles (using several methods based on information available), quadrilaterals, circles and composite shapes, including arcs
* formulas for the volumes and surface areas of solids (spheres, cylinders, pyramids, prisms) and their application to composite objects
* similarity and scaling, and the linear scale factor and its extension to areas and volumes
* scientific notation, exact and approximate answers, significant figures and rounding

Key skills

* solve practical problems involving right-angled triangles in the dimensions including the use of angles of elevation and depression, Pythagoras’ theorem trigonometric ratios sine, cosine and tangent and the use of three-figure (true) bearings in navigation
* solve practical problems requiring the calculation of side lengths or angles in non-right-angled triangles using the sine rule or the cosine rule as appropriate
* calculate the perimeter and areas of triangles (calculating the areas of triangles in practical situations using the rules or where
* use quadrilaterals, circles and composite shapes including arcs ( and sectors () in practical situations
* calculate the perimeter, areas, volumes and surface areas of solids (spheres, cylinders, pyramids and prisms and composite objects) in practical situations, including simple uses of Pythagoras’ in three dimensions
* use a linear scale factor to scale lengths, areas and volumes of similar figures and shapes in practical situations
* distinguish between exact and approximate answers and write approximate answers correct to a given number of decimal places or significant figures

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the facts, concepts and techniques associated with the topics studied
* the standard mathematical models used in the topics studied
* the facts, concepts, techniques and/or models suitable to solve extended application problems or to conduct a structured investigation in context
* assumptions and conditions underlying the facts, concepts, techniques and models used when solving a problem or conducting a mathematical investigation

Key skills

* identify, recall and select the mathematical facts, concepts and techniques needed to solve an extended problem or conduct an investigation in a variety of contexts
* recall, select and use standard mathematical models to represent practical situations
* use specific models to comment on particular situations being analysed and to make predictions
* interpret and report the results of applying these models in terms of the context of the problem being solved, including discussing the assumptions underlying the use of such models

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* the difference between exact numerical and approximate numerical answers when using technology to perform computation, and rounding to a given number of decimal places or significant figures
* domain and range requirements for specification of graphs, and the role of parameters in specifying general forms of models, relations and equations
* the relation between numerical, graphical and symbolic forms of information about models, relations and equations and the corresponding features of those functions, relations and equations
* the similarities and differences between formal mathematical expressions and their representation by technology

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
* use technology to carry out numerical, graphical and symbolic computation as applicable
* produce results, using a technology, which identify examples or counter-examples for propositions
* produce tables of values, families of graphs and collections of other results using technology, which support general analysis in investigative, modelling and problem-solving contexts
* use appropriate domain and range specifications to illustrate key features of graphs of models and relations
* identify the connection between numerical, graphical and symbolic forms of information about functions, relations and equations, and the corresponding features of those functions, relations and equations
* specify the similarities and differences between formal mathematical expressions and their representation by technology
* select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
* apply suitable constraints and conditions, as applicable, to carry out required computations
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass the areas of study in the unit.

Suitable tasks for assessment in this unit may be selected from the following.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

* assignments
* tests
* solutions to sets of worked questions
* summary notes or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

* modelling tasks
* problem-solving tasks
* mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for computational thinking and the effective and appropriate use of technology.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Unit 1: Mathematical Methods

Mathematical Methods Units 1 and 2 provide an introductory study of simple elementary functions of a single real variable, algebra, calculus, probability and statistics and their applications in a variety of practical and theoretical contexts. The units are designed as preparation for Mathematical Methods Units 3 and 4 and contain assumed knowledge and skills for these units.

The focus of Unit 1 is the study of simple algebraic functions, and the areas of study are ‘Functions, relations and graphs’, ‘Algebra, number and structure’, ‘Calculus’ and ‘Data analysis, probability and statistics’. At the end of Unit 1, students are expected to have covered the content outlined in each area of study, with the exception of ‘Algebra, number and structure’ which extends across Units 1 and 2. This content should be presented so that there is a balanced and progressive development of skills and knowledge from each of the four areas of study with connections between and across the areas of study being developed consistently throughout both Units 1 and 2.

In undertaking this unit, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algorithms, algebraic manipulation, equations, graphs and differentiation, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout the unit as applicable.

Area of Study 1

Functions, relations and graphs

In this area of study students cover the graphical representation of simple algebraic functions (polynomial and power functions) of a single real variable and the key features of functions and their graphs such as axis intercepts, domain (including the concept of maximal, natural or implied domain), co-domain and range, stationary points, asymptotic behaviour and symmetry. The behaviour of functions and their graphs is to be explored in a variety of modelling contexts and theoretical investigations.

This area of study includes:

* functions and function notation, domain, co-domain and range, representation of a function by rule, graph and table, inverse functions and their graphs
* qualitative interpretation of features of graphs of functions, including those of real data not explicitly represented by a rule, with approximate location of any intercepts, stationary points and points of inflection
* graphs of power functionsfor , and transformations of these graphs
to the form where  and 
* graphs of polynomial functions of low degree, and interpretation of key features of these graphs.
*

Area of Study 2

Algebra, number and structure

This area of study supports students’ work in the ‘Functions, relations and graphs’, ‘Calculus’ and ‘Data analysis, probability and statistics’ areas of study, and content is to be distributed between Units 1 and 2.
In Unit 1 the focus is on the algebra of polynomial functions of low degree and transformations of the plane.

This area of study includes:

* use of symbolic notation to develop algebraic expressions and represent functions, relations, equations, and systems of simultaneous equations
* substitution into, and manipulation of, these expressions
* recognition of equivalent expressions and simplification of algebraic expressions involving different forms of polynomial and power functions, the use of distributive and exponent laws applied to these functions, and manipulation from one form of expression to an equivalent form
* use of parameters to represent families of functions and determination of rules of simple functions and relations from given information
* transformations of the plane and application to basic functions and relations by simple combinations of dilations (students should be familiar with both ‘parallel to an axis’ and ‘from an axis’ descriptions), reflections in an axis and translations (matrix representation may be used but is not required)
* the connection between the roots of a polynomial function, its factors and the horizontal axis intercepts of its graph, including the remainder, factor and rational root theorems
* solution of polynomial equations of low degree, numerically, graphically and algebraically, including numerical approximation of roots of simple polynomial functions using the bisection method algorithm
* solution of a set of simultaneous linear equations (geometric interpretation only required for two variables) and equations of the form  numerically, graphically and algebraically.

Area of Study 3

Calculus

In this area of study students cover constant and average rates of change and an introduction to instantaneous rate of change of a function in familiar contexts, including graphical and numerical approaches to estimating and approximating these rates of change.

This area of study includes:

* average and instantaneous rates of change in a variety of practical contexts and informal treatment of instantaneous rate of change as a limiting case of the average rate of change
* interpretation of graphs of empirical data with respect to rate of change such as temperature or pollution levels over time, motion graphs and the height of water in containers of different shapes that are being filled at a constant rate, with informal consideration of continuity and smoothness
* use of gradient of a tangent at a point on the graph of a function to describe and measure instantaneous rate of change of the function, including consideration of where the rate of change is positive, negative or zero, and the relationship of the gradient function to features of the graph of the original function.

Area of Study 4

Data analysis, probability and statistics

In this area of study students cover the concepts of experiment (trial), outcome, event, frequency, probability and representation of finite sample spaces and events using various forms such as lists, grids, Venn diagrams and tables. They also cover introductory counting principles and techniques and their application to probability.

This area of study includes:

* random experiments, sample spaces, outcomes, elementary and compound events, random variables and the distribution of results of experiments
* simulation using simple random generators such as coins, dice, spinners and pseudo-random generators using technology, and the display and interpretation of results, including informal consideration of proportions in samples
* addition and multiplication principles for counting
* combinations including the concept of a selection and computation of  and the application of counting techniques to probability.

Mathematical investigation

This comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes.

Investigation is to be incorporated in the development of concepts, skills and processes for the unit, and can be used to assess the outcomes.

There are three components to mathematical investigation:

Formulation

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest.

Exploration

Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Communication

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

Possible contexts or scenarios

Some possible contexts or scenarios for the mathematical investigation in Unit 1 include:

* A practical or theoretical situation in which a curve passing through points on the plane is modelled by the graph of a polynomial function of low degree. This could involve exploring what information can be used to determine the equation of a linear, quadratic or cubic function, and what graphs can be found to fit particular sets of information. Key features of families of graphs that fit a particular set of information could also be explored. For example, a family of quadratic functions with a leading coefficient equal to 1, whose graph contain the points with coordinates and . By systematically varying , the effect of on the behaviour of the graph of the quadratic function could be explored.
* Combinatorial counting problems and their application to probability in games of chance. This could involve using technology to simulate simple card game problems and comparing the simulation results with theoretical probabilities obtained using combinatorial counting techniques. For example, simulating drawing two cards at random from a standard pack and estimating the probability that they are of the same suit. Alternatively, using simulation to estimate probabilities that are more complicated to calculate theoretically, such as (i) derangements (the rearrangement of the elements of a set such that no element appears in its original position), or (ii) the probability of selecting a particular ‘poker hand’ when five cards are drawn at random from a standard pack.
* Exploration of graphical, algebraic and numerical methods and algorithms for finding the roots of a polynomial function and their accuracy and efficiency. This could include using graphical, spreadsheet and dynamic geometry functionalities of technology to observe multiple representations of convergence towards the root of a polynomial function by successive iterations of the bisection method. For each iteration, an interval is bisected and a subinterval in which a root must lie is selected for further processing. A possible extension is to analyse the pseudocode for the bisection method algorithm and use it to assist in writing a program to run the bisection method algorithm on a selected technology.

Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the definition of a function, the concepts of domain (including maximal, natural or implied domain),
co-domain and range, notations for specification of the domain, co-domain and range and rule of a function
* the key features and properties of power and polynomial functions and their graphs, including any vertical or horizontal asymptotes
* the effect of transformations of the plane, dilation, reflection in axes, translation and simple combinations of these transformations, on the graphs of functions
* the relation between the graph of a one-to-one function, its inverse function and reflection in the line
* representations of points and transformations
* factorisation patterns, the quadratic formula and discriminant, the remainder, factor and rational root theorems and the null factor law
* the exponent laws
* average and instantaneous rates of change and their interpretation with respect to the graphs of functions
* forms of representation of sample spaces and events
* the properties that probabilities for a given sample space are non-negative and the sum of these probabilities is one
* counting techniques and their application to probability

Key skills

* specify the rule, domain (including maximal, natural or implied domain), co-domain, and range of a function and identify whether or not a relation is a function
* substitute integer, simple rational and irrational numbers in exact form into expressions, including rules of functions and relations, and evaluate these by hand
* re-arrange and solve simple algebraic equations and inequalities by hand
* expand and factorise linear and simple quadratic expressions with integer coefficients by hand
* express  in completed square form where and , by hand
* express a cubic polynomial, with integer coefficients, in the form and determine , by hand
* use algebraic, graphical and numerical approaches, including the factor theorem and the bisection method algorithm, to determine and verify solutions to equations over a specified interval
* apply distributive and exponent laws to manipulate and simplify expressions involving polynomial and power function, by hand in simple cases
* set up and solve systems of simultaneous linear equations involving up to four unknowns, including by hand for a system of two equations in two unknowns
* sketch by hand graphs of linear, quadratic and cubic polynomial functions, and quartic polynomial functions in factored form (approximate location of stationary points only for cubic and quartic functions), including cases where an x-axis intercept is a touch point or a stationary point of inflection
* sketch by hand graphs of power functions where and simple transformations of these, and identify any vertical or horizontal asymptotes
* draw graphs of polynomial functions of low degree, simple power functions and simple relations that are not functions
* describe the effect of transformations on the graphs of relations and functions
* sketch the graph of the inverse function of a one-to-one function given its graph
* use graphical, numerical and algebraic approaches to find an approximate value or the exact value
(as appropriate) for the gradient of a secant or tangent to a curve at a given point
* set up probability simulations, and describe the notions of randomness and variability, and their relation to events
* apply counting techniques to solve probability problems and calculate probabilities for compound events, by hand in simple cases

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* key mathematical content from one or more areas of study related to a given context
* specific and general formulations of concepts used to derive results for analysis within a given context
* the role of examples, counter-examples and general cases in working mathematically
* key elements of algorithm design: sequencing, decision-making, repetition and representation including the use of pseudocode
* inferences from analysis and their use to draw valid conclusions related to a given context

Key skills

* specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
* identify important information, variables, constraints and other key features to the investigation of various questions in a given context
* develop mathematical formulations of specific and general cases used to derive results for analysis within a given context
* use a variety of techniques to verify results
* make inferences from analysis and use these to draw valid conclusions related to a given context
* communicate results and conclusions using both mathematical expression and everyday language,
in particular, the interpretation of mathematics with respect to a context

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of technology
* domain and range requirements for specification of graphs of functions and relations when using technology
* the role of parameters in specifying general forms of functions and equations
* the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
* the similarities and differences between formal mathematical expressions and their representation by technology
* the purpose and effect of sequencing, decision-making and repetition statements on relevant functionalities of technology, and their role in the design of algorithms and simulations
* the appropriate functionality of technology for a variety of mathematical contexts

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
* use technology to carry out numerical, graphical and symbolic computation as applicable
* produce results, using technology, which identify examples or counter-examples for propositions
* produce tables of values, families of graphs and collections of other results using technology, which support general analysis in investigative, modelling and problem-solving contexts
* use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
* identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* specify the similarities and differences between formal mathematical expressions and their representation by technology, in particular, equivalent forms of symbolic expressions
* select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
* design and implement simulations and algorithms using appropriate functionalities of technology
* apply suitable constraints and conditions, as applicable, to carry out required computations
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass the areas of study in the unit.

Suitable tasks for assessment in this unit may be selected from the following.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

* assignments
* tests
* solutions to sets of worked questions
* summary notes or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on mathematical investigations and a selection of modelling or problem-solving tasks.

Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for computational thinking and the effective and appropriate use of technology.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Unit 2: Mathematical Methods

The focus of Unit 2 is the study of simple transcendental functions, the calculus of polynomial functions and related modelling applications. The areas of study are ‘Functions, relations and graphs’, ‘Algebra, number and structure’, ‘Calculus’ and ‘Data analysis, probability and statistics’. At the end of Unit 2, students are expected to have covered the content outlined in each area of study.

Material from the areas of study should be organised so that there is a clear progression of skills and knowledge from Unit 1 to Unit 2 in each area of study.

In undertaking this unit, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algorithms, algebraic manipulation, equations, graphs, differentiation and anti-differentiation, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment,
is to be incorporated throughout the unit as applicable.

Area of Study 1

Functions, relations and graphs

In this area of study students cover graphical representation of circular, exponential and logarithmic functions of a single real variable and the key features of graphs of functions such as axis intercepts, domain (including maximal, natural or implied domain), co-domain and range, asymptotic behaviour, periodicity and symmetry. The behaviour of functions and their graphs is to be explored in a variety of modelling contexts and theoretical investigations.

This area of study includes:

* the unit circle, radians, arc length and sine, cosine and tangent as functions of a real variable
* the relationships  for small values of ,  and 
* exact values for sine, cosine and tangent of and , 
* symmetry properties, complementary relations and periodicity properties for sine, cosine and tangent functions
* circular functions of the form  and their graphs, where  is the sine, cosine or tangent function, and  with 
* simple applications of sine and cosine functions of the above form, with examples from various modelling contexts, the interpretation of period, amplitude and mean value in these contexts and their relationship to the parameters  and 
* exponential functions of the form  and their graphs, where ,, 
* logarithmic functions of the form , where , and their graphs, and as the inverse function of , including the relationships  and 
* simple applications of exponential functions of the above form, with examples from various modelling contexts, and the interpretation of initial value, rate of growth or decay, half-life, doubling time and long run value in these contexts and their relationship to the parameters  and.

Area of Study 2

Algebra, number and structure

This area of study supports students’ work in the ‘Functions, relations and graphs’, ‘Calculus’ and ‘Data analysis, probability and statistics’ areas of study. In Unit 2 the focus is on the algebra of some simple transcendental functions and transformations of the plane. This area of study provides an opportunity for the consolidation and revision, further development and application of content prescribed in Unit 1, as well as the study of additional algebra material introduced in the other areas of study in Unit 2 as follows:

* use of inverse functions and transformations to solve equations of the form , where  for , and  is sine, cosine, tangent or , using exact or approximate values on
a given domain
* exponent laws and logarithm laws, including their application to the solution of simple exponential equations
* numerical approximation of roots of cubic polynomial functions using the Newton’s method algorithm.

Area of Study 3

Calculus

In this area of study students cover differentiation and anti-differentiation of polynomial functions by rule, different notations, and related applications including the analysis of graphs.

This area of study includes:

* informal treatment of the gradient of the tangent to a curve at a point as a limit, and the limit definition of the derivative of a function
* the central difference approximation and its graphical interpretation
* the derivative as the gradient of the graph of a function at a point and its representation by a gradient function, and as a rate of change
* differentiation of polynomial functions by rule
* applications of differentiation, including finding instantaneous rates of change, stationary values of functions, local maxima or minima, points of inflection, analysing graphs of functions including motion graphs, and solving maximum and minimum problems with consideration of modelling domain and local and global maxima and minima
* anti-differentiation as the inverse process of differentiation and identification of families of curves with the same gradient function, and the use of a boundary condition to determine a specific anti-derivative of a given function.

Area of Study 4

Data analysis, probability and statistics

In this area of study students cover the use of lists, tables and diagrams to calculate probabilities, including consideration of complementary, mutually exclusive, conditional and independent events involving one, two or three events (as applicable), including rules for computation of probabilities for compound events.

This area of study includes:

* probability of elementary and compound events and their representation as lists, grids, Venn diagrams, tables and tree diagrams
* the addition rule for probabilities, , and the relation that for mutually exclusive events , hence 
* conditional probability in terms of reduced sample space, the relations  and



* the law of total probability for two events 
* the relations for pairwise independent events  and , ,  and 
* simulation to estimate probabilities involving selection with and without replacement.

Mathematical investigation

This comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes.

Investigation is to be incorporated in the development of concepts, skills and processes for the unit, and can be used to assess the outcomes.

There are three components to mathematical investigation:

Formulation

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest.

Exploration

Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Communication

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

Possible contexts or scenarios

Some possible contexts or scenarios for mathematical investigation in Unit 2 include:

* Variability in random sampling, including sample-to-sample variability when using sampling to estimate the proportion of a population with a particular characteristic. This could involve using technology to simulate drawing repeated random samples from a large population where the proportion of the population with a particular characteristic is known. The sample proportion of that characteristic for each sample could be captured and used to create a graphical display of the distribution of sample proportions and provide a visual representation of variability between samples of a given sample size. The effect of changing the sample size or the population proportion on the distribution of sample proportions could also be explored.
* Representation and application of transformations of the plane involving sets of points, geometric shapes, graphs of functions and graphics images. This could include using dynamic geometry functionality of technology to visualise, in a dynamic environment, the effect of each successive reflection, translation or dilation in a compound transformation. This approach could also be used to explore a sequence of transformations needed to transform a geometric object or graph of a function from its initial state to its final state.
* Modelling and qualitative analysis of rates of change from real data graphically, the relationship between the graph of a modelling function and its rate of change function and vice-versa. A variety of real-world examples could be explored, such as rate of fuel consumption, run rate in cricket (runs per over), postage rate and speed. Dynamic functionalities of technology could be used to facilitate simultaneous comparison of key features of the graphs of the rate of change function and the modelling function. For example, by interactively creating a plot of the graph of the rate of change function while tracing the graph of the modelling function.

Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the unit circle and exact values of sine, cosine and tangent for  (and their degree equivalents) and integer multiples of these
*  for small values of 
* the key features and properties of the circular functions sine, cosine and tangent, and their graphs, including any vertical asymptotes
* the effect of transformations of the plane on the graphs of sine, cosine, tangent and exponential functions
* characteristics of data which suggest the use of sine, cosine, exponential or logarithmic functions as an appropriate type of model for a given context
* the key features and properties of the exponential functions, logarithmic functions and their graphs, including any vertical or horizontal asymptotes
* the relationship between an exponential function to a given base and the logarithmic function to the same base as inverse functions
* the limit definition of the derivative of a function, the central difference approximation, and the derivative as the rate of change or gradient function of a given function
* informal concepts of limit, continuity and differentiability
* the sign of the gradient at and near a point and its interpretation in terms of key features of a graph of simple polynomial functions
* the rules for finding derivatives and anti-derivatives of simple polynomial functions
* representations of compound events, the addition rule for probability, the concepts of mutually exclusive and independent events, conditional probability and the law of total probability

Key skills

* sketch by hand the unit circle, graphs of the sine, cosine and exponential functions, and simple transformations of these to the form , sketch by hand graphs of and the tangent function, and identify any vertical or horizontal asymptotes
* draw graphs of circular, exponential and simple logarithmic functions over a given domain and identify and discuss key features and properties of these graphs, including any vertical or horizontal asymptotes
* describe the effect of transformations of the plane on the graphs of the sine, cosine, tangent and exponential functions
* solve simple equations over a specified interval related to circular, exponential and simple logarithmic functions using graphical, numerical and analytical approaches
* use the algorithm for Newton’s method to find a numerical approximation to a root of a cubic polynomial function
* recognise characteristics of data which suggest that a circular or exponential function is an appropriate model for the data
* evaluate limit values of a function
* use a variety of approaches to find the value of the derivative of a function at a given point
* find by hand the derivative function and an anti-derivative function for a polynomial function of low degree
* use derivatives to assist in the sketching of graphs of simple polynomial functions and to solve simple maximum and minimum optimisation problems
* find a family of anti-derivative functions for a given polynomial function, and determine a specific antiderivative given a boundary condition
* calculate probabilities for compound events using rules and tree diagrams, by hand in simple cases
* solve probability problems involving tree diagrams, by hand in simple cases
* set up probability simulations to estimate probabilities for selection with and without replacement

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* key mathematical content from one or more areas of study related to a given context
* specific and general formulations of concepts used to derive results for analysis within a given context
* the role of examples, counter-examples and general cases in working mathematically
* key elements of algorithm design: sequencing, decision-making, repetition, and representation including the use of pseudocode
* inferences from analysis and their use to draw valid conclusions related to a given context

Key skills

* specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
* identify important information, variables, constraints and other key features to the investigation of various questions in a given context
* develop mathematical formulations of specific and general cases used to derive results for analysis within a given context
* use a variety of techniques to verify results
* make inferences from analysis and use these to draw valid conclusions related to a given context
* communicate results and conclusions using both mathematical expression and everyday language, in particular, the interpretation of mathematics with respect to a context

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of technology
* domain and range requirements for specification of graphs of functions and relations when using technology
* the role of parameters in specifying general forms of functions and equations
* the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
* the similarities and differences between formal mathematical expressions and their representation by technology
* the purpose and effect of sequencing, decision-making and repetition statements on relevant functionalities of technology, and their role in the design of algorithms and simulations
* the appropriate functionality of technology for a variety of mathematical contexts

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
* use technology to carry out numerical, graphical and symbolic computation as applicable
* produce results, using technology, which identify examples or counter-examples for propositions
* produce tables of values, families of graphs and collections of other results using technology, which support general analysis in investigative, modelling and problem-solving contexts
* use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
* identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* specify the similarities and differences between formal mathematical expressions and their representation by technology, in particular, equivalent forms of symbolic expressions
* select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
* design and implement simulations and algorithms using appropriate functionalities of technology
* apply suitable constraints and conditions, as applicable, to carry out required computations
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass the areas of study in the unit.

Suitable tasks for assessment in this unit may be selected from the following.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

* assignments
* tests
* solutions to sets of worked questions
* summary notes or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on mathematical investigations and a selection of modelling or problem-solving tasks.

Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for computational thinking and the effective and appropriate use of technology.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Unit 1: Specialist Mathematics

Specialist Mathematics Units 1 and 2 provide a course of study for students who wish to undertake an
in-depth study of mathematics, with an emphasis on concepts, skills and processes related to mathematical structure, modelling, problem-solving, reasoning and proof. This study has a focus on interest in the discipline of mathematics and investigation of a broad range of applications, as well as development of a sound background for further studies in mathematics and mathematics related fields.

Mathematical Methods Units 1 and 2 and Specialist Mathematics Units 1 and 2, taken in conjunction, provide a comprehensive preparation for Specialist Mathematics Units 3 and 4. Study of Specialist Mathematics Units 3 and 4 also assumes concurrent study or previous completion of Mathematical Methods Units 3 and 4.

The areas of study for Specialist Mathematics Units 1 and 2 are ‘Algebra, number and structure’, ‘Data analysis, probability and statistics’, ‘Discrete mathematics’, ‘Functions, relations and graphs’ and ‘Space and measurement’.

At the end of Unit 1 students are expected to have covered the material in the areas of study: ‘Algebra, number and structure’ and ‘Discrete mathematics’. Concepts from these areas of study will be further developed and used in Unit 2 and also in Units 3 and 4.

In undertaking this unit, students are expected to be able to apply techniques, routines and processes involving rational, real and complex arithmetic, sets, lists, tables and matrices, diagrams, graphs, logic gates and geometric constructions, algorithms, algebraic manipulation, recurrence relations, equations and graphs, with and without the use of technology. They are expected to be able to construct proofs and develop and interpret algorithms to solve problems. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Algebra, number and structure

In this area of study students cover the development of formal mathematical notation, definition, reasoning and proof applied to number systems, graph theory, sets, logic, and Boolean algebra, and the development of algorithms to solve problems.

Proof and number

This topic includes:

* number systems for the natural numbers, *N*, integers, *Z*, rational numbers, *Q*, real numbers, *R*, and complex numbers, *C*, and their fundamental properties and structure
* set notation and operations including element, intersection, union, complement sub-set and power set
* prime numbers, the fundamental theorem of arithmetic, and proof that there are infinitely many prime numbers
* conversion between fraction and decimal forms of rational numbers
* introduction to principles of proof including propositions and quantifiers, examples and counter-examples, direct proof, proof by contradiction, and proof using the contrapositive and mathematical induction
* simple proofs involving, for example, divisibility, sequences and series, inequalities and irrationality.

Graph theory

This topic includes:

* vertices and edges for undirected graphs, including multiple edges and loops, and their representation using lists, diagrams and adjacency matrices
* examples of graphs from a range of contexts such as molecular structure, electrical circuits, social networks, utility connections and their use to discuss types of problems in graph theory including existence problems, construction problems, counting problems and optimisation problems
* the degree of a vertex and the result that the sum of all the vertex degrees is equal to twice the number of edges (the handshaking lemma)
* simple graphs, isomorphism, subgraphs, connectedness, complete graphs and the complement of a graph
* bi-partite graphs, trees, and regular graphs (including the Platonic graphs)
* planar graphs and related proofs and applications such as:
* Euler’s formula for simple connected planar graphs
* the complete graph on vertices has edges
* a regular graph with vertices each of degree has edges
* the planarity of various types of graphs, including all trees, complete graph on vertices, if , and the complete bipartite graph with vertices, if or
* equivalent conditions for a simple graph with 𝑛 vertices to be a tree
* trails and circuits, Euler circuits and Euler trails, Hamiltonian cycles and paths, and the Konigsberg bridge problem.

Logic and algorithms

This topic includes:

* propositions, connectives, truth values, truth tables and Karnaugh maps
* Boolean algebras
* binary number systems
* tautologies, validity and proof patterns and the application of these to proofs in natural language and
laws and properties of Boolean algebra, the algebra of sets and propositions
* logic gates and circuits, and simplification of circuits
* definition of an algorithm and the fundamental constructs needed to describe algorithms: sequence, decision (selection, choice, if … then … blocks) and repetition (iteration and loops)
* construction and implementation of basic algorithms incorporating the fundamental constructs using pseudocode.

Area of Study 2

Discrete mathematics

In this area of study students cover the study of sequences, series, and first-order linear difference equations, combinatorics, including the pigeon-hole principle, the inclusion-exclusion principle, permutations and combinations, combinatorial identities, and matrices.

Sequences and series

This topic includes:

* definitions of sequences and series, arithmetic and geometric sequences and their partial sums
* the limiting behaviour as of the terms in a geometric sequence and dependence on the value of the common ratio
* sequences generated by recursion
* solution of first order linear recurrence relations of the form with constant coefficients and their application to financial problems and population modelling.

Combinatorics

This topic includes:

* the pigeon-hole principle and its use in solving problems and proving results
* the inclusion-exclusion principle for the union of two sets and the union of three sets
* permutations and combinations and their use in solving problems involving arrangements and selections with or without repeated elements
* derivation and application of simple combinatorial identities.

Matrices

This topic includes:

* matrix notation, dimension and the use of matrices to represent data
* matrix operations and algebra
* determinants and matrix equations, and simple applications.

Mathematical investigation

This comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes.

Investigation is to be incorporated in the development of concepts, skills and processes for the unit, and can be used to assess the outcomes.

There are three components to mathematical investigation:

Formulation

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest.

Exploration

Investigation and analysis of the context or scenario with respect to the questions of interest, conjecture
or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Communication

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

Possible contexts or scenarios

Some possible contexts or scenarios for mathematical investigation in Unit 1 include:

* Using first-order linear recurrence relations to investigate situations such as financial modelling, population growth, predator prey models and their sensitivity to boundary conditions. Use of coding or spreadsheets to investigate the result of varying parameters. Situations such as three different businesses sharing a market and how the behaviour of each business influences the other two or how the growth of a parasite affects the growth of a plant and vice versa. The Collatz sequence and Fibonacci sequences also provide sequence investigations which can involve limiting behaviour.
* Considering partial sums of sequences including proofs by mathematical induction of identities involving partial sums and the use of partial sums to find approximations to numbers such as or to approximate functions such as sine and cosine. Coding, spreadsheets and CAS can be used to support the investigation.
* Simplifying electronic circuits using Boolean algebra and Karnaugh maps with two and three variables. Using ‘or’, ‘and’, ‘not’, ‘nor’ (not or) and ‘nand’ (not and) gates in design of circuits, and simulating circuits.
* Using technology and coding to aid systematic counting, for example, after tossing 10 dice and adding together the values on the uppermost faces, asking how many ways a particular sum or range of sums can be obtained; or counting problems that arise in Biology from considering DNA strings and the occurrence of particular molecules in a particular string.

Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* sets of numbers: representations of natural numbers, integers, rational numbers and real numbers
* set notations and operations
* principles of proof and proof techniques: contradiction, contrapositive and counter-examples, the method of proof by mathematical induction
* first-order linear recurrence relations and in particular arithmetic and geometric sequences and their partial sums
* the pigeon-hole principle as a problem-solving technique, techniques of counting such as permutations and combinations, the inclusion-exclusion principle and simple combinatorial identities
* matrix notation and algebra
* notation, definitions and representations of graphs, types of graphs and their properties
* applications of graph theory and constructions and graphs
* elementary graph theory theorems and their proofs
* concepts of proposition, truth value, tautology, Karnaugh maps, set and set operations
* concepts of validity, argument and proof
* Boolean operators, and axioms of Boolean algebra
* logic gates and circuit diagrams
* key elements of algorithm design: sequence, decision and repetition
* the ordered steps for an algorithm from the related pseudocode
* algorithms in a variety of contexts as a precise set of instructions that can be specified using pseudocode

Key skills

* apply deductive reasoning, including mathematical induction, and use appropriate language, in the construction of mathematical arguments and proofs involving concepts from topics in this area of study
* identify assumptions, give definitions and provide examples and counter-examples using appropriate mathematical language, diagrams and models
* define and represent number in various structures and contexts such as integer, rational, real and complex number systems, ordered sets of numbers such as sequences and series
* solve problems involving first order linear recurrence relations using both recursive and formula-based techniques
* solve problems which involve techniques of counting
* use deductive reasoning to solve problems involving counting techniques, the pigeon-hole principle and Pascal’s triangle
* use matrix algebra including solving matrix equations
* construct graphs and use them to model situations
* use algorithms to construct subsets of graphs according to conditions and solve related problems
* develop and understand results, including planar graphs, trails and circuits
* solve problems and prove theorems involving graphs
* represent and test the truth of propositions using truth tables
* develop proofs of propositions in natural language and mathematics
* perform exact and approximate computations and apply algorithms including reading and interpreting algorithms described by pseudocode in various structures and contexts

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the application of mathematical content from one or more areas of study in a given context for investigation
* specific and general formulations of concepts used to derive results for analysis within a given context for investigation
* the role of proof in establishing results
* the role of examples, counter-examples and general cases in developing mathematical analysis
* the role of developing algorithms and expressing these through pseudocode to help determine and understand mathematical ideas and results
* inferences from analysis and their use to draw valid conclusions related to a given context for investigation

Key skills

* specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
* develop mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation
* use algorithms to solve or help solve problems
* use proof to establish results rigorously in different contexts
* make inferences from analysis and use these to draw valid conclusions related to a given context for investigation
* communicate conclusions using both mathematical expression and everyday language; in particular,
the interpretation of mathematics with respect to the context for investigation

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques
or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* exact and approximate technological specification of mathematical information such as numerical data, graphical forms and the solutions of equations
* domain and range requirements for the technological specification of graphs of functions and relations
* the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* the similarities and differences between formal mathematical expressions and their representation in various technology applications
* the appropriate selection of a technology application for a variety of mathematical contexts

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results, and interpret these results to a specified degree of accuracy
* produce results, using technology, which identify examples or counter-examples for propositions
* produce tables of values, families of graphs or collections of other results which support general analysis in investigative, modelling or problem-solving contexts
* carry out analysis in investigative, modelling or problem-solving contexts
* use appropriate domain and range technological specifications to illustrate key features of graphs of functions and relations
* design and implement simulations and algorithms using appropriate functionalities of technology
* identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* specify the similarities and differences between formal mathematical expressions and their representation in various technology applications
* make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass the areas of study in the unit.

Suitable tasks for assessment in this unit may be selected from the following.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

* assignments
* tests
* solutions to sets of worked questions
* summary notes or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on mathematical investigations and a selection of modelling or problem-solving tasks.

Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for computational thinking and the effective and appropriate use of technology.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Unit 2: Specialist Mathematics

The areas of study for Specialist Mathematics Units 1 and 2 are ‘Algebra, number and structure’, ‘Data analysis, probability and statistics’, ‘Discrete mathematics’, ‘Functions, relations and graphs’ and ‘Space and measurement’.

At the end of Unit 2 students are expected to have covered the material in the areas of studies: ‘Data analysis, probability and statistics’, ‘Space and measurement’, ‘Algebra, number and structure’ and ‘Functions, relations and graphs’.

In undertaking this unit, students are expected to be able to apply techniques, routines and processes involving rational, real and complex arithmetic, sets, lists, tables, vectors and matrices, diagrams and geometric constructions, algorithms, algebraic manipulation, equations and graphs, with and without the use of technology. They are expected to be able to construct proofs and develop and interpret algorithms to solve problems. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Data analysis, probability and statistics

In this area of study students cover the study of linear combinations of random variables and the distribution of sample means of a population, with the use of technology to explore variability of sample means.

Simulation, sampling and sampling distributions

This topic includes:

*Distribution of* *sums of discrete random variables*

* the mean, variance and standard deviation of a discrete random variable *X*
* the distribution of the sum of identically distributed independent discrete random variables
* comparison of the distribution of 2*X*, where *X* is a discrete random variable and the sum of 2 independent discrete random variables that are each identically distributed as *X*.

*Simulation*

* random experiments, events and event spaces
* use of simulation to generate a random sample.

*Sampling distributions*

* the distinction between a population parameter and a sample statistic and the use of a sample statistic
* (sample mean ) as an estimate of the associated population parameter (mean )
* the concept of a sampling distribution
* the distribution of sample means considered empirically including comparing the distributions of different size samples from the same population in terms of centre and spread
* display of variation in sample means through dot plots and other displays and considering the centre and spread of these distributions
* consideration of the mean and standard deviation of the distribution of sample means and the effect of taking larger samples.

Area of Study 2

Space and measurement

In this area of study students cover trigonometry and identities, rotation and reflection transformations of the plane and vectors for working with position, shape, direction and movement in the plane and related applications.

Trigonometry

This topic includes:

* radian measure, arc length, sectors and segments
* the sine rule and cosine rule applied to two and three-dimensional situations, including problems involving angles between planes
* compound and double angle formulas for sine, cosine and tangent and the identities:
and
* proof and application of identities between and or where is in the first quadrant, the identities for products of sines and cosines expressed as sums and differences, and the identities for addition and differences of sines and cosines expressed as products.

Transformations

This topic includes:

* points in the plane, coordinates and their representation as 2 × 1 matrices (column vectors)
* translations of the plane
* linear transformations of the plane as a map of the plane onto itself, rotations about the origin and reflection in a line through the origin
* the effect of translation, linear transformations and their inverse transformations, and compositions of these transformations on subsets of the plane such as points, lines, shapes and graphs of functions and relations
* invariance of properties under transformation, and the relationship between the determinant of a transformation matrix and the effect of the linear transformation on the area of a bounded region of the plane.

Vectors in the plane

This topic includes:

* the representation of plane vectors as directed lines segments, magnitude and direction of a plane vector, and unit vectors
* geometric representation of addition, subtraction (triangle and/or parallelogram rules), scalar multiple of a vector and linear combination of plane vectors
* the representation of a plane vector as an ordered pair in the form 
* simple vector algebra (addition, subtraction, multiplication by a scalar, linear combination) using these forms
* scalar (dot) product of two plane vectors, perpendicular and parallel vectors, projection of one vector onto another vector, and angle between two vectors
* geometric proofs with vectors
* application of vectors to displacement, velocity, resultant velocity, relative velocity, statics and motion under a constant force.

Area of Study 3

Algebra, number and structure

In this area of study students cover the arithmetic and algebra of complex numbers, including polar form, regions and curves in the complex plane and introduction to factorisation of quadratic functions over the complex field.

Complex numbers

This topic includes:

* definition and properties of the complex numbers, *C*, arithmetic, modulus of a complex number, and the representation of complex numbers on an Argand diagram
* general solution of quadratic equations (with real coefficients) of a single variable over *C*, and conjugate roots
* lines, rays, circles and ellipses
* regions defined in the complex plane using combinations of the above
* use of the modulus of a complex number and the argument of a non-zero complex number to prove basic identities
* conversion between Cartesian and polar form of complex numbers
* multiplication, division, and powers of complex numbers in polar form and their geometric interpretation.

Area of Study 4

Functions, relations and graphs

In this area of study students cover an introduction to partial fractions; reciprocal and inverse circular functions and their graphs and simple transformations of these graphs; locus definitions of lines, parabolas, circles, ellipses and hyperbolas and the Cartesian, parametric and polar forms of these relations.

This area of study includes:

* identities from equating coefficients of polynomials, rational functions and their decompositions into partial fractions with denominators expressed as products of linear and irreducible quadratic terms
* graphs of simple reciprocal functions, including graphs of the reciprocal circular functions cosecant, secant and cotangent, and simple transformations of these
* graphs of the restricted circular functions of sine, cosine and tangent over principal domains and their respective inverse functions sin–1, cos-1 and tan–1 (students should be familiar with alternative notations, arcsin, arccos and arctan), and simple transformations of these graphs
* locus definition and construction in the plane of lines, parabolas, circles, ellipses and hyperbolas and their Cartesian, polar and parametric forms and graphs
* the absolute value function, its graph and simple transformations of its graph.

Mathematical investigation

This comprises one to two weeks of investigation into one or two practical or theoretical contexts or scenarios based on content from areas of study and application of key knowledge and key skills for the outcomes.

Investigation is to be incorporated in the development of concepts, skills and processes for the unit, and can be used to assess the outcomes.

There are three components to mathematical investigation:

Formulation

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest.

Exploration

Investigation and analysis of the context or scenario with respect to the questions of interest, conjectures or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

Communication

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

Possible contexts or scenarios

Some possible contexts or scenarios for mathematical investigation in Unit 2 include:

* Using simulation to investigate the sample mean distribution of an experiment. Simulation can be carried out with technology through built-in statistics packages or coding to provide tabular and graphical representations of the means of multiple samples of the same size from the same population. The mean, variance and standard deviation of the resulting distribution of sample means can be considered. Further simulations of samples of a different size can be undertaken.
* Solving problems involving resultant displacement, velocity and relative velocity. For resultant velocity, consider the flight of an object with wind acting on it or the effect of currents or flow on the progress of a boat and the implications for navigation. For relative velocity, consider the relative velocity of one moving object relative to another moving object and the implications of this on observing or colliding.
* Exploring loci, curves and shapes in the plane generated using different forms of specification. For example, suitably using parametric, polar and Cartesian forms and locus definitions with the relations of Area of Study 4 and applying the transformations of Area of Study 2. Considering inequalities involving these functions and the associated regions of the plane.
* Transformations of the plane, considering questions such as: How can a given transformation (translation rotation, glide reflection) be expressed as a composition of reflections? How can composition of transformations be used to describe rotations about any point in the plane or reflection
in any straight line?
* Applying trigonometric identities to find exact values of sine, cosine and tangent of as many acute angles as possible and constructing corresponding table of values.

Outcomes

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* linear combination of random variables
* methods of simulation applied to sampling distributions and the effect of taking larger samples from a fixed population
* arc length of circles, areas of sectors and segments of circles
* the reciprocal circular functions and their graphs and the restricted circular functions and their graphs
* trigonometric identities
* sine and cosine rules applied in two and three dimensions, including angles between planes, and congruence conditions for their application
* coordinate and matrix representation of points and transformations
* translations, dilation, rotations and reflections, invariance properties, inverse transformations and composition of transformations
* locus definitions of curves in the plane
* Cartesian, polar and parametric coordinate systems and graphs, including exact values for circular functions
* standard notation and representations for vectors in the plane, including arrows, coordinates and matrices
* the definition of arithmetic operations for vectors in the plane
* geometrical representation of vectors and geometric and coordinate definition of the scalar product of two vectors
* representation of complex numbers and the conventions for arithmetic of complex numbers in Cartesian and polar forms
* regions of the complex plane
* reciprocals and their properties
* representation of relations by Cartesian, parametric and polar equations
* the absolute value function

Key skills

* simulate sampling from a population and display the results of taking multiple samples of the same size from a fixed population
* consider measures of central tendency and spread of the distribution of sample means
* prove trigonometric identities and use them to solve problems
* prove and use the sine and cosine rules and solve problems in both two and three dimensions, including angle between planes problems
* define and apply transformations to the plane and specify their effect on subsets of the plane
* identify the set of points that are invariant under a given transformation
* find and apply inverse transformations and composite transformations, and interpret their effects on subsets of the plane
* use vectors to model situations involving direction and magnitude
* apply vector operations of scalar multiples, addition, subtraction and scalar product
* use vectors to solve problems involving position, displacement, velocity and force
* use vectors to solve geometric problems and prove theorems
* construct the graph of a reciprocal function from the graph of the original function
* use the distance formula and locus definitions to obtain the rule of a relation and draw the corresponding curve in the plane
* represent complex numbers and the effect of arithmetic of complex numbers in both Cartesian and polar form
* graph non-linear relations in the plane from their Cartesian, polar and parametric representations, and identify and interpret their key features, including any linear asymptotes

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the application of mathematical content from one or more areas of study in a given context for investigation
* specific and general formulations of concepts used to derive results for analysis within a given context for investigation
* the role of examples, counter-examples and general cases in developing mathematical analysis
* the role of proof involving the communication and justification of an argument for a mathematical statement in a clear, concise and precise manner
* the role of developing algorithms and expressing these through pseudocode to help determine and understand mathematical ideas and results
* inferences from analysis and their use to draw valid conclusions related to a given context for investigation

Key skills

* specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
* develop mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation
* use algorithms to solve problems
* use proof to establish mathematical results
* make inferences from analysis and use these to draw valid conclusions related to a given context for investigation
* communicate conclusions using both mathematical expression and everyday language; in particular,
the interpretation of mathematics with respect to the context for investigation

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* exact and approximate technological specification of mathematical information such as numerical data, graphical forms and the solutions of equations
* domain and range requirements for the technological specification of graphs of functions and relations
* the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* the similarities and differences between formal mathematical expressions and their representation in various technology applications
* the appropriate selection of a technology application for a variety of mathematical contexts

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results, and interpret these results to a specified degree of accuracy
* produce results, using technology, which identify examples or counter-examples for propositions
* produce tables of values, families of graphs or collections of other results which support general analysis in investigative, modelling or problem-solving contexts
* use appropriate domain and range technological specifications which illustrate key features of graphs of functions and relations
* identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* specify the similarities and differences between formal mathematical expressions and their representation in various technology applications
* make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections
* specify the purpose and effect of sequencing, decision-making and iterative statements on applicable functionalities of technology, and their role in the design of algorithms and simulations
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass the areas of study in the unit.

Suitable tasks for assessment in this unit may be selected from the following.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

* assignments
* tests
* solutions to sets of worked questions
* summary notes or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on mathematical investigations and a selection of modelling or problem-solving tasks.

Demonstration of achievement of Outcome 3 should be based on the student’s performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for computational thinking and the effective and appropriate use of technology.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Units 3 and 4: Foundation Mathematics

Foundation Mathematics Units 3 and 4 focus on providing students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning, community and global settings relevant to contemporary society. The areas of study for Units 3 and 4 are ‘Algebra, number and structure’, ‘Data analysis, probability and statistics’, ‘Discrete mathematics’ and ‘Space and measurement’. All four areas of study are to be completed over the two units, and content equivalent to two areas of study covered in each unit. The selected content for each unit should be developed using contexts present in students’ other studies, work and personal or other familiar situations, and in national and international contexts, events and developments.

Assumed knowledge and skills for Foundation Mathematics Units 3 and 4 are contained in Foundation Mathematics Units 1 and 2, and will be drawn on, as applicable, in the development of related content from the areas of study, and key knowledge and key skills for the outcomes.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, contemporary data displays, diagrams, plans, geometric objects and constructions, algebra, algorithms, measures, equations and graphs, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment,
is to be incorporated throughout each unit as applicable.

Area of Study 1

Algebra, number and structure

In this area of study students cover estimation, the use and application of different forms of numbers and calculations, algorithmic and computational thinking, and the representation of formal mathematical expressions and processes including formulas and other algebraic expressions to solve practical problems in community, business and industry contexts.

This area of study includes:

* mathematical conventions notations for number and number operations
* rational numbers and irrational numbers related to measurement, ratios and proportions in a practical context
* direct and indirect variation
* symbolic expressions, equations and formulas
* graphical and algebraic analysis of relations including transposition of formulas and finding a break-even point using simultaneous equations
* estimation and approximation including interval estimates, rounding, significant figures, leading-digit approximations, floor and ceiling values and percentage error.

Area of Study 2

Data analysis, probability and statistics

In this area of study students cover collection, presentation and analysis of gathered and provided data from community, work, recreation and media contexts, including consideration of suitable forms of representation and summaries. This area of study incorporates the ability to critically reflect on statistical data and results, and to be able to communicate and report on the outcomes and any implications.

This area of study includes:

* development and specification of data collection requirements and methods, including consideration of audience and purpose of data collection, errors and misrepresentations in statistics
* collection and modelling of data, including the construction of tables or spreadsheets and graphs to represent data and correct representations
* contemporary representations of data and graphs derived from technology including reviewing appropriateness of graphical representations, including pictograms, bubble, Mekko, radar, sunburst, heat map and stacked area charts
* long-term data and relative frequencies in practical situation such as in relation to epidemics, climate, environment, sport and marketing
* interpolation and extrapolation of data, predictions, limitations, inferences and conclusions comparing and interpreting data sets and graphs, including using measures of central tendency and spread (percentiles and standard deviation) and cumulative frequency.

Area of Study 3

Discrete mathematics

Financial and consumer mathematics

In this area of study students cover the use and application of different forms of numbers and calculations, relationships and formulae, and their application in relation to the analysis of, and critical reflection on, personal, local, national and global financial, consumer and global matters.

This area of study includes:

* money management including investments and loans, credit and debit, comparing mortgages versus rental costs and debt consolidation
* taxation systems at the personal and business level
* income and expenditure calculations such as GST, invoicing and BAS
* comparison of financial products and services such as insurance
* informal consideration of financial risk at the national and global level (short, medium and long term)
* analysis and interpretation of financial information and data sets, trends and economic indicators and their impact (at the personal, community, national or global level) such as gender pay gap, career trends and interruption, currency fluctuations and inflation, stock market movements and recessions.

Area of Study 4

Space and measurement

In this area of study students cover the use and application of the metric system and related measurement in a variety of domestic, societal, industrial and commercial contexts, including consideration of accuracy, precision and error.

This area of study includes:

* spatial and geometric constructions including transformations, similarity, symmetry and projections
* calculations of enlargement and reduction using scaling techniques for two-dimensional and three-dimensional plans, diagrams and models
* measurements and related quantities including derived quantities, metric and relevant non-metric measures
* conventions, properties and measurement of perimeter, area, surface area and volume of compound shapes and objects
* calibration and error in measurement, including tolerance, accuracy and precision.

Outcomes

For each unit the student is required to demonstrate achievement of all three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For Units 3 and 4, the outcomes apply to the content from the areas of study selected for that unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures to solve practical problems from a range of everyday and real-life contexts.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Algebra, number and structure

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 1.

Key knowledge

* conventions of formal mathematical terminology and notations in calculations, symbolic expressions and formulas
* rational numbers and measurement related irrational numbers
* algorithmic, algebraic and computational strategies
* number facts, operations and relationships for calculations
* ratios, proportions and percentages, direct and indirect variation
* manipulation and transposition of formulas to find unknown values
* simultaneous equations and their graphical and algebraic solutions
* estimations and approximations including interval estimates, rounding, significant figures, leading-digit approximations, floor and ceiling values, and percentage error
* contextual and real-world meaning of numerical results

Key skills

* use and apply the conventions of mathematical notations, terminology and representations
* make estimates and carry out relevant calculations using mental and by-hand methods
* use different technologies effectively for accurate, reliable and efficient calculations
* solve practical problems which require the use and application of a range of numerical and algebraic computations involving rational and real values of variables
* solve practical problems requiring graphical and algebraic processes and applications, including substitution into, and transposition of, formulas and finding a break-even point using simultaneous equations
* use estimation and other approaches to check the outcomes, including for accuracy and reasonableness of results
* evaluate the mathematics used and the outcomes obtained relative to personal, contextual and real-world implications

Data analysis, probability and statistics

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 2.

Key knowledge

* categorical data and numerical data, including continuous data
* purposes for data collection and nature of audience for communication of results
* data collection and organisation, including the development and production of surveys
* the purposes and effectiveness of different forms of data representation and types of graphs and data scales, including contemporary digital representations
* features of sets of data such as measures of central tendency (mean, median and mode) and spread (such as standard deviation, quantile intervals, range and percentiles), shape of distribution and outliers
* likelihood and chance of events in relation to relative and long-term data and frequencies such as in relation to epidemics, the climate and the environment
* characteristics and properties of data sets, the shape of their distribution and representations and the terminology for comparison and analysis of data sets, graphs and summary statistics

Key skills

* collect, organise, collate and represent categorical data and numerical data, including continuous data
* use technology effectively and appropriately for accurate, reliable and efficient collation and representation of data sets
* accurately read and interpret charts, tables and graphs including prediction, interpolation and extrapolation of data
* calculate summary statistical data using common measures of central tendency and spread, including standard deviation
* use statistical language to describe, compare and analyse data sets, in terms of centre, spread, relationship and sample size
* draw inferences and conclusions, and explain any limitations and implications of a statistical study
* identify and interpret any errors and misrepresentations in data sets
* use long-term data and relative frequencies in practical situation to make informed interpretations and decisions about the likelihood of events or outcomes with respect to financial data, epidemics, climate data, or environmental data

Discrete mathematics

Financial and consumer mathematics

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 3.

Key knowledge

The numerical, data and algebraic knowledge that underpin the following financial, consumer and global topics and issues.

* investment and borrowing services offered by financial institutions such as investments, loans and mortgages
* the Australian taxation and superannuation system at a local (rates), state and national level
* routine income and expenditure transactions for businesses such as invoicing, BAS, superannuation and leave entitlements
* non-bank financial products, services and contracts such as insurances, mobile phone/internet plans, hire purchase and club membership contracts
* common data and information on financial and economic measures and indicators at the state, national or global level
* national and global health and environmental long-term relative data and frequencies such as epidemics, climate and environmental data/records
* financial risk and returns at personal, community and national level such as, tenancy agreements, roles of different types of insurance, diversification strategies and recessions

Key skills

* compare, contrast and undertake routine calculations for credit options for personal financial management such as personal borrowing (a car, holiday) or for housing
* describe and explain taxation scales and related rates and how taxes and superannuation are calculated and distributed at a local, state and national level, and perform calculations with taxation or superannuation
* undertake routine calculations or transactions related to the income and expenditure for businesses such as invoicing, BAS, superannuation and leave entitlements
* compare, contrast and undertake calculations for non-banking financial products and services such as insurances and mobile phone/internet plans
* interpret and analyse available financial and economic data at the community, national or global level and report on trends and outcomes such as health or environmental statistical data

Space and measurement

To achieve this outcome the student will draw on key knowledge and key skills outlined in Area of Study 4.

Key knowledge

* the names and properties of shapes and objects and their representations
* transformations, symmetry, similarity and common angle properties
* formal language, symbols and labelling and drawing conventions for diagrams, maps, plans, models, transformations, symmetry, similarity, perspective and common angle properties
* metric and other related quantity measures including derived and routine non-metric measures
* routine digital and analogue measurement tools and instruments and scales
* formulas for calculating length, area, surface area, volume and capacity
* measurement error, tolerance, accuracy and precision

Key skills

* interpret and describe shapes and objects and their representations using geometric and spatial language and conventions
* use triangles and other polygons to solve problems
* explain and apply transformations, including similarity and symmetry of shapes
* create and modify diagrams, plans, maps or designs using drawing equipment and packages/ applications, including undertaking standard transformations
* estimate and accurately measure different quantities using appropriate measurement tools using metric and routine non-metric measures
* convert between a range of metric units, including derived and routine non-metric measures
* calculate and interpret area, surface area, volume, capacity and density of routine, including compound, shapes and objects
* calculate and make informal considerations of errors in measurement in relation to specified or appropriate tolerance, accuracy and precision
* use estimation, rounding and approximation strategies to check the outcomes and interpret the results and to reflect on the outcomes obtained relative to personal, contextual and real-world implications

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine practical contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* uses and applications of mathematics and numerical data and information in aspects of contemporary life and the embedded nature of this mathematics in work, social and personal contexts
* relevant and appropriate mathematics in areas relating to student’s study, work, social or personal contexts
* common methods of presenting and communicating mathematics in everyday life, for example charts, graphs, maps, plans, tables, algebraic expressions and diagrams

Key skills

* identify and recognise how mathematics is used in everyday situations and contexts, making connections between mathematics and the real world
* extract the mathematics embedded in everyday situations and contexts and formulate what mathematics can be used to solve practical problems in both familiar and new contexts
* represent the mathematical information in a form that is personally useful as an aid to problem-solving, such as a table, summary, chart, numeric or algebraic representation, physical model or sketch
* undertake a range of mathematical tasks, applications and processes to solve practical problems, such as drawing, measuring, counting, estimating, calculating, generalising and modelling
* use estimation and other assessment skills to check the outcomes and decide on the appropriate accuracy for the outcome
* interpret results and outcomes of the application of mathematics in a context, including how appropriately and accurately they fit the situation, and to critically reflect on and evaluate the mathematics used and the outcomes obtained relative to personal, contextual and real-world implications
* represent, communicate and discuss the results and outcomes of the application of mathematics in a range of contexts

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in practical situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* the conventions for the representations of mathematical and statistical information, objects and processes, using different technologies and digital media
* contemporary technology and online and digital media, including software and applications based on computers, tablets, calculators and hand-held devices
* relevance, appropriateness and validity of the use and application of technology
* the numerical, graphical, symbolic, geometric statistical and financial functionalities of this range of technologies for working mathematically
* the conditions and settings for effective application of a given technology and its functionality

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* use and apply appropriate technology and devices to carry out calculations and analysis, and produce summaries, diagrams, tables, charts and graphs which model situations and solve practical problems
* make decisions on the appropriateness of graphical and digital representations derived from technology
* interpret, evaluate and discuss the inputs and outputs of technology, including critically reflecting on and evaluating the technology used and the outcomes obtained relative to personal, contextual and real-world implications
* use technology to communicate the results of working mathematically
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

School-based assessment

Satisfactory completion

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks to provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and key knowledge and key skills listed for the outcomes should be used for course design and the development of learning activities and assessment tasks.

Assessment of levels of achievement

The student’s level of achievement for Units 3 and 4 will be determined by a combination of School-assessed Coursework and an External assessment. The School-assessed Coursework will contribute 60 per cent and the examination will contribute 40 per cent to the study score.

School-assessed Coursework tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe.

Where teachers provide a range of options for the same School-assessed Coursework task, they should ensure that the options are of comparable scope and demand.

The types and range of forms of School-assessed Coursework for the outcomes are prescribed within the study design. The VCAA publishes Support materials for this study, which includes advice on the design of assessment tasks and the assessment of student work for a level of achievement.

Teachers will provide to the VCAA a numerical score representing an assessment of the student’s level of achievement. The score must be based on the teacher’s assessment of the performance of each student on the tasks set out in the following table.

Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Contribution to final assessment

Unit 3

School-assessed Coursework for Unit 3 will contribute 40 per cent to the study score. Each area of study is to be covered in at least one of the three mathematical investigations across Units 3 and 4.

There are three components to mathematical investigation:

***Formulation***

Overview of the context or scenario, and related background, including historical or contemporary background as applicable, and the mathematisation of questions, conjectures, hypotheses, issues or problems of interest.

***Exploration***

Investigation and analysis of the context or scenario with respect to the questions of interest, conjecture or hypotheses, using mathematical concepts, skills and processes, including the use of technology and application of computational thinking.

***Communication***

Summary, presentation and interpretation of the findings from the mathematical investigation and related applications.

***Possible contexts or scenarios***

Some possible contexts or scenarios for investigation include: analysis of pandemic data, climate change modelling/analysis, financial life stage planning, financial operations of a community organisation, workplace or industry related modelling or investigation of mathematics used on-the-job, financial and economic risk, gender pay gaps, public health outcomes such as vaccination programs, construction of an object according to a design brief, and production numbers and costs, and sales income of a product in terms of break-even points.

|  |  |  |
| --- | --- | --- |
| **Outcomes** | **Marks allocated** | **Assessment tasks** |
| **Outcome 1**Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures to solve practical problems from a range of everyday and real-life contexts. | **15** |

|  |  |
| --- | --- |
| 7 | Mathematical Investigation 1 |
| 8 | Mathematical Investigation 2 |

 |
| **Outcome 2**Apply mathematical processes in non-routine practical contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics. | **30** |

|  |  |
| --- | --- |
| 15 | Mathematical Investigation 1 |
| 15 | Mathematical Investigation 2 |

 |
| **Outcome 3**Apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in practical situations requiring investigative, modelling or problem-solving techniques or approaches. | **15** |

|  |  |
| --- | --- |
| 8 | Mathematical Investigation 1 |
| 7 | Mathematical Investigation 2 |

 |
| **Total marks** | **60** | Each mathematical investigation is to address content from two or more areas of study and is to be of 4–6 hours’ duration over a period of 1–2 weeks. |

Unit 4

School-assessed Coursework for Unit 4 will contribute 20 per cent to the study score. Each area of study is to be covered in at least one of the three mathematical investigations across Units 3 and 4.

|  |  |  |
| --- | --- | --- |
|  **Outcomes** | **Marks allocated** | **Assessment tasks** |
| **Outcome 1**Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures to solve practical problems from a range of everyday and real-life contexts. | **7** |

|  |  |
| --- | --- |
|  | Mathematical Investigation 3 |

 |
| **Outcome 2**Apply mathematical processes in non-routine practical contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics. | **15** |

|  |  |
| --- | --- |
|  | Mathematical Investigation 3 |

 |
| **Outcome 3**Apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in practical situations requiring investigative, modelling or problem-solving techniques or approaches. | **8** |

|  |  |
| --- | --- |
|  | Mathematical Investigation 3 |

 |
| **Total marks** | **30** | The mathematical investigation is to address content from two or more areas of study and is to be of 4–6 hours’ duration over a period of 1–2 weeks. |

External assessment

The level of achievement for Units 3 and 4 is also assessed by an end-of-year examination.

Contribution to final assessment

The examination will contribute 40 per cent to the study score.

End-of-year examination

Description

The examination will be set by a panel appointed by the VCAA. All of the content from the areas of study and the key knowledge and key skills that underpin the outcomes in Units 3 and 4 are examinable.

This examination comprises multiple-choice questions and written response questions covering all areas of the study in relation to all three outcomes.

The examination is designed to assess students’ knowledge of mathematical concepts, models and techniques and their ability to reason, interpret, and apply this knowledge in a range of practical contexts. The examination also assesses students’ ability to understand and communicate mathematical ideas.

The examination will be of two hours’ duration and student access to a scientific calculator will be assumed. [One bound reference](https://www.vcaa.vic.edu.au/assessment/vce-assessment/materials/Pages/index.aspx) text (which may be annotated) or lecture pad may be brought into the examination. VCAA examination rules will apply.

Conditions

The examination will be completed under the following conditions:

* Duration: 2 hours.
* Student access to a scientific calculator will be assumed.
* One [bound reference](https://www.vcaa.vic.edu.au/assessment/vce-assessment/materials/Pages/index.aspx) text (which may be annotated) or lecture pad may be brought into the examination.
* Date: end-of-year, on a date to be published annually by the VCAA.
* VCAA examination rules will apply. Details of these rules are published annually in the [*VCE and VCAL Administrative Handbook*](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx)*.*
* The examination will be marked by assessors appointed by the VCAA.

Further advice

The VCAA publishes specifications for all VCE examinations on the VCAA website. Examination specifications include details about the sections of the examination, their weighting, the question format(s) and any other essential information. The specifications are published in the first year of implementation of the revised Unit 3 and 4 sequence together with any sample material.

Units 3 and 4: General Mathematics

General Mathematics Units 3 and 4 focus on real-life application of mathematics and consist of the areas of study ‘Data analysis, probability and statistics’ and ‘Discrete mathematics’.

Unit 3 comprises *Data analysis* and *Recursion and financial modelling*, and Unit 4 comprises *Matrices* and *Networks and decision mathematics*.

Assumed knowledge and skills for General Mathematics Units 3 and 4 are contained in General Mathematics Units 1 and 2, and will be drawn on, as applicable, in the development of related content from the areas of study, and key knowledge and key skills for the outcomes of General Mathematics Units 3 and 4.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists, tables and matrices, diagrams, networks, algorithms, algebraic manipulation, recurrence relations, equations and graphs. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic statistical and financial functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Data analysis, probability and statistics

Data analysis

Students cover data types, representation and distribution of data, location, spread, association, correlation and causation, response and explanatory variables, linear regression, data transformation and goodness of fit, times series, seasonality, smoothing and prediction.

Investigating data distributions

This topic includes:

* types of data
* representation, display and description of the distributions of categorical variables: data tables, two-way frequency tables and their associated segmented bar charts
* representation, display and description of the distributions of numerical variables: dot plots, stem plots, histograms; the use of a logarithmic (base 10) scale to display data ranging over several orders of magnitude and their interpretation in terms of powers of ten
* use of the distribution(s) of one or more categorical or numerical variables to answer statistical questions
* summary of the distributions of numerical variables; the five-number summary and boxplots (including the use of the lower fence (Q1 – 1.5 × IQR) and upper fence (Q3 + 1.5 × IQR) to identify and display possible outliers); the sample mean and standard deviation and their use in comparing data distributions in terms of centre and spread
* the normal model for bell-shaped distributions and the use of the 68–95–99.7% rule to estimate percentages and to give meaning to the standard deviation; standardised values (-scores) and their use in comparing data values across distributions.

Investigating association between two variables

This topic includes:

* response and explanatory variables and their role in investigating associations between variables
* contingency (two-way) frequency tables, their associated bar charts (including percentage segmented bar charts) and their use in identifying and describing associations between two categorical variables
* back-to-back stem plots, parallel dot plots and boxplots and their use in identifying and describing associations between a numerical variable and a categorical variable
* scatterplots and their use in identifying and qualitatively describing the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)
* answering statistical questions that require a knowledge of the associations between pairs of variables
* Pearson correlation coefficient, , and its calculation and interpretation
* cause and effect; the difference between observation and experimentation when collecting data and the need for experimentation to definitively determine cause and effect.

Investigating and modelling linear associations

This topic includes:

* least squares line of best fit , where represents the explanatory variable, and represents the response variable; the determination of the coefficients and using technology, and the formulas and
* modelling linear association between two numerical variables, including the:
* identification of the explanatory and response variables
* use of the least squares method to fit a linear model to the data
* interpretation of the slope and intercepts of the least squares line in the context of the situation being modelled, including:
* use of the rule of the fitted line to make predictions being aware of the limitations of extrapolation
* use of the coefficient of determination, , to assess the strength of the association in terms of explained variation
* use of residual analysis to check quality of fit
* data transformation and its use in transforming some forms of non-linear data to linearity using a square, logarithmic (base 10) or reciprocal transformation (applied to one axis only)
* interpretation and use of the equation of the least squares line fitted to the transformed data to make predictions.

Investigating and modelling time series data

This topic includes:

* qualitative features of time series plots; recognition of features such as trend (long-term direction), seasonality (systematic, calendar related movements) and irregular fluctuations (unsystematic, short-term fluctuations); possible outliers and their sources, including one-off real-world events, and signs of structural change such as a discontinuity in the time series
* numerical smoothing of time series data using moving means with consideration of the number of terms required (using centring when appropriate) to help identify trends in time series plot with large fluctuations
* graphical smoothing of time series plots using moving medians (involving an odd number of points only) to help identify long-term trends in time series with large fluctuations
* seasonal adjustment including the use and interpretation of seasonal indices and their calculation using seasonal and yearly means
* modelling trend by fitting a least squares line to a time series with time as the explanatory variable (data de-seasonalised where necessary), and the use of the model to make forecasts (with re-seasonalisation where necessary) including consideration of the possible limitations of fitting a linear model and the limitations of extending into the future.

Area of Study 2

Discrete mathematics

Recursion and financial modelling

Students cover the use of first-order linear recurrence relations and the time value of money (TVM) to model and analyse a range of financial situations, and using technology to solve related problems involving interest, appreciation and depreciation, loans, annuities and perpetuities.

Depreciation of assets

This topic includes:

* use of a first-order linear recurrence relation of the formwhere and are constants to generate the terms of a sequence
* use of a recurrence relation to model and compare (numerically and graphically) flat rate, unit cost and reducing balance depreciation of the value of an asset with time, including the use of a recurrence relation to determine the depreciating value of an asset after depreciation periods for the initial sequence
* use of the rules for the future value of an asset after depreciation periods for flat rate, unit cost and reducing balance depreciation and their application.

Compound interest investments and loans

This topic includes:

* the concepts of simple and compound interest
* use of a recurrence relation to model and analyse (numerically and graphically) a compound interest investment or loan, including the use of a recurrence relation to determine the value of the compound interest loan or investment after compounding period for an initial sequence from first principles
* the difference between nominal and effective interest rates and the use of effective interest rates to compare investment returns and the cost of loans when interest is paid or charged, for example, daily, monthly, quarterly
* the future value of a compound interest investment or loan after compounding periods and its use to solve practical problems.

Reducing balance loans

This topic includes:

* use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortisation of a reducing balance loan, including the use of a recurrence relation to determine the value of the loan or investment after payments for an initial sequence from first principles
* use of a table to investigate and analyse the amortisation of a reducing balance loan on a step-by-step basis, the payment made, the amount of interest paid, the reduction in the principal and the balance of the loan
* use of technology with financial modelling functionality to solve problems involving reducing balance loans, such as repaying a personal loan or a mortgage, including the impact of a change in interest rate on repayment amount, time to repay the loan, total interest paid and the total cost of the loan.

Annuities and perpetuities

This topic includes:

* use of a first-order linear recurrence relation to model and analyse (numerically and graphically) the amortisation of an annuity, including the use of a recurrence relation to determine the value of the annuity after payments for an initial sequence from first principles
* use of a table to investigate and analyse the amortisation of an annuity on a step-by-step basis, the payment made, the interest earned, the reduction in the principal and the balance of the annuity
* use of technology to solve problems involving annuities including determining the amount to be invested in an annuity to provide a regular income paid, for example, monthly, quarterly
* simple perpetuity as a special case of an annuity that lasts indefinitely.

Compound interest investment with periodic and equal additions to the principal

This topic includes:

* use of a first-order linear recurrence relation to model and analyse (numerically and graphically) annuity investment, including the use of a recurrence relation to determine the value of the investment after payments have been made for an initial sequence from first principles
* use of a table to investigate and analyse the growth of an annuity investment on a step-by-step basis after each payment is made, the payment made, the interest earned and the balance of the investment
* use of technology with financial modelling functionality to solve problems involving annuity investments, including determining the future value of an investment after a number of compounding periods, the number of compounding periods for the investment to exceed a given value and the interest rate or payment amount needed for an investment to exceed a given value in a given time.

Matrices

Students cover the definition of matrices, different types of matrices, matrix operations, transition matrices and the use of first-order linear matrix recurrence relations to model a range of situations and solve related problems.

Matrices and their applications

This topic includes:

* matrix arithmetic: the order of a matrix, types of matrices (row, column, square, diagonal, symmetric, triangular, zero, binary and identity), the transpose of a matrix, and elementary matrix operations (sum, difference, multiplication of a scalar, product and power)
* inverse of a matrix, its determinant, and the condition for a matrix to have an inverse
* use of matrices to represent numerical information presented in tabular form, and the use of a rule for the element of a matrix to construct the matrix
* binary and permutation matrices, and their properties and applications
* communication and dominance matrices and their use in analysing communication systems and ranking players in round-robin tournaments.

Transition matrices

This topic includes:

* use of the matrix recurrence relation: initial state matrix, or where is a transition matrix, is a Leslie matrix, and is a column state matrix, to generate a sequence of state matrices (assuming the next state only relies on the current state)
* informal identification of the equilibrium state matrix in the case of regular transition matrices (no noticeable change from one state matrix to the next state matrix)
* use of transition diagrams, their associated transition matrices and state matrices to model the transitions between states in discrete dynamical situations and their application to model and analyse practical situations such as the modelling and analysis of an insect population comprising eggs, juveniles and adults
* use of the matrix recurrence relation initial state matrix, to extend modelling to populations that include culling and restocking.

Networks and decision mathematics

Students cover the definition and representation of different kinds of undirected and directed graphs, Eulerian trails, Eulerian circuits, bridges, Hamiltonian paths and cycles, and the use of networks to model and solve problems involving travel, connection, flow, matching, allocation and scheduling.

Graphs and networks

This topic includes:

* the concepts, conventions and terminology of graphs including planar graphs and Euler’s rule, and directed (digraphs) and networks
* use of matrices to represent graphs, digraphs and networks and their application.

Exploring and travelling problems

This topic includes:

* the concepts, conventions and notations of walks, trails, paths, cycles and circuits
* Eulerian trails and Eulerian circuits: the conditions for a graph to have a Eulerian trail or a Eulerian circuit, properties and applications
* Hamiltonian paths and cycles: properties and applications.

Trees and minimum connector problems

This topic includes:

* trees and spanning trees
* minimum spanning trees in a weighed connected graph and their determination by inspection or by Prim’s algorithm
* use of minimal spanning trees to solve minimal connector problems.

Flow problems

This topic includes:

* use of networks to model flow problems: capacity, sinks and sources
* solution of small-scale network flow problems by inspection and the use of the ‘maximum-flow minimum-cut’ theorem to aid the solution of larger scale problems.

Shortest path problems

This topic includes:

* determination of the shortest path between two specified vertices in a graph, digraph or network by inspection
* Dijkstra’s algorithm and its use to determine the shortest path between a given vertex and each of the other vertices in a weighted graph or network.

Matching problems

This topic includes:

* use of a bipartite graph and its tabular or matrix form to represent a matching problem
* determination of the optimum assignment(s) of people or machines to tasks by inspection or by use of the Hungarian algorithm for larger scale problems.

Scheduling problems and critical path analysis

This topic includes:

* construction of an activity network from a precedence table (or equivalent) including the use of dummy activities where necessary
* use of forward and backward scanning to determine the earliest starting times (EST) and latest starting times (LST) for each activity
* use of earliest starting times and latest starting times to identify the critical path in the network and determine the float times for non-critical activities
* use of crashing to reduce the completion time of the project or task being modelled.

Outcomes

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes as
a set apply to the content from the areas of study covered in that unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Area of Study 1 – Data analysis, probability and statistics

Key knowledge

* types of data: categorical (nominal and ordinal) and numerical (discrete and continuous)
* frequency tables, bar charts including segmented bar charts, histograms, stem plots, dot plots, and their application in the context of displaying and describing distributions
* logarithmic (base 10) scales, and their purpose and application
* the five-number summary and boxplots (including the designation and display of possible outliers)
* mean and sample standard deviation
* the normal model and the 68–95–99.7% rule, and standardised values (-scores)
* response and explanatory variables
* two-way frequency tables, segmented bar charts, back-to-back stem plots, parallel boxplots, and scatterplots, and their application in the context of identifying and describing associations
* correlation coefficient, , its interpretation, the issue of correlation and cause and effect
* coefficient of determination, its interpretation
* least squares line and its use in modelling linear associations
* data transformation and its purpose
* time series data and its analysis

Key skills

* construct frequency tables and bar charts and use them to describe and interpret the distributions of categorical variables
* answer statistical questions that require a knowledge of the distribution(s) of one or more categorical variables
* construct stem and dot plots, boxplots, histograms and appropriate summary statistics and use them to describe and interpret the distributions of numerical variables
* answer statistical questions that require a knowledge of the distribution(s) of one or more numerical variables
* solve problems using -scores and the 68–95–99.7% rule
* construct two-way tables and use them to identify and describe associations between two categorical variables
* construct parallel boxplots and use them to identify and describe associations between a numerical variable and a categorical variable
* construct scatterplots and use them to identify and describe associations between two numerical variables
* calculate the correlation coefficient, , and interpret it in the context of the data
* answer statistical questions that require a knowledge of the associations between pairs of variables
* determine the equation of the least squares line giving the coefficients correct to a required number of decimal places or significant figures as specified, and distinguish between correlation and causation
* use the least squares line of best fit to model and analyse the linear association between two numerical variables and interpret the model in the context of the association being modelled
* calculate the coefficient of determination, , and interpret in the context of the association being modelled and use the model to make predictions, being aware of the problem of extrapolation
* construct a residual analysis to test the assumption of linearity and, in the case of clear non-linearity, transform the data to achieve linearity and repeat the modelling process using the transformed data
* identify key qualitative features of a time series plot including trend (using smoothing if necessary), seasonality, irregular fluctuations and outliers, and interpret these in the context of the data
* calculate, interpret and apply seasonal indices
* model linear trends using the least squares line of best fit, interpret the model in the context of the trend being modelled, use the model to make forecasts with consideration of the limitations of extending forecasts too far into the future

Area of Study 2 – Discrete mathematics

Recursion and financial modelling

Key knowledge

* the use of first-order linear recurrence relations to model growth and decay problems in financial contexts
* the use of first-order linear recurrence relations to model flat rate and unit cost, and reduce balance depreciation of an asset over time, including the rule for the future value of the asset after depreciation periods
* the concepts of financial mathematics including simple and compound interest, nominal and effective interest rates, the present and future value of an investment, loan or asset, amortisation of a reducing balance loan or annuity and amortisation tables
* the use of first-order linear recurrence relations to model compound interest investments and loans, and the flat rate, unit cost and reducing balance methods for depreciating assets, reducing balance loans, annuities, perpetuities and annuity investments

Key skills

* model and analyse growth and decay in financial contexts using a first-order linear recurrence relation of the form:  where and are constants
* demonstrate the use of a recurrence relation to determine the depreciating value of an asset or the future value of an investment or a loan after time periods for the initial sequence
* use a rule for the future value of a compound interest investment or loan, or a depreciating asset, to solve practical problems
* use a table to investigate and analyse on a step–by-step basis the amortisation of a reducing balance loan or an annuity, and interpret amortisation tables
* use technology with financial mathematics capabilities, to solve practical problems associated with compound interest investments and loans, reducing balance loans, annuities and perpetuities, and annuity investments

Matrices

Key knowledge

* the order of a matrix, types of matrices (row, column, square, diagonal, symmetric, triangular, zero, binary, permutation and identity), the transpose of a matrix, and elementary matrix operations (sum, difference, multiplication of a scalar, product and power)
* the inverse of a matrix and the condition for a matrix to have an inverse, including determinant
* for transition matrices, assuming the next state only relies on the current state with a fixed population
* communication and dominance matrices and their application
* transition diagrams and transition matrices and regular transition matrices and their identification

Key skills

* use matrix recurrence relations to generate a sequence of state matrices, including an informal identification of the equilibrium or steady state matrix in the case of regular state matrices
* construct a transition matrix from a transition diagram or a written description and vice versa
* construct a transition matrix to model the transitions in a population with an equilibrium state
* use matrix recurrence relations to model populations with culling and restocking

Networks and decision mathematics

Key knowledge

* the conventions, terminology, properties and types of graphs; edge, face, loop, vertex, the degree of
a vertex, isomorphic and connected graphs, and the adjacency matrix, and Euler’s formula for planar graphs and its application
* the exploring and travelling problem, walks, trails, paths, Eulerian trails and circuits, and Hamiltonian paths and cycles
* the minimum connector problem, trees, spanning trees and minimum spanning trees and Prim’s algorithm
* the flow problem, and the minimum cut/maximum flow theorem
* the shortest path problem and Dijkstra’s algorithm
* the matching problem and the Hungarian algorithm
* the scheduling problem and critical path analysis

Key skills

* construct graphs, digraphs and networks and their matrix equivalents to model and analyse practical situations
* recognise the exploring and travelling problem and to solve it by utilising the concepts of walks, trails, paths, Eulerian trails and circuits, and Hamiltonian paths and cycles
* recognise the minimum connector problem and solve it by utilising the properties of trees, spanning trees and by determining a minimum spanning tree by inspection or using Prim’s algorithm for larger scale problems
* recognise the flow problem, use networks to model flow problems and determine the minimum flow problem by inspection, or by using the minimum cut/maximum flow theorem for larger scale problems
* recognise the shortest path problem and solve it by inspection or using Dijkstra’s algorithm for larger scale problems
* recognise the matching problem and solve it by inspection or using the Hungarian algorithm for larger scale problems
* recognise the scheduling problem and solve it by using critical path analysis

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the facts, concepts and techniques associated with data analysis, recursion and financial modelling, matrices and networks and decision mathematics
* standard models studied in data analysis, recursion and financial modelling, matrices, and networks and decision mathematics, and their areas of application
* general formulation of the concepts, techniques and models studied in data analysis, recursion and financial modelling, matrices, and networks and decision mathematics
* assumptions and conditions underlying the use of the concepts, techniques and models associated with data analysis, recursion and financial modelling, matrices, and networks and decision mathematics

Key skills

* identify, recall and select facts, concepts, models and techniques needed to investigate and analyse statistical features of a data set with several variables that can include time series data
* select and implement standard financial models to investigate and analyse a financial or mathematically equivalent non-financial situation that requires the use of increasingly sophisticated models to complete the analysis
* identify, recall and select the mathematical concepts, models and techniques needed to solve an extended problem or conduct an investigation in a variety of contexts related to matrices and networks and decision mathematics
* interpret and report the results of a statistical investigation or of completing a modelling or problem-solving task in terms of the context under consideration, including discussing the assumptions in application of these models

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* the difference between exact numerical and approximate numerical answers when using technology to perform computation, and rounding to a given number of decimal places or significant figures
* domain and range requirements for specification of graphs of models and relations, when using technology
* the role of parameters in specifying general forms of models and equations
* the relation between numerical, graphical and symbolic forms of information about models and equations, and the corresponding features of those models and equations
* the similarities and differences between formal mathematical expressions and their representation by technology
* the appropriate functionality of technology for a variety of mathematical contexts

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy in terms of a given number
of decimal places or significant figures
* use technology to carry out numerical, graphical and symbolic computation as applicable
* produce results, using a technology, which identify examples or counter-examples for propositions
* produce tables of values, families of graphs and collections of other results using technology, which support general analysis in investigative, modelling and problem-solving contexts
* use appropriate domain and range specifications to illustrate key features of graphs
* identify the relation between numerical, graphical and symbolic forms of information about models and equations, and the corresponding features of those models and equations
* specify the similarities and differences between formal mathematical expressions and their representation by technology
* select an appropriate functionality of technology in a variety of mathematical contexts, related to data analysis, recurrence relations and financial modelling, and provide a rationale for these selections
* apply suitable constraints and conditions, as applicable, to carry out required computations
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

School-based assessment

Satisfactory completion

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks to provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and key knowledge and key skills listed for the outcomes should be used for course design and the development of learning activities and assessment tasks.

Assessment of levels of achievement

The student’s level of achievement for Units 3 and 4 will be determined by School-assessed Coursework.

School-assessed Coursework tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe.

Where teachers provide a range of options for the same School-assessed Coursework task, they should ensure that the options are of comparable scope and demand.

The types and range of forms of School-assessed Coursework for the outcomes are prescribed within the study design. The VCAA publishes Support materials for this study, which includes advice on the design of assessment tasks and the assessment of student work for a level of achievement.

Teachers will provide to the VCAA a numerical score representing an assessment of the student’s level of achievement. The score must be based on the teacher’s assessment of the performance of each student on the tasks set out in the following table.

Contribution to final assessment

Unit 3

School-assessed Coursework for Unit 3 will contribute 24 per cent to the study score.

The **Application task** is a guided investigation of a given data set with several variables. The task has three components of increasing complexity:

* the construction, description and interpretation of data plots, including smoothed plots where time series data is used
* the calculation and interpretation of summary statistics, including seasonal indices and their application where time series data is used
* the modelling of linear associations, or trends where time series data is used, including the use of data transformation as appropriate.

The application task is to be of 4–6 hours’ duration over a period of 1–2 weeks.

|  |  |  |
| --- | --- | --- |
| **Outcomes** | **Marks allocated** | **Assessment tasks** |
| **Outcome 1**Define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures. | **15** |

|  |  |
| --- | --- |
| 10 | Application task |
| 5 | Modelling or problem-solving task 1 |

 |
| **Outcome 2**Apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics. | **30** |

|  |  |
| --- | --- |
| 20 | Application task |
| 10 | Modelling or problem-solving task 1 |

 |
| **Outcome 3**Apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches. | **15** |

|  |  |
| --- | --- |
| 10 | Application task |
| 5 | Modelling or problem-solving task 1 |

 |
| **Total marks** | **60** | The Modelling or problem-solving task 1 is to relate to **Recursion and financial modelling.** The modelling or problem-solving task is to be of 2–3 hours’ duration over a period of 1 week. |

Unit 4

School-assessed Coursework for Unit 4 will contribute 16 per cent to the study score.

|  |  |  |
| --- | --- | --- |
| **Outcomes** | **Marks allocated** | **Assessment tasks** |
| **Outcome 1**Define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures. | **10** |

|  |  |
| --- | --- |
| 5 | Modelling or problem-solving task 2 |
| 5 | Modelling or problem-solving task 3 |

 |
| **Outcome 2**Apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics. | **20** |

|  |  |
| --- | --- |
| 10 | Modelling or problem-solving task 2 |
| 10 | Modelling or problem-solving task 3 |

 |
| **Outcome 3**Apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches. | **10** |

|  |  |
| --- | --- |
| 5 | Modelling or problem-solving task 2 |
| 5 | Modelling or problem-solving task 3 |

 |
| **Total marks** | **40** | One of the modelling or problem-solving tasks is to relate to **Matrices** and the other modelling or problem-solving task is to relate to **Networks and decision mathematics.**Each modelling or problem-solving task is to be of 2–3 hours’ duration over a period of 1 week. |

External assessment

The level of achievement for Units 3 and 4 is also assessed by two end-of-year examinations.

Contribution to final assessment

The examinations will contribute 60 per cent to the study score. Each examination will contribute 30 per cent to the study score.

End-of-year examinations

Description

The examinations will be set by a panel appointed by the VCAA. All of the content from the areas of study and the key knowledge and key skills that underpin the outcomes in Units 3 and 4 are examinable.

Examination 1

This examination comprises multiple-choice questions covering all areas of study. The examination is designed to assess students’ knowledge of mathematical concepts, models and techniques and their ability to reason, interpret and apply this knowledge in a range of contexts.

Examination 2

This examination comprises written response questions covering all areas of study. The examination will be designed to assess students’ ability to select and apply mathematical facts, concepts, models and techniques to solve extended application problems in a range of contexts.

Conditions

The examinations will be completed under the following conditions:

* Duration: 1.5 hours.
* Student access to an approved technology with numerical, graphical, symbolic, financial and statistical functionality will be assumed. One [bound reference](https://www.vcaa.vic.edu.au/assessment/vce-assessment/materials/Pages/index.aspx) text (which may be annotated) or lecture pad may be brought into the examination.
* Date: end-of-year, on a date to be published annually by the VCAA.
* VCAA examination rules will apply. Details of these rules are published annually in the *[VCE and VCAL Administrative Handbook.](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx)*
* The examination will be marked by assessors appointed by the VCAA.

Further advice

The VCAA publishes specifications for all VCE examinations on the VCAA website. Examination specifications include details about the sections of the examination, their weighting, the question format(s) and any other essential information. The specifications are published in the first year of implementation of the revised Unit 3 and 4 sequence together with any sample material.

Units 3 and 4: Mathematical Methods

Mathematical Methods Units 3 and 4 extend the introductory study of simple elementary functions of a single real variable, to include combinations of these functions, algebra, calculus, probability and statistics, and their applications in a variety of practical and theoretical contexts. Units 3 and 4 consist of the areas of study ‘Algebra, number and structure’, ‘Data analysis, probability and statistics’, ‘Calculus’, and ‘Functions, relations and graphs’, which must be covered in progression from Unit 3 to Unit 4, with an appropriate selection of content for each of Unit 3 and Unit 4. Assumed knowledge and skills for Mathematical Methods Units 3 and 4 are contained in Mathematical Methods Units 1 and 2, and will be drawn on, as applicable, in the development of related content from the areas of study, and key knowledge and key skills for the outcomes of Mathematical Methods Units 3 and 4.

For Unit 3 a selection of content would typically include the areas of study ‘Functions, relations and graphs’ and ‘Algebra, number and structure’, applications of derivatives and differentiation, and identifying and analysing key features of the functions and their graphs from the ‘Calculus’ area of study. For Unit 4, a corresponding selection of content would typically consist of remaining content from ‘Functions, relations and graphs’, ‘Algebra, number and structure’ and ‘Calculus’ areas of study, and the study of random variables, discrete and continuous probability distributions, and the distribution of sample proportions from the ‘Data analysis, probability and statistics’ area of study. For Unit 4, the content from the ‘Calculus’ area of study would be likely to include the treatment of anti-differentiation, integration, the relation between integration and the area of regions specified by lines or curves described by the rules of functions, and simple applications of this content, including to probability distributions of continuous random variables.

The selection of content from the areas of study should be constructed so that there is a development in the complexity and sophistication of problem types and mathematical processes used (modelling, transformations, graph sketching and equation solving) in application to contexts related to these areas of study. There should be a clear progression of skills and knowledge from Unit 3 to Unit 4 in an area of study.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, sets, lists and tables, diagrams and geometric constructions, algorithms, algebraic manipulation, equations, graphs, differentiation, anti-differentiation, integration and inference, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Functions, relations and graphs

In this area of study students cover transformations of the plane and the behaviour of some elementary functions of a single real variable, including key features of their graphs such as axis intercepts, stationary points, points of inflection, domain (including maximal, implied or natural domain), co-domain and range, asymptotic behaviour and symmetry. The behaviour of functions and their graphs is to be explored in a variety of modelling contexts and theoretical investigations.

This area of study includes:

* graphs of polynomial functions and their key features
* graphs of the following functions: power functions, ; exponential functions, , in particular ; logarithmic functions,  and ; and circular functions,
 and and their key features
* transformation from  to , where  and , , and  is one of the functions specified above, and the inverse transformation
* the relation between the graph of an original function and the graph of a corresponding transformed function (including families of transformed functions for a single transformation parameter)
* graphs of sum, difference, product and composite functions involving functions of the types specified above (not including composite functions that result in reciprocal or quotient functions)
* modelling of practical situations using polynomial, power, circular, exponential and logarithmic functions, simple transformation and combinations of these functions, including simple piecewise (hybrid) functions.

Area of Study 2

Algebra, number and structure

In this area of study students cover the algebra of functions, including composition of functions, inverse functions and the solution of equations. They also study the identification of appropriate solution processes for solving equations, and systems of simultaneous equations, presented in various forms. Students also cover recognition of equations and systems of equations that are solvable using inverse operations or factorisation, and the use of graphical and numerical approaches for problems involving equations where exact value solutions are not required, or which are not solvable by other methods. This content is to be incorporated as applicable to the other areas of study.

This area of study includes:

* solution of polynomial equations with real coefficients of degree having up to real solutions, including numerical solutions
* functions and their inverses, including conditions for the existence of an inverse function, and use of inverse functions to solve equations involving exponential, logarithmic, circular and power functions
* composition of functions, where  composite , is defined by given 
* solution of equations of the form  over a specified interval, where  and  are functions of the type specified in the ‘Functions, relations and graphs’ area of study, by graphical, numerical and algebraic methods, as applicable
* solution of literal equations and general solution of equations involving a single parameter
* solution of simple systems of simultaneous linear equations, including consideration of cases where no solution or an infinite number of possible solutions exist (geometric interpretation only required for two equations in two variables).

Area of Study 3

Calculus

In this area of study students cover graphical treatment of limits, continuity and differentiability of functions of a single real variable, and differentiation, anti-differentiation and integration of these functions. This material is to be linked to applications in practical situations.

This area of study includes:

* deducing the graph of the derivative function from the graph of a given function and deducing the graph of an anti-derivative function from the graph of a given function
* derivatives of for , , , , and
* derivatives of , ,  and where  and  are polynomial functions exponential, circular, logarithmic or power functions and transformations or simple combinations of these functions
* application of differentiation to graph sketching and identification of key features of graphs, including stationary points and points of inflection, and intervals over which a function is strictly increasing or strictly decreasing
* identification of local maximum/minimum values over an interval and application to solving optimisation problems in context, including identification of interval endpoint maximum and minimum values
* anti-derivatives of polynomial functions and functions of the form  where  is , for , , ,  and linear combinations of these
* informal consideration of the definite integral as a limiting value of a sum involving quantities such as area under a curve and approximation of definite integrals using the trapezium rule
* anti-differentiation by recognition that  implies  and informal treatment of the fundamental theorem of calculus,
* properties of anti-derivatives and definite integrals
* application of integration to problems involving finding a function from a known rate of change given a boundary condition, calculation of the area of a region under a curve and simple cases of areas between curves, average value of a function and other situations.

Area of Study 4

Data analysis, probability and statistics

In this area of study students cover discrete and continuous random variables, their representation using tables, probability functions (specified by rule and defining parameters as appropriate); the calculation and interpretation of central measures and measures of spread; and statistical inference for sample proportions. The focus is on understanding the notion of a random variable, related parameters, properties and application and interpretation in context for a given probability distribution.

This area of study includes:

* random variables, including the concept of a random variable as a real function defined on a sample space and examples of discrete and continuous random variables
* discrete random variables:
* specification of probability distributions for discrete random variables using graphs, tables and probability mass functions
* calculation and interpretation of mean, , variance, and standard deviation of a discrete random variable and their use
* Bernoulli trials and the binomial distribution, , as an example of a probability distribution for a discrete random variable
* effect of variation in the value(s) of defining parameters on the graph of a given probability mass function for a discrete random variable
* calculation of probabilities for specific values of a random variable and intervals defined in terms of a random variable, including conditional probability
* continuous random variables:
* construction of probability density functions from non-negative functions of a real variable
* specification of probability distributions for continuous random variables using probability density functions
* calculation and interpretation of mean, , variance, and standard deviation of a continuous random variable and their use
* standard normal distribution, , and transformed normal distributions, , as examples of a probability distribution for a continuous random variable
* effect of variation in the value(s) of defining parameters on the graph of a given probability density function for a continuous random variable
* calculation of probabilities for intervals defined in terms of a random variable, including conditional probability (the cumulative distribution function may be used but is not required)
* statistical inference, including definition and distribution of sample proportions, simulations and confidence intervals:
* distinction between a population parameter and a sample statistic and the use of the sample statistic to estimate the population parameter
* simulation of random sampling, for a variety of values of and a range of sample sizes, to illustrate the distribution of and variations in confidence intervals between samples
* concept of the sample proportion as a random variable whose value varies between samples,

where is a binomial random variable which is associated with the number of items that have a particular characteristic and is the sample size

* approximate normality of the distribution of for large samples and, for such a situation, the mean

 (the population proportion) and standard deviation,

* determination and interpretation of, from a large sample, an approximate confidence interval , for a population proportion where is the appropriate quantile for the standard normal distribution, in particular the 95% confidence interval as an example of such an interval where (the term standard error may be used but is not required).

Outcomes

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes as a set apply to the content from the areas of study covered in that unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the key features and properties of a function or relation and its graph and of families of functions and relations and their graphs
* the effect of transformations on the graphs of a function or relation
* representations of points and transformations of the plane
* the concepts of domain, maximal domain, range and asymptotic behaviour of functions
* the concept of an inverse function, connection between domain and range of the original function and its inverse relation and the conditions for existence of an inverse function, including the form of the graph of the inverse function for specified functions
* the concept of combined functions, and the connection between domain and range of the functions involved and the domain and range of the combined functions
* the features which enable the recognition of general forms of possible models for data presented in graphical or tabular form
* exponent laws and logarithm laws
* analytical, graphical and numerical approaches to solving equations and the nature of corresponding solutions (real, exact or approximate) and the effect of domain restrictions
* features which link the graph of a function to the graph of the corresponding gradient function or its numerical values, the tangent to a curve at a given point and how the sign and magnitude of the derivative of a function can be used to describe key features of the function and its derivative function
* the sum, difference, chain, product and quotient rules for differentiation
* the properties of anti-derivatives and definite integrals
* the concept of approximation to the area under a curve using the trapezium rule, the ideas underlying the fundamental theorem of calculus and the relationship between the definite integral and area
* the concepts of a random variable (discrete and continuous), Bernoulli trials and probability distributions, the parameters used to define a distribution and properties of probability distributions and their graphs
* the conditions under which a Bernoulli trial or a probability distribution may be selected to suitably model various situations
* the definition of sample proportion as a random variable and key features of the distribution of sample proportions
* the concept of confidence intervals for proportions, variation in confidence intervals between samples and confidence intervals for estimates

Key skills

* identify key features and properties of the graph of a function or relation and draw the graphs of specified functions and relations, clearly identifying their key features and properties, including any vertical or horizontal asymptotes
* describe the effect of transformations on the graphs of a function or relation
* find the rule of an inverse function and give its domain and range
* find the rule of a composite function and give its domain and range
* sketch by hand graphs of polynomial functions up to degree 4; simple power functions, where , , (using key points , and ); ; ; and simple transformations
of these
* apply a range of analytical, graphical and numerical processes (including the algorithm for Newton’s method), as appropriate, to obtain general and specific solutions (exact or approximate) to equations (including literal equations) over a given domain and be able to verify solutions to a particular equation or equations over a given domain
* solve by hand equations of the form ,  and with exact value solutions over a given interval
* apply algebraic, logarithmic and circular function properties to the simplification of expressions and the solution of equations
* evaluate derivatives of basic, transformed and combined functions and apply differentiation to curve sketching and related optimisation problems
* find derivatives of polynomial functions and power functions, functions of the form  where is , for , sine, cosine; tangent, , or  and simple linear combinations of these, using pattern recognition, or by hand
* apply the product, chain and quotient rules for differentiation to simple combinations of functions by hand
* find derivatives of basic and more complicated functions and apply differentiation to curve sketching and optimisation problems
* find anti-derivatives of polynomial functions and power functions, functions of the form where
 is , for , , sine or cosine, and simple linear combinations of these, using pattern recognition, or by hand
* evaluate approximations to the area under a curve using the trapezium rule, find and verify anti-derivatives of specified functions and evaluate definite integrals
* apply definite integrals to the evaluation of the area under a curve and between curves over a specified interval
* analyse a probability mass function or probability density function and the shape of its graph in terms of the defining parameters for the probability distribution and the mean and variance of the probability distribution
* calculate and interpret the probabilities of various events associated with a given probability distribution, by hand in cases where simple arithmetic computations can be carried out
* apply probability distributions to modelling and solving related problems
* simulate repeated random sampling and interpret the results, for a variety of population proportions and a range of sample sizes, to illustrate the distribution of sample proportions and variations in confidence intervals
* calculate sample proportions and approximate confidence intervals for population proportions

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* key mathematical content from one or more areas of study related to a given context
* specific and general formulations of concepts used to derive results for analysis within a given context
* the role of examples, counter-examples and general cases in working mathematically
* key elements of algorithm design, including sequencing, decision-making and repetition, and representations of the ordered steps for an algorithm including through the use of pseudocode
* inferences from analysis and their use to draw valid conclusions related to a given context

Key skills

* specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
* identify important information, variables, constraints and other key features to the investigation of various questions in a given context
* develop mathematical formulations of specific and general cases used to derive results for analysis within a given context
* use algorithms, patterns, models and simulation to solve problems related to a given context
* use a variety of techniques to verify results
* make inferences from analysis and use these to draw valid conclusions related to a given context
* communicate results and conclusions using both mathematical expression and everyday language, in particular, the interpretation of mathematics with respect to the context

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques
or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of technology
* domain and range requirements for specification of graphs of functions and relations when using technology
* the role of parameters in specifying general forms of functions and equations
* the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions and equations
* the similarities and differences between formal mathematical expressions and their representation by technology
* the purpose and effect of sequencing, decision-making and repetition statements on relevant functionalities of technology, and their role in the design of algorithms and simulations
* the appropriate functionality of technology for a variety of mathematical contexts

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
* use technology to carry out numerical, graphical and symbolic computation as applicable
* produce results, using technology, which identify examples or counter-examples for propositions
* produce tables of values, families of graphs and collections of other results using technology, which support general analysis in investigative, modelling and problem-solving contexts
* use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
* identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* specify the similarities and differences between formal mathematical expressions and their representation by technology, in particular, equivalent forms of symbolic expressions
* select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
* design and implement simulations and algorithms using appropriate functionalities of technology
* apply suitable constraints and conditions, as applicable, to carry out required computations
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling, or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

School-based assessment

Satisfactory completion

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks to provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and key knowledge and key skills listed for the outcomes should be used for course design and the development of learning activities and assessment tasks.

Assessment of levels of achievement

The student’s level of achievement for Units 3 and 4 will be determined by School-assessed Coursework.

School-assessed Coursework tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe.

Where teachers provide a range of options for the same School-assessed Coursework task, they should ensure that the options are of comparable scope and demand.

The types and range of forms of School-assessed Coursework for the outcomes are prescribed within the study design. The VCAA publishes Support materials for this study, which includes advice on the design of assessment tasks and the assessment of student work for a level of achievement.

Teachers will provide to the VCAA a numerical score representing an assessment of the student’s level of achievement. The score must be based on the teacher’s assessment of the performance of each student on the tasks set out in the following table.

Contribution to final assessment

School-assessed Coursework for Unit 3 will contribute 20 per cent to the study score.

Unit 3

|  |  |  |
| --- | --- | --- |
| **Outcomes** | **Marks allocated** | **Assessment task** |
| **Outcome 1**Define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures. | **15** | **Application task**A function and calculus-based mathematical investigation of a practical or theoretical context involving content from two or more areas of study, with the following three components of increasing complexity:* introduction of the context through specific cases or examples
* consideration of general features of the context
* variation or further specification of assumption or conditions involved in the context to focus on a particular feature or aspect related to the context.
 |
| **Outcome 2**Apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics. | **20** |
| **Outcome 3**Apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches. | **15** |
| **Total marks** | **50** | The application task is to be of 4–6 hours’ duration over a period of 1–2 weeks. |

School-assessed Coursework for Unit 4 will contribute 20 per cent to the study score.

Unit 4

|  |  |  |
| --- | --- | --- |
| **Outcomes** | **Marks allocated** | **Assessment tasks** |
| **Outcome 1**Define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures. | **15** |

|  |  |
| --- | --- |
| 8 | Modelling or problem-solving task 1 |
| 7 | Modelling or problem-solving task 2 |

 |
| **Outcome 2**Apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics. | **20** |

|  |  |
| --- | --- |
| 10 | Modelling or problem-solving task 1 |
| 10 | Modelling or problem-solving task 2 |

 |
| **Outcome 3**Apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches. | **15** |

|  |  |
| --- | --- |
| 7 | Modelling or problem-solving task 1 |
| 8 | Modelling or problem-solving task 2 |

 |
| **Total marks** | **50** | One of the modelling or problem-solving tasks is to address the **Data analysis, probability and statistics** area of study.Each modelling or problem-solving task is to be of 2–3 hours’ duration over a period of 1 week. |

External assessment

The level of achievement for Units 3 and 4 is also assessed by two end-of-year examinations.

Contribution to final assessment

Examination 1 will contribute 20 per cent to the study score and Examination 2 will contribute 40 per cent to the study score.

End-of-year examinations

Description

The examinations will be set by a panel appointed by the VCAA. All of the content from the areas of study and the key knowledge and key skills that underpin the outcomes in Units 3 and 4 are examinable.

Examination 1

This examination comprises short-answer and some extended-answer questions covering all areas of study in relation to Outcome 1. It is designed to assess students’ knowledge of mathematical concepts, their skills in carrying out mathematical algorithms without the use of technology and their ability to apply concepts and skills.

Conditions

The examination will be completed under the following conditions:

* Duration: 1 hour.
* No technology (calculators or software) or notes of any kind are permitted. A sheet of formulas will be provided with the examination.
* Date: end-of-year, on a date to be published annually by the VCAA.
* VCAA examination rules will apply. Details of these rules are published annually in the *[VCE and VCAL Administrative Handbook.](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx)*
* The examination will be marked by assessors appointed by the VCAA.

Examination 2

This examination comprises multiple-choice questions and extended-answer questions covering all areas of the study in relation to all three outcomes, with an emphasis on Outcome 2. The examination is designed to assess students’ ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems.

Conditions

The examination will be completed under the following conditions:

* Duration: 2 hours.
* Student access to an approved technology with numerical, graphical, symbolic and statistical functionality will be assumed.
* One [bound reference](https://www.vcaa.vic.edu.au/assessment/vce-assessment/materials/Pages/index.aspx) text (which may be annotated) or lecture pad may be brought into the examination.
* Date: end-of-year, on a date to be published annually by the VCAA.
* VCAA examination rules will apply. Details of these rules are published annually in the *[VCE and VCAL Administrative Handbook](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx)*[.](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx)
* The examination will be marked by assessors appointed by the VCAA.

Further advice

The VCAA publishes specifications for all VCE examinations on the VCAA website. Examination specifications include details about the sections of the examination, their weighting, the question format(s) and any other essential information. The specifications are published in the first year of implementation of the revised Unit 3 and 4 sequence together with any sample material.

Units 3 and 4: Specialist Mathematics

Specialist Mathematics Units 3 and 4 consist of the areas of study: ‘Algebra, number and structure’, ‘Calculus’, ‘Data analysis, probability and statistics’, ‘Discrete mathematics’, ‘Functions, relations and graphs’, and ‘Space and measurement’. The development of course content should highlight mathematical structure, reasoning and proof and applications across a range of modelling contexts with an appropriate selection of content for each of Unit 3 and Unit 4. The selection of content for Unit 3 and Unit 4 should be constructed so that there is a balanced and progressive development of knowledge and skills with connections among the areas of study being developed as appropriate across Unit 3 and Unit 4.

Specialist Mathematics Units 3 and 4 assumes familiarity with the key knowledge and key skills from Mathematical Methods Units 1 and 2; the key knowledge and key skills from Specialist Mathematics Units 1 and 2; and concurrent study or previous completion of Mathematical Methods Units 3 and 4. Together these cover the assumed knowledge and skills for Specialist Mathematics Units 3 and 4, which are drawn on as applicable in the development of content from the areas of study and key knowledge and key skills for the outcomes.

For Unit 3 a selection of content would typically include content from the ‘Discrete mathematics’, ‘Functions, relations and graphs’, ‘Algebra, number and structure’, ‘Space and measurement’ and ‘Calculus’ areas of study. In Unit 4 the corresponding selection of content would typically consist of the remaining content from the ‘Discrete mathematics’, ‘Calculus’, and ‘Space and measurement’ areas of study and the content from the ‘Data analysis, probability and statistics’ area of study.

In undertaking these units, students are expected to be able to apply techniques, routines and processes involving rational, real and complex arithmetic, sets, lists, tables and vectors, diagrams and geometric constructions, algorithms, algebraic manipulation, equations, graphs, differentiation, anti-differentiation and integration and inference, with and without the use of technology. They should have facility with relevant mental and by-hand approaches to estimation and computation. The use of numerical, graphical, geometric, symbolic and statistical functionality of technology for teaching and learning mathematics, for working mathematically, and in related assessment, is to be incorporated throughout each unit as applicable.

Area of Study 1

Discrete mathematics

Logic and proof

In this area of study students cover the development of mathematical argument and proof. This includes conjectures, connectives, quantifiers, examples and counter-examples, and proof techniques including mathematical induction. Proofs will involve concepts from topics such as: divisibility, inequalities, graph theory, combinatorics, sequences and series including partial sums and partial products and related notations, complex numbers, matrices, vectors and calculus. The concepts, skills and processes from this area of study are to be applied in the other areas of study.

This area of study includes:

* conjecture – making a statement to be proved or disproved
* implications, equivalences and if and only if statements (necessary and sufficient conditions)
* natural deduction and proof techniques: direct proofs using a sequence of direct implications, proof by cases, proof by contradiction, and proof by contrapositive
* quantifiers ‘for all’ and ‘there exists’, examples and counter-examples
* proof by mathematical induction.

Area of Study 2

Functions, relations and graphs

In this area of study students cover rational functions and other simple quotient functions, curve sketching of these functions and relations, and the analysis of key features of their graphs including intercepts, asymptotic behaviour and the nature and location of stationary points and points of inflection and symmetry.

This area of study includes:

* rational functions and the expression of rational functions of low degree as sums of partial fractions
* graphs of rational functions of low degree, their asymptotic behaviour, and the nature and location of stationary points and points of inflection
* graphs of simple quotient functions, their asymptotic behaviour, and the nature and location of stationary points and points of inflection.

Area of Study 3

Algebra, number and structure

Complex numbers

In this area of study students cover the algebra of complex numbers, including polar form, factorisation of polynomial functions over the complex field and an informal treatment of the fundamental theorem of algebra.

This area of study includes:

* De Moivre’s theorem, proof for integral powers, powers and roots of complex numbers in polar form, and their geometric representation and interpretation
* the roots of unity and other complex numbers and their location in the complex plane
* factors over *C*, of polynomials; and introduction to the fundamental theorem of algebra, including
its application to factorisation of polynomial functions of a single variable over *C*, for example,
 or
* solution over *C* of polynomial equations by completing the square, use of the quadratic factorisation and the conjugate root theorem.

Area of Study 4

Calculus

In this area of study students cover the advanced calculus techniques for analytical and numerical differentiation and integration of a broad range of functions, and combinations of functions; and their application in a variety of theoretical and practical situations, including curve sketching, evaluation of arc length, area and volume, differential equations and kinematics, and modelling with differential equations drawing from a variety of fields such as biology, economics and science.

Differential calculus and integral calculus

This topic includes:

* the relationship between the graph of a function and the graphs of its anti-derivative functions
* derivatives of inverse circular functions
* second derivatives, use of notations and , and their application to the analysis of graphs of

functions, including points of inflection and concavity

* applications of chain rule to related rates of change and implicit differentiation; for example, implicit differentiation of the relations and
* techniques of anti-differentiation and for the evaluation of definite integrals:
* anti-differentiation of to obtain
* anti-differentiation of and for by recognition that they are derivatives of

corresponding inverse circular functions

* use of the substitution to anti-differentiate expressions
* use of the trigonometric identities and ) in

anti-differentiation techniques

* anti-differentiation using partial fractions of rational functions
* integration by parts
* numerical and symbolic integration using technology
* application of integration, areas of regions bounded by curves, arc lengths for parametrically determined curves, surface area of solids of revolution, volumes of solids of revolution of a region about either coordinate axis.

Differential equations

This topic includes:

* formulation of differential equations from contexts in, for example, chemistry, biology and economics, in situations where rates are involved (including some differential equations whose analytic solutions are not required, but can be solved numerically using technology)
* the logistic differential equation
* verification of solutions of differential equations and their representation using direction (slope) fields
* solution of simple differential equations of the form and in general differential equations of the form using separation of variables and differential equations of the form
* numerical solution by Euler’s method (first order approximation).

Kinematics: rectilinear motion

This topic includes:

* use of velocity–time graphs to describe and analyse rectilinear motion
* application of differentiation, anti-differentiation and solution of differential equations to rectilinear motion of a single particle, including the different derivative forms for acceleration .

Area of Study 5

Space and measurement

In this area of study students cover the arithmetic and algebra of vectors; linear dependence and independence of a set of vectors; proof of geometric results using vectors; vector representation of curves in the plane and their parametric and Cartesian equations; vector kinematics in one, two and three dimensions; vector, parametric and Cartesian equations of lines and planes.

Vectors

This topic includes:

* addition and subtraction of vectors and their multiplication by a scalar, position vectors
* linear dependence and independence of a set of vectors and geometric interpretation
* magnitude of a vector, unit vector, the orthogonal unit vectors 
* resolution of a vector into rectangular components
* scalar (dot) product of two vectors, deduction of dot product for the  vector system and its use to find scalar resolute and vector resolute
* vector (cross) product of two vectors in three dimensions, including the determinant form
* parallel and perpendicular vectors
* vector proofs of simple geometric results, such as ‘the diagonals of a rhombus are perpendicular’, ‘the medians of a triangle are concurrent’ and ‘the angle subtended by a diameter in a circle is a right angle’.

Vector and Cartesian equations

This topic includes:

* vector equations and parametric equations of curves in two or three dimensions involving a parameter (and the corresponding Cartesian equation in the two-dimensional case)
* vector equation of a straight line, given the position of two points, or equivalent information, in both two and three dimensions
* vector cross product, normal to a plane and vector, parametric and Cartesian equations of a plane.

Vector calculus

This topic includes:

* position vector as a function of time and sketching the corresponding path given the function, including circles, ellipses and hyperbolas in Cartesian or parametric forms
* the positions of two particles each described as a vector function of time, and whether their paths cross or if the particles meet
* differentiation and anti-differentiation of a vector function with respect to time and applying vector calculus to motion in a plane and in three dimensions.

Area of Study 6

Data analysis, probability and statistics

In this area of study students cover the study of linear combinations of random variables and introductory statistical inference with respect to the mean of a single population, the determination of confidence intervals, and hypothesis testing for the mean using the distribution of sample means.

Distribution of linear combinations of random variables

This topic includes:

* for independent identically distributed random variables each with mean and variance :
* for independent random variables and real numbers :
* for normally distributed independent random variables and real numbers the random variable is also normally distributed.

Distribution of the sample mean

This topic includes:

* the concept of the sample mean as a random variable whose value varies between samples where
is a random variable with mean and the standard deviation
* simulation of repeated random sampling, from a variety of distributions and a range of sample sizes,
to illustrate properties of the distribution of across samples of a fixed size including its mean its standard deviation (where and are the mean and standard deviation of respectively) and its approximate normality if is large.

Confidence intervals for the population mean

This topic includes:

* determination of confidence intervals for means and the use of simulation to illustrate variations in confidence intervals between samples and to show that the likelihood of a confidence interval containing depends on the level of confidence chosen in the determination of the interval
* construction of an approximate confidence interval, where is the population standard deviation and is the appropriate quantile for the standard normal distribution or construction of an approximate confidence interval ) where is the sample standard deviation and is the appropriate quantile for the standard normal distribution, and is large ( ≥ 30 in many practical contexts).

Hypothesis testing for a population mean with a sample drawn from a normal distribution of known variance, or for a large sample

This topic includes:

* concepts of null hypothesis, , and alternative hypotheses, , test statistic
* level of significance and -value
* formulation of hypotheses and making a decision concerning a population mean based on:
* a random sample from a normal population of known variance
* a large random sample from any population
* 1-tail and 2-tail tests
* interpretation of the results of a hypothesis test in the context of the problem
* hypothesis test, relating the formulation, conduct, errors and results in terms of conditional probability.

Outcomes

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes as a set apply to the content from the areas of study covered in that unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* principles of proof and deduction techniques
* the proof scheme and method for mathematical induction
* functions and relations, the form of their sketch graphs and their key features, including linear asymptotes
* complex numbers, Cartesian and polar forms, operations and properties and representation in the complex plane
* geometric interpretation of vectors in the plane and of complex numbers in the complex plane
* differentiation techniques and the meaning of first and second derivatives of a function
* anti-differentiation techniques, the relationship between the graph of a function and the graph of its anti-derivative functions, and graphical interpretation of definite integrals
* analytical, graphical and numerical techniques for setting up and solving equations involving functions and relations
* modelling contexts for formulation of differential equations and associated solution techniques, including numerical approaches
* definition and properties of vectors, vector operations, the geometric representation of vectors and the geometric interpretation of linear dependence and independence of a set of vectors
* standard contexts for the application of vectors to the motion of a particle and to geometric problems
* techniques for solving kinematics problems in one, two and three dimensions
* the vector product and methods determining vector equations of lines and planes
* systems of equations with two and three variables and their geometric interpretation
* the distribution of sample means

Key skills

* apply deductive reasoning and language, including mathematical induction, to mathematical arguments and proofs involving concepts and contexts from the areas of study
* sketch by hand and describe behaviour of the graphs of specified functions and relations, and identify their key features, including the use of the first and second derivative
* perform operations on complex numbers expressed in Cartesian form or polar form and interpret them geometrically
* Interpret and apply algorithms in a variety of contexts including the use of pseudocode for representation
* represent regions of an Argand diagram using complex relations
* apply implicit differentiation, by hand in simple cases
* use analytic techniques to find derivatives and anti-derivatives by pattern recognition, and apply anti-differentiation to evaluate definite integrals
* set up and evaluate definite integrals to calculate areas, volumes, curve lengths (where described parametrically) and surface areas
* set up and solve differential equations of specified forms
* perform operations on vectors and interpret them geometrically
* apply vectors to motion of a particle and to geometric problems
* solve kinematics problems using a variety of techniques
* formulate problems which require solutions with systems of linear equations
* apply a range of analytical, graphical and numerical processes to obtain solutions (exact or approximate) to equations
* set up and solve problems involving the distribution of sample means
* construct confidence intervals, and approximate confidence intervals, for sample means

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* key mathematical content from one or more areas of study related to a given context
* specific and general formulations of concepts used to derive results for analysis within a given context
* the role of examples, counter-examples and general cases in working mathematically
* the role of proof in establishing a general result
* key elements of algorithm design, including sequencing, decision-making and repetition, and representations of the ordered steps for an algorithm including the use of pseudocode
* inferences from analysis and their use to draw valid conclusions related to a given context

Key skills

* specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context
* identify important information, variables, constraints and other key features to the investigation of various questions in a given context
* develop mathematical formulations of specific and general cases used to derive results for analysis within a given context and establish proofs for general case results
* use algorithms, patterns, models and simulation to solve problems related to a given context
* use a variety of techniques to verify results
* make inferences from analysis and use these to draw valid conclusions related to a given context
* communicate results and conclusions using both mathematical expression and everyday language,
in particular, the interpretation of mathematics with respect to the context

Outcome 3

On completion of this unit the student should be able to apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques
or approaches.

To achieve this outcome the student will draw on key knowledge and key skills outlined in all the areas of study.

Key knowledge

* the role of computational thinking (abstraction, decomposition, pattern and algorithm) in problem-solving, and its application to mathematical investigation
* exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by technology
* domain and range requirements for specification of graphs of functions and relations, when using technology
* the role of parameters in specifying general forms of functions and equations
* the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* the similarities and differences between formal mathematical expressions and their representation by technology
* the appropriate functionality of technology for a variety of mathematical contexts

Key skills

* use computational thinking, algorithms, models and simulations to solve problems related to a given context
* distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy
* produce results, using a technology, which identify examples or counter-examples for propositions
* produce tables of values, symbolic expressions, families of graphs or collections of other results using technology, which support general analysis in investigative, modelling or problem-solving contexts
* use appropriate domain and range specifications to illustrate key features of graphs of functions and relations
* identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions and equations
* select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
* design and implement simulations and algorithms using appropriate functionalities of technology
* apply suitable constraints and conditions, as applicable, to carry out required computations
* relate the results from a particular technology application to the nature of a particular mathematical task (investigative, modelling or problem-solving) and verify these results
* specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process

School-based assessment

Satisfactory completion

The award of satisfactory completion for a unit is based on whether the student has demonstrated achievement of the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks to provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes.

The areas of study and key knowledge and key skills listed for the outcomes should be used for course design and the development of learning activities and assessment tasks.

Assessment of levels of achievement

The student’s level of achievement for Units 3 and 4 will be determined by School-assessed Coursework.

School-assessed Coursework tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe.

Where teachers provide a range of options for the same School-assessed Coursework task, they should ensure that the options are of comparable scope and demand.

The types and range of forms of School-assessed Coursework for the outcomes are prescribed within the study design. The VCAA publishes Support materials for this study, which includes advice on the design of assessment tasks and the assessment of student work for a level of achievement.

Teachers will provide to the VCAA a numerical score representing an assessment of the student’s level of achievement. The score must be based on the teacher’s assessment of the performance of each student on the tasks set out in the following table.

Contribution to final assessment

School-assessed Coursework for Unit 3 will contribute 20 per cent to the study score.

Unit 3

|  |  |  |
| --- | --- | --- |
| **Outcomes** | **Marks allocated** | **Assessment task** |
| **Outcome 1**Define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures. | **15** | **Application task**A mathematical investigation of a practical or theoretical context involving content from two or more areas of study, with the following three components of increasing complexity:* introduction of the context through specific cases or examples
* consideration of general features of the context
* variation or further specification of assumption or conditions involved in the context to focus on a particular feature or aspect related to the context.
 |
| **Outcome 2**Apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics. | **20** |
| **Outcome 3**Apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches. | **15** |
| **Total marks** | **50** | The application task is to be of 4–6 hours’ duration over a period of 1–2 weeks. |

School-assessed Coursework for Unit 4 will contribute 20 per cent to the study score.

Unit 4

|  |  |  |
| --- | --- | --- |
| **Outcomes** | **Marks allocated** | **Assessment tasks** |
| **Outcome 1**Define and explain key concepts as specified in the content from the areas of study and apply a range of related mathematical routines and procedures. | **15** |

|  |  |
| --- | --- |
| 8 | Modelling or problem-solving task 1 |
| 7 | Modelling or problem-solving task 2 |

 |
| **Outcome 2**Apply mathematical processes in non-routine contexts, including situations with some open-ended aspects requiring investigative, modelling or problem-solving techniques or approaches, and analyse and discuss these applications of mathematics. | **20** |

|  |  |
| --- | --- |
| 10 | Modelling or problem-solving task 1 |
| 10 | Modelling or problem-solving task 2 |

 |
| **Outcome 3**Apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring investigative, modelling or problem-solving techniques or approaches. | **15** |

|  |  |
| --- | --- |
| 7 | Modelling or problem-solving task 1 |
| 8 | Modelling or problem-solving task 2 |

 |
| **Total marks** | **50** | One of the modelling or problem-solving tasks is to address the **Data analysis, probability and statistics** area of study.Each modelling or problem-solving task is to be of 2–3 hours’ duration over a period of 1 week. |

External assessment

The level of achievement for Units 3 and 4 is also assessed by two end-of-year examinations.

Contribution to final assessment

Examination 1 will contribute 20 per cent to the study score and Examination 2 will contribute 40 per cent to the study score.

End-of-year examinations

Description

The examinations will be set by a panel appointed by the VCAA. All of the content from the areas of study and the key knowledge and key skills that underpin the outcomes in Units 3 and 4 are examinable.

Examination 1

This examination comprises short-answer and some extended-answer questions covering all areas of study in relation to Outcome 1. It is designed to assess students’ knowledge of mathematical concepts, their skills in carrying out mathematical algorithms without the use of technology and their ability to apply concepts and skills.

Conditions

The examination will be completed under the following conditions:

* Duration: 1 hour.
* No technology (calculators or software) or notes of any kind are permitted. A sheet of formulas will be provided with the examination.
* Date: end-of-year, on a date to be published annually by the VCAA.
* VCAA examination rules will apply. Details of these rules are published annually in the *[VCE and VCAL Administrative Handbook.](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx)*
* The examination will be marked by assessors appointed by the VCAA.

Examination 2

This examination comprises multiple-choice questions and extended-answer questions covering all areas of the study in relation to all three outcomes, with an emphasis on Outcome 2. The examination is designed to assess students’ ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems.

Conditions

The examination will be completed under the following conditions:

* Duration: 2 hours.
* Student access to an approved technology with numerical, graphical, symbolic and statistical functionality will be assumed.
* One [bound reference](https://www.vcaa.vic.edu.au/assessment/vce-assessment/materials/Pages/index.aspx) text (which may be annotated) or lecture pad may be brought into the examination.
* Date: end-of-year, on a date to be published annually by the VCAA.
* VCAA examination rules will apply. Details of these rules are published annually in the *[VCE and VCAL Administrative Handbook.](https://www.vcaa.vic.edu.au/administration/vce-vcal-handbook/Pages/index.aspx)*
* The examination will be marked by assessors appointed by the VCAA.

Further advice

The VCAA publishes specifications for all VCE examinations on the VCAA website. Examination specifications include details about the sections of the examination, their weighting, the question format(s) and any other essential information. The specifications are published in the first year of implementation of the revised Unit 3 and 4 sequence together with any sample material.