VCE Mathematical Methods Unit 4

Sample modelling or problem-solving task: Close to normal

The modelling or problem-solving task is to be of 2–3 hours’ duration over a period of 1 week.

Introduction

A context such as the following could be used to develop a two-part problem-solving task that involves a composition of functions leading to the standard normal distribution, and the use of simulations to investigate the distribution of proportions in probability experiments.



Part 1

Let $f:R\rightarrow R, f\left(x\right)=e^{x}$ and$ g:R\rightarrow R, g\left(x\right)= -x^{2}$.

1. Plot the graphs of *f*(x) and *g(x)* and explain how the shape of the graph of *f*(*g*(*x*)) can be deduced from these.
2. Plot the graph of *f*(*g*(*x*)) and clearly identify its key features.
3. Use sets of rectangles to form a sequence of under-estimates and over-estimates for the area bounded by the graph of *f*(*g*(*x*)) and the horizontal axis between *x* = –10 and *x* = 10.
4. Use the results from step c. to find approximate values for *a* and *b* such that

 $h:R \rightarrow R, h\left(x\right)=ae^{-bx^{2}}$ forms a probability density function, with mean 0 and standard deviation 1.

1. Plot the graph from step d. on the same set of axes as the standard normal distribution and comment on similarities and differences.

Part 2

1. A pair of standard dice are rolled simultaneously. Use technology to simulate this experiment for 60 rolls of the dice and record the set of outcomes. What is the proportion of rolls for which the two dice had the same value?
2. Run the simulation 100 times and plot the distribution of this proportion. Describe the distribution.
3. Now consider two identical packs of 10 cards numbered 1 to 10. Both packs are shuffled thoroughly. The first card is turned over from each pack and the result is recorded. This is then repeated for the second card from each pack, the third card from each pack and so on, through to the 10th and final card of each pack. Use technology to simulate this experiment and record the set of outcomes. How many pairs of cards in the experiment had the same value?
4. Run this simulation 100 times and plot the distribution of proportions for the number of times when the pair of cards had the same value. Describe this distribution.
5. Give an estimate for the probability that there is at least one pair of matching cards and explain how this estimate was obtained.

Areas of study

The following content from the areas of study is addressed through this task.

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| **Area of study** | **Content dot points** |
| Functions, relations and graphs | 2, 3, 4, 5 |
| Algebra, number and structure | 3, 5 |
| Calculus | 3, 4, 7, 9 |
| Data analysis, probability and statistics | 1, 2, 3 |

Outcomes

The following outcomes, key knowledge and key skills are addressed through this task.

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| **Outcome** | **Key knowledge dot points** | **Key skills dot points** |
| 1 | 1, 2, 4, 6, 7, 11, 12, 13, 14, 17 | 1, 2, 4, 5, 12, 14, 15, 16, 17, 18, 19, 20 |
| 2 | 1, 2, 4, 5 | 1, 2, 3, 4, 5, 6, 7 |
| 3 | 1, 2, 3, 4, 7, 8 | 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13 |